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Young Children's 21st Century Mathematical Skills and the Role of the Early Childhood Environment in Developing these Skills

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YOUNG CHILDREN'S 21ST CENTURY MATHEMATICAL SKILLS AND THE ROLE OF THE EARLY CHILDHOOD ENVIRONMENT IN DEVELOPING THESE SKILLS

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Submitted in fulfilment of the requirements for the Master of Philosophy

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Declaration

To the best of the candidate's knowledge, this thesis contains no material previously published by another person, except where due acknowledgement has been made. This thesis is the candidate's own work and contains no material which has been accepted for the award of any other degree or diploma in any institution.

Human Ethics

The research presented and reported in this thesis was conducted in accordance with the National Health and Medical Research Council National Statement on Ethical Conduct in Human Research (2007, updated 2018). The proposed research study received human research ethics approval from the University of Notre Dame Australia Human Research Ethics Committee (EC00418), Approval Number 019169F.

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Abstract

This research examined the preparation of young children in an Early Years school environment in the development of 21st Century mathematical skills including critical thinking, problem-solving, innovation, collaboration, creativity, and communication. The research used a field-based case study using qualitative data collection methods, and the data were reviewed using thematic analysis. The research explores the pedagogical strategies employed by an Early Years teacher and the interplay of various stakeholders in nurturing 21st Century mathematics competencies among young learners. Key findings of the research indicated that there were eight elements that Early Years teachers should apply in their mathematics lessons as well as embedding 21st Century skills across all curriculum areas. It was also found that strong partnerships between school and home as well as collaboration between teachers and the leadership team contribute to the cultivation of essential mathematical skills necessary for the challenges of the 21st Century.

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I dedicate this research to the late Dr. Derek Hurrell, whose memory remains an enduring source of inspiration. Your wisdom, humour, encouragement, and unwavering belief in my abilities has enabled me to complete this thesis. It is a great privilege that I had the opportunity to call you my boss, colleague, supervisor and most importantly friend.
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Contents

Declaration ii
Abstract iii
Acknowledgements.. iv
Contents.. v
List of Tables.. x
List of Figures.. xi

Chapter One: Introduction... 1

1.1 Introduction.. 1
1.2 Research Question.. 1
1.2.1 Defining the Early Years in Western Australia.................................... 1
1.2.2 Research Questions 1
1.3 Background to the Research.. 2
1.3.1 Defining 21st Century Skills.. 2
1.3.2 Defining 21st Century Mathematical Skills.. 6
1.4 Significance of the Research.. 8
1.5 Structure of the Thesis.. 9
1.6 Conclusion.. 10

Literature Review... 11

2.1 Introduction.. 11
2.2 Why 21st Century Mathematical Skills Are Crucial for Young Children's Future Success... 12
2.3 Effective Mathematical Teaching... 15
2.3.1 Pedagogical Content Knowledge... 15
2.3.2 Mathematical Knowledge for Teachers.. 16
2.3.3 Western Australian Curriculum Mathematics 17
2.3.4 Pedagogical Practices and Principles... 19
2.3.5 Mathematical Lesson Structure... 22
2.3.6 Mathematical Dispositions.. 24

2.3.7	Mathematical Engagement..	25
2.4	Young Children's Early Years Mathematical Learning.........................	26
2.4.1	Early Years Mathematical Learning...	26
2.4.2	Mathematics Pedagogical Practices for the Early Years........................	28
2.5	Early Years School Environment...	29
2.5.1	Early Years Environment..	29
2.5.2	Building a Strong Early Years Environment with All Key Stakeholders...	29
2.5.3	The Importance of School and Home Partnerships..............................	30
2.5.4	The Importance of Parents as the First Educators................................	31
2.5.5	Early Childhood Teachers and the Key Role They Provide in Laying the Foundations for Future Success...	32
2.5.6	The Importance of an Effective Leadership Team...............................	33
2.6	Conclusion..	34

Chapter Three: Research Plan.. 35

3.1	Introduction..	35
3.2	Theoretical Framework..	35
3.2.1	Epistemology...	36
3.2.2	Theoretical Perspective..	37
3.3	Methodology..	37
3.4	Methods...	38
3.4.1	Framework...	38
3.4.2	Data Collection Methods..	39
3.4.2.1	Semi-Structured Interviews..	40
3.4.2.2	Student Focus Groups...	42
3.4.2.3	Classroom Observations..	42
3.4.2.4	Parental Survey...	43
3.5	Research Participants..	43
3.6	Data Analysis...	44
3.6.1	Semi-Structured Teacher Interviews...	44
3.6.2	Student Focus Groups..	45
3.6.3	Classroom Observations..	46

3.6.4 Parental Surveys.......... 48
3.6.5 Principal Interview.......... 49
3.6.6 Triangulation of Data.......... 49
3.7 Ethical Consideration.......... 49
3.8 Conclusion.......... 52

Chapter Four: Presentation of Research Results.......... **53**
4.1 Introduction.......... 53
4.2 Teacher Interview.......... 53
4.3 Classroom Observations.......... 55
4.4 Student Focus Groups.......... 63
4.5 Parental Surveys.......... 67
4.6 Principal Interview.......... 69
4.7 Conclusion.......... 72

Chapter Five: Discussion of Results.......... **74**
5.1 Introduction.......... 74
5.2 How do Early Years Teachers Support Mathematical Practice that Develops 21st Century Mathematical Skills? 76
5.2.1 Eight Elements an Early Years Teacher Should Regularly Apply in a Mathematics Lesson.......... 76
5.2.1.1 Element One: 21st Century Mathematical Skills Are Embedded in A Constructivist Approach.......... 76
5.2.1.2 Element Two: A Problem-Solving Approach to Teaching Where Learning Experiences Are Challenging, Rich, Engaging, Meaningful and Appropriate to The Learners.......... 79
5.2.1.3 Element Three: A Strong Emphasis on Crucial Mathematical Learning.......... 80
5.2.1.4 Element Four: Lessons Conclude with a Whole Class Summary.......... 82
5.2.1.5 Element Five: Wait Time is Provided and Encouraged.......... 83
5.2.1.6 Element Six: Positive Dispositions are Valued and Encouraged.......... 84
5.2.1.7 Element Seven: An Emphasis on Mathematical Drawings to Represent Ideas and Solutions.......... 84
5.2.1.8 Element Eight: Picture Books to Engage Students in

	Mathematics Learning..	85
5.2.2	Effective Numeracy Education Naturally Embed 21st Century Skills Across All Curriculum Areas..	86
5.3	Research Sub-Question One: How Do the Elements of The Early Years School Environment Align in the Development of 21st Century Mathematical Skills?	87
5.3.1	Establishing Strong School/Home Partnerships..................................	87
5.3.2	Positive Collaboration Between Classroom Teachers and the Leadership Team...	88
5.3.3	Inconsistent Perceptions of 21st Century Mathematical Skills...............	89
5.4	Responding to the Overarching Research Question..............................	90
5.4.1	Ensuring Early Years Teachers Naturally Embed 21st Century Learning Across All Curriculum Areas and Continually Provide the Eight Pedagogically Elements in Their Mathematics Lessons Will Provide the Most Effective Context For 21st Century Mathematically Proficient Students..	90
5.4.2	Establishing Trusting and Strong Relationships Between All Key Stakeholders is Essential in Developing Mathematically Proficient Students With 21st Century Mathematical Skills...	91
5.5	Conclusion..	92
Chapter Six: Limitations and Recommendations..		**93**
6.1	Findings Presented..	93
6.2	Research Questioned Answered..	95
6.3	Limitations of the Research...	96
6.4	Implications..	96
6.5	Recommendations	97
6.5.1	Recommendations for Practice...	97
6.5.1.1	Classroom-based Practical Recommendations....................................	97
6.5.1.2	Professional Learning Based on The Eight Pedagogical Practices.........	98
6.5.1.3	Creating Culturally Responsive Classrooms.......................................	98
6.5.2	Recommendations for Future Research...	98

6.5.2.1 Diversification of School Settings.. 98

6.5.2.2 Expanded Teacher Participation... 99

6.5.2.3 Longitudinal studies... 99

6.5.2.4 Instinctive versus Developing Teacher's.. 99

6.6 Conclusion... 100

References.. 101

Appendices

Appendix A Research Instrument.. 112

Appendix B Parental Consent Form. 114

Appendix C Child Classroom Observations Consent Form...................................... 116

Appendix D Child Focus Group Consent Form... 117

Appendix E Participation Sheet... 118

Appendix F Parental Survey... 121

Appendix G Lesson Observation One Matrix.. 123

Appendix H Pre-Semi Structured Teacher Interview Questions 125

Appendix I Post-Semi Structured Teacher Interview Questions 127

Appendix J. Principal Interview Questions 130

List of Tables

Table

Table 1.1 A List of 21st Century Skills.. 5
Table 1.2 Overview of Thesis Chapters.. 9
Table 2.1 A Set of Pedagogical Practices.. 20
Table 2.2 Six Principles for Effective Teaching of Mathematics 21
Table 4.1 Teacher Interview.. 53
Table 4.2 Frequency of the 21st Century Mathematical Skills and
Sub-themes Observed During Classroom Observations................ 56
Table 4.3 The Sub-themes of the Observed 21st Century Mathematical
Skills with the Highest Frequency.. 59
Table 4.4 Field Notes.. 60
Table 4.5 Students Focus Group Questions.. 64
Table 4.6 5 Focus Questions and the Key Themes from Parental Survey.............. 68
Table 4.7 Mathematics at Home, the Responses from Students and Parents.......... 69
Table 4.8. Principal Interview Results... 70
Table 6.1 Overall Findings of Research Questions... 95

List of Figures

Figure

Figure 1.1 21st Century Student Outcomes and Support Systems.......................... 7

Figure 2.1 Conceptual Framework.. 12

Figure 2.2 Shulman's (1986) Domains of Pedagogical Content Knowledge……... 16

Figure 2.3 Domains of Mathematical Knowledge for Teaching 17

Figure 3.1 Overview of Research Design.. 36

Figure 3.2 Framework Underpinning the Data Collection...................................... 39

Figure 3.3 Data Collection Timeline... 40

Figure 3.4 Teacher Interview Analysis Sample... 45

Figure 3.5 Mathematical Lesson Structure.. 46

Figure 3.6 Matrix Coding Sample... 47

Figure 3.7 A Sample of Field Notes and Coding... 48

Figure 4.1 Student Work Samples………………………………………………… 66

Figure 5.1 Structure of Discussion... 75

Figure 5.2 Eight Elements an Early Years Teacher Should Regularly Apply in a Mathematics Lesson to Support the Development of Proficient Students with 21st Century Mathematical Skills... 77

Figure 6.1 Research Framework.. 94

Chapter One: Introduction

Introduction

Twenty-first Century learning is the development of a highly valuable skill set for the future (Trilling & Fadel, 2009; Voogt & Roblin, 2012). Instead of specific subject knowledge, living, learning, and working in the 21st Century requires an expanded set of skills, competencies, and flexibility, as demanded by Trudinger (2017). It is essential that teachers prepare children to thrive in tomorrow's world. It is important to address how we can guarantee that every student achieves numeracy by the time they finish school. This ensures they have the necessary mathematical knowledge, skills, and confidence to participate fully in further learning, enter the workforce, and actively contribute to their communities.

To ensure that students are mathematically prepared to live and learn in the 21st Century, a focus needs to be placed on pedagogy not just specific mathematical knowledge (Trudinger, 2017). Pedagogy generally refers to instructional strategies or actions and is the method or process of teaching (Gervasoni et al., 2012). It is crucial that pedagogical models are developed to align with 21st Century learning in which the skills of mathematical creativity, critical thinking, problem-solving, communication and collaboration are developed (Battelle for Kids, 2019; Lindeman & Anderson, 2015).

According to Trudinger (2017) 75 per cent of Australian employment growth areas require Science, Technology, Engineering and Mathematics [STEM]; therefore, a strong foundation in mathematics is imperative to maximise young people's opportunities for healthy, productive, and rewarding futures. Mathematical proficiency is one of the most important capabilities needed by people of Australia in the 21st Century (Askew, 2013). In the midst of this ever-changing educational environment and the necessity for 21st Century skills, this research aims to make a meaningful contribution, with the goal to advance effective pedagogical approaches that align with the evolving requirements of both students and society.

1.2 Research Questions

1.2.1 Defining the Early Years in Western Australia

Western Australia's schooling is unique to other states in Australia. In Western Australia, children typically commence their education journey within a school setting in Kindergarten at age 4, with compulsory schooling beginning the subsequent year in Pre-primary at age 5, and then extending through Year 12. Kindergarten a non-compulsory year is universally available on school sites and administered in the primary school system. Pre-primary is the first compulsory year of school in Western Australia (Foundation equivalent) where children make the transition from informal home or Early Childhood Learning Centres [ECLC] environments. The Early Years Learning Framework [EYLF] (Australia Government Department of Education [AGDE], 2022) is mandated for use with children from Kindergarten to Year 2 and placed alongside The Western Australian Curriculum and Assessment Outline (School Curriculum and Standards Authority [SCSA], 2018). The children in this study are in Pre-primary (Foundation) year.

Education is state based, The Australian Curriculum and Assessment and Reporting Authority [ACARA] (Australian Curriculum Assessment and Reporting Authority, 2010) has been subsumed into SCSA that outlines Western Australian curriculums for all schools from Pre-primary to Year 12. In addition, the state Education Minister mandated in 2013 that all schools would implement the seven Quality Areas of the National Quality Standards [NQS] (Australian Children's Education and Care Quality Authority [ACECQA], 2011) and follow the Principles and Practices of the EYLF from Kindergarten to Year 2. So, in the Early Years of school (Kindergarten and Pre-primary) teachers use the EYLF, the WA Curriculum and Assessment Outline that highlights the content and benchmarks of the Australian Curriculum (ACARA, 2010) and NQS (ACECQA, 2011). This research is referring to Western Australia's education system in which Early Years teachers encompass Kindergarten to Year 2.

1.2.2 Research Questions

The overarching research question that guided this investigation was:
How does the Early Years school environment prepare young children with regards to 21st Century mathematical skills? The research sub-questions that emerged from the review of literature and which were developed to respond to the overarching research question were:

1. How do Early Years teachers support mathematical practice that develops 21st Century mathematical skills?
2. How do the elements of the Early Years school environment align in the development of 21st Century mathematical skills?

1.3 Background to the Research

1.3.1 Defining 21st Century Skills

This research is significant because there is little existing research evidence in exploring Early Years teachers' perspectives of 21st Century mathematical skills and effective practices to prepare students with these skills. Twenty-first Century skills are defined as the skills needed for education and the workplace in the current economy (van Laar et al., 2020). Gervasoni (2018) contended “ensuring that all children thrive mathematically is recognised as important for children’s future citizenship and opportunities for work and further education, and ultimately for contributing to the economic and cultural prosperity of a society” (p. 115). Gelmez Burakgazi et al. (2019) stated that the “advancements of the twenty-first century have led to the emergence of new economic and social indicators that capture technological progress and changes to competition in the labour market” (p. 451). The 21st Century is being determined by advancements in technology, global competition, high levels of knowledge, and swift societal changes (Gelmez Burakgazi et al., 2019).

Today, automation is replacing lower-skilled jobs, and it is estimated that by 2030 the fastest-growing occupations will require higher-level cognitive skills in areas such as collaboration, problem-solving, critical thinking, and creativity (Binkley et al., 2012; Holzapfel, 2018; van Laar et al., 2020). Success in the future job market will require innovative and adaptable responses to new demands and changing circumstances (Binkley et al., 2012; Holzapfel, 2018; van Laar et al., 2020). In their professional lives, individuals must effectively manage the rising complexity of tasks by leveraging relevant knowledge and cultivating skills such as problem-solving and creative thinking. These abilities are essential for overcoming the challenges inherent in the corporate world. (Grootenboer, 2018).

Young people of today are the first generation in history to know more than their elders (parents and teachers) about the powerful tools of digital information and communications technologies, which are leading change in society (Trilling & Fadel, 2009). Therefore, young people today have a unique advantage in navigating and leveraging the transformative potential of digital technologies compared to previous generations. This shift in the holders of knowledge is switching the roles of adults and children and shifting the dynamics, where students are becoming digital mentors, while teachers and parents are becoming students (Trilling & Fadel, 2009). Fast-paced technological change has had a significant impact on the structure of the current workforce, therefore placing new demands on the education system (Lamb et al., 2017). This rapid technological advancement provides instances to open new opportunities and career paths, which means that many of today’s primary school students will end up in careers not yet invented (Lamb et al., 2017; Trudinger, 2017).

In the 21st century, Australia's ability to ensure a high quality of life for its citizens hinges on its competitiveness in the global economy. This assertion is supported by Organisation for Economic Co-operation and Development [OCED] (2018) indicating a strong correlation between a nation's economic performance and its citizens' overall well-being. The driving force for economic growth and job opportunities are workers who create and innovate new products and solve real problems. The shortage of well-trained workers in STEM fields poses a significant concern due to its potential impact on innovation and economic competitiveness (Trilling & Fadel, 2009). As industries increasingly rely on technological advancements and expertise in STEM disciplines to drive growth and productivity, the scarcity of skilled workers in these areas could hinder Australia's ability to maintain a competitive edge in the global marketplace (Trilling & Fadel, 2009). Consequently, the reliance on foreign workers or outsourcing to lower-wage countries may increase this challenge, highlighting the importance of STEM skills in education.

Binkley et al. (2012) claimed that students can no longer look for success in work that can be accomplished by machines, such as manual labour or the use of routine skills. The Australian education systems should ensure that young people are equipped with the necessary knowledge, understanding, skills, and values to seize opportunities and confront challenges with confidence in this era (Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA], 2008). According to Hannay and Earl (2012) this upskilling does not mean that traditional skills are deemed obsolete, but that teachers need the skills to challenge past processes and create new practices. Twenty-first Century skills are not a new idea but have gained much greater importance over time in the education system (Trilling & Fadel, 2009). Schools operating in the 21st Century need to be radically different from schools of the past (Hannay & Earl, 2012).

As previously asserted, global and technological trends have prompted educational reformers to argue that the traditionally taught skills are not enough, and that education systems must provide students with a broad set of 21st Century skills, if they want students to thrive in a technology-saturated and rapidly evolving world (Jerald, 2009). Hannay and Earl (2012) along with Trilling and Fadel (2009) claimed that 21st Century learning is imperative for future student success. Education is about preparing students to live well in a world worth living in (Grootenboer, 2018). Thus, education should be concerned with the development of the skills necessary to do this (Grootenboer, 2018).

Binkley et al. (2012) analysed 12 relevant frameworks drawn from several countries to define ten important 21st Century skills in four broad categories, listed in Table 1.1 below. The table presents a comprehensive list of 21st Century Skills categorised into four domains: Ways of Thinking, Ways of Working, Tools for Working, and Living in the World. The table underscores the multifaceted nature of 21st Century Skills, emphasising not only cognitive abilities but also interpersonal, technological, and societal competencies essential for success in contemporary society. These skills are vital for individuals to thrive in an increasingly interconnected and rapidly evolving world, both professionally and personally.

Table 1.1

A list of 21st Century Skills (Binkley et al., 2012, pp. 18-19)

Ways of Thinking	1. Creativity and innovation 2. Critical thinking, problem solving, decision making 3. Learning to learn, Metacognition
Ways of Working	4. Communication 5. Collaboration (teamwork)
Tools for Working	6. Information literacy 7. ICT Literacy
Living in the World	8. Citizenship-local and global 9. Life and career 10. Personal and social responsibility-including cultural awareness and competence

In a society abundant in information and technology, it is necessary to support teachers in continuously enhancing their skills. This support is important for effectively imparting the essential skills and fundamental knowledge required for young Australians to thrive in a modern society and economy (Education Council, 2019). The Australian education system has a mandated national curriculum, the Australian Curriculum (ACARA, 2010) that aims to support students to “become responsible local and global members of the community in an interconnected world and to engage with complex ethical issues and concepts such as sustainability” (Education Council, 2019, p. 10). The Australian Curriculum assists in students developing Information and Communication Technology [ICT] capabilities, as students learn to use ICT effectively and appropriately through the skills of critical and creative thinking, intercultural understanding and problem-solving. These areas are central to contributing to Australia’s knowledge-based economy as they support students in developing imagination, discovery, innovation, empathy, and the capacity to devise creative solutions to complex problems (Education Council, 2019).

In addition to the Australian Curriculum, it is also essential to consider the Early Years Learning Framework for Australia [EYLF]. The EYLF provides a guiding framework for early childhood teachers, emphasising the importance of holistic development in children from birth to five years old, as well as their transition to school (AGDE, 2022). By incorporating the principles of the EYLF into the Early Years environment, teachers can better support the foundational learning experiences of young Australians. The EYLF focuses on key areas such as belonging, being, and becoming, emphasising the significance of responsive and respectful relationships, play-based learning, and intentional teaching practices (AGDE, 2022).

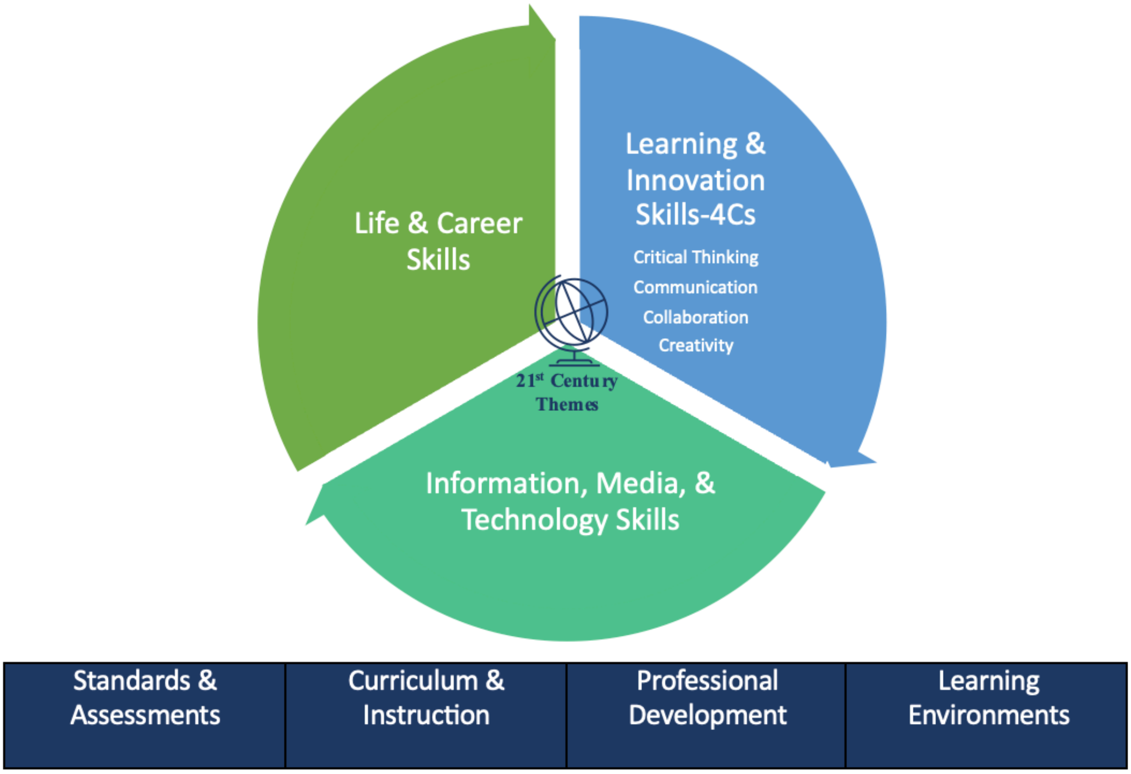
Furthermore, integrating ICT capabilities within the context of the EYLF can enrich Early Years education by fostering curiosity, creativity, and problem-solving skills from an early age (AGDE, 2022). Teachers can leverage technology as a tool for exploration and expression, supporting children's development across all areas of learning, including literacy, numeracy, and social-emotional skills (ADGE, 2022). In summary, by aligning efforts to enhance teacher skills with both the Australian Curriculum and the EYLF, teachers can create learning environments that promote the holistic development of young Australians, preparing them to thrive in a rapidly evolving society and economy.

1.3.2 Defining 21st Century Mathematical Skills

By integrating 21st Century skills into mathematics education, teachers can prepare students more effectively to succeed in a world that increasingly relies on mathematical thinking, problem-solving, and data analysis. In the context of mathematics education, defining 21st Century mathematical skills involves identifying the key competencies that students need to develop to thrive in today's rapidly changing world. The Partnership for 21st Century Skills [P21] (Battelle for Kids, 2019) formulated a framework pertaining to 21st Century learning. This framework was constructed to help integrate necessary skills into core academic disciplines, as shown in Figure 1.1. The P21 Framework for 21st Century Learning consists of four major elements: life and career skills; learning and innovation skills; information, media, and technology skills; and core subjects and 21st Century themes (Battelle for Kids, 2019). The framework was designed to describe “….the skills, knowledge and expertise students must master to succeed in work and life; it is a blend of content knowledge, specific skills, expertise and literacies” (p. 1). Mathematics was listed as one of the core subjects (Battelle for Kids, 2019).

Figure 1.1

21st Century Student Outcomes and Support Systems



Adapted from, "Framework for 21st century learning. *Partnership For 21st Century Learning*." by Battelle for Kids, 2019,

According to the P21 Framework (Figure 1.1), the 21st Century curriculum and instruction;

- teaches 21st century skills discreetly in the context of core subjects and 21st century interdisciplinary themes,
- focuses on providing opportunities for applying 21st century skills across content areas and for a competency-based approach to learning,
- enables innovative learning methods that integrate the use of supportive technologies, inquiry- and problem-based approaches, and higher order thinking skills, and
- encourages the integration of community resources beyond school walls.

(Battelle for Kids, 2019, p. 7)

Among the stated skills (listed in Appendix A) are learning and innovation skills, crucial for navigating the complexities of the 21st Century (Battelle for Kids, 2019). Researchers have identified these skills as encompassing creativity and innovation, critical thinking and problem-solving, communication, and collaboration (Battelle for Kids, 2019; Lindeman & Anderson, 2015; Voogt & Roblin, 2012). These skills find alignment in the domain of early childhood education, particularly within mathematics. For instance, critical thinking and problem-solving are fundamental aspects of mathematical thinking, involving the ability to approach problems logically and find effective solutions (Battelle for Kids, 2019). Furthermore, mathematical communication plays a vital role in early childhood mathematics education, as it involves students making sense of mathematical concepts and articulating their understanding to others (Ata Baran & Kabael, 2021).

Fostering a culture of creativity and innovation within early childhood classrooms contributes to the growth and development of mathematical thinking (Battelle for Kids, 2019). Teachers play a pivotal role in creating an environment where students feel encouraged to explore ideas and take risks, fostering creativity and innovation in mathematical problem-solving (Binkley et al., 2012).

Collaboration, another key skill, is not only vital for working effectively within diverse teams but also for promoting a supportive learning environment in early childhood classrooms (Battelle for Kids, 2019). Encouraging collaboration among young learners helps them develop essential social and teamwork skills. Developing shared responsibility and recognising the unique contributions of each team member is an important aspect of collaboration (Battelle for Kids, 2019).

1.4 Significance of the Research

As outlined in the background and context section of this Chapter, the world and the requisite skills for success and adaptability within it, have changed. In response, teachers, institutions, and sectors must be equipped to meet the evolving needs of students. This research endeavours to address this imperative for educational adaptation by exploring innovative methodologies that empower teachers, schools, and sectors to effectively cater to the dynamic requirements of students within this shifting landscape. The impetus for the research was a recognised need by the participant school to consider an Early Years teacher’s perspective of 21st Century mathematical skills. At present, there is a paucity of research around 21st Century skills for mathematics in education, particularly in the Early Years. A majority of the current research is in the fields of 21st Century skills that focus on Technology, or 21st Century skills related to the Secondary school context. It is crucial to conduct research that will inform sectors and schools about how the changes in the world influence and impact education in general, and early childhood education in particular. This research aims to inform and influence sector and schools’ policies and practices, to ensure that early years’ students are enabled to develop 21st Century mathematical skills that will equip them to evolve with the changing world.

1.5 Structure of the Thesis

The thesis is structured into six chapters and an overview is provided below in Table 1.2.

Table 1.2

Overview of Thesis Chapters

Chapter	Title
Chapter One:	Introduction
Chapter Two	Literature Review
Chapter Three:	Research Plan
Chapter Four:	Presentation of Research Results
Chapter Five:	Discussion of Results
Chapter Six:	Conclusion and Recommendations

Chapter One introduces the research, providing essential context to the background and outlining the significance of the research. In Chapter Two, the Literature Review delves into relevant literature, discussing the concept of 21st Century mathematical skills and emphasising their importance in student development. A pivotal aspect of this exploration is the examination of effective mathematics teaching, covering elements such as Pedagogical Content Knowledge [PCK], Mathematical Knowledge for Teaching [MKT], pedagogical practices and principles, mathematics lesson structure, and mathematical disposition. The chapter also investigates the role of the Early Years learning environment, highlighting the influential contributions of parents, Early Years teachers, and the school leadership team in nurturing 21st Century mathematical skills. Chapter Three, the Research Plan outlines the qualitative theoretical framework that underpins the research, encompassing its epistemology, methodology, and the methods employed for data collection and analysis.

Chapter Four provides a detailed presentation of the research outcomes, utilising tables and figures to visually represent the collected data. Alongside these visual tools, thorough data analysis is conducted to interpret and contextualise the findings. This approach ensures that the information is not only clearly communicated but also understood within the broader context of the study, highlighting the significance of the results and their implications. In Chapter Five, the discussion contextualises the findings, relating them to the research questions and drawing connections to the existing body of literature. Finally, Chapter Six presents the conclusion, and offers a review of the research findings, provides a response to the research questions, proposes implications, and provides insightful recommendations for further inquiry.

1.6 Conclusion

This investigation aims to address the overarching research question: How does the Early Years school environment equip young children in the development of 21st Century mathematical skills? This chapter has framed the investigation by situating the research within the current policy context. The next chapter will delve into existing literature, focusing on 21st Century mathematical skills and their importance for students' lifelong success and skill development.

Chapter Two: Literature Review

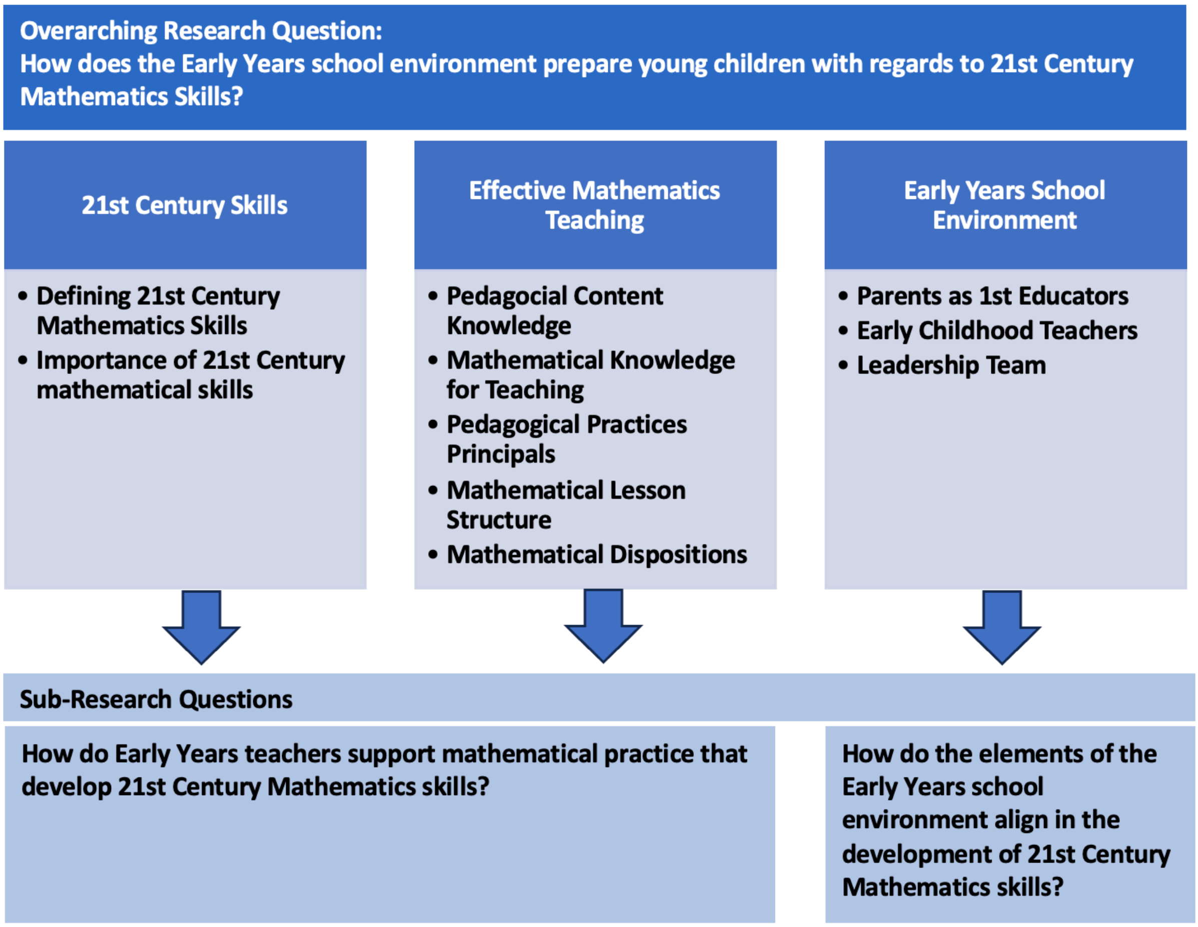
2.1 Introduction

It is the role of teachers to ensure that "all young Australians become confident and creative individuals, successful lifelong learners, and active and informed members of the community" (Education Council, 2019, p. 4). This position is supported by The Australian Curriculum, Assessment and Reporting Authority stated that the role of the Australian Curriculum is equipping young Australians to live and work successfully in the 21st Century (ACARA, 2010). This context frames the current research which was designed to focus on the perspectives of key stakeholders in preparing young children with 21st Century mathematical skills, specifically in an Early Years context.

This chapter reviews the existing literature on 21st Century skills, effective mathematics teaching, and the Early Years school environment from which the research sub-questions subsequently emerged. To investigate 21st Century mathematical skills it was important to define the terms 21st Century skills and 21st Century mathematical skills. Defining these two terms aids in understanding and distinguishing the general skills from those specific to mathematics. In addition, the research was designed to focus on an Early Years class. Therefore, it was necessary to explore the key elements of the Early Years environment and what key role each stakeholder plays in contributing to young children's development of 21st Century mathematical skills. Deliberation of young children's early mathematical skills and the factors attributing to the effective teaching and learning of these skills is also examined, as illustrated in Figure 2.1.

Figure 2.1

Conceptual Framework



2.2 Why 21st Century Mathematical Skills Are Crucial for Young Children's Future Success

The Alice Springs (Mparntwe) Education Declaration stated that the educational vision for Australia's education system "is for a world class education system that encourages and supports every student to be the very best they can be, no matter where they live or what kind of learning challenges they may face" (Education Council, 2019, p. 3). To achieve this vision of a world class education system, The Alice Springs (Mparntwe) Education Declaration claimed that numeracy is essential to allow students to become successful learners at school, and in life beyond school, in preparing them for their future roles as family, community, and workforce members (Education Council, 2019). Pound (2006) stated that children need to learn to recognise mathematics as a powerful tool of communication if they are to become competent users of mathematics in the 21st Century. The Melbourne Declaration (MCEETYA, 2008) which preceded The Alice Springs (Mparntwe) Education Declaration, supported Pound's (2006) statement claiming, literacy, numeracy, and knowledge of fundamental disciplines continue to be the cornerstone of education for young Australians. Education should also foster the growth of skills in areas like social interaction, interdisciplinary thinking, and the utilisation of digital media, which are indispensable in all occupations of the 21st Century.

In essence, a strong foundation in 21st Century mathematical skills is crucial for young children's future success in navigating various aspects of life and contributing meaningfully to society. In line with the Alice Springs (Mparntwe) Education Declaration, the teacher's role should be more than just teaching students for school success. The teacher's role must also include developing life-skills that prepare students for the future (Bell, 2016; Stipek et al., 2001). Traditionally education was primarily focused on acquiring the essential content within each subject area, followed by evaluating this content knowledge through tests administered at the conclusion of a unit (Bray & Tangney, 2016). However, living, learning, and working in the 21st Century requires an expanded set of skills, competencies, and flexibility. There is an acknowledgement that there exists a gap between what students learn, and what students need to learn to succeed in the workplace (Smit, 2016). Knaus (2017) supported this claim, stating that the traditional approach of teaching based on a model of one-size-fits-all and rote-learning methods in mathematics has been proven inappropriate.

A new paradigm is emerging that shifts the teaching focus from a traditional approach to a constructivist approach. This constructivist approach emphasises inquiry mathematics, as learning in a constructivist environment requires active student engagement, inquiry, problem-solving and collaboration with others (O'Shea & Leavy, 2013). Conventional teaching models must shift to a "transformative" style of education for the 21st Century where student-driven learning flourishes and teachers become facilitators of learning rather than simply sources of all information (Bell, 2016). If teachers aim to equip students for success in the evolving landscape of tomorrow's world, adopting a transformative teaching style could be considered a strategic and forward-thinking approach (Bell, 2016).

An essential challenge in mathematics education is ensuring that all students attain proficiency in mathematics, enabling them to possess the necessary knowledge, skills, and confidence to actively engage in further learning, employment, and community participation by the end of their schooling (Gervasoni et al., 2012). Consequently, it is essential that pedagogical models that align with 21st Century learning, incorporating skills such as mathematical creativity and innovation, critical thinking and problem-solving, communication, and collaboration (Battelle for Kids, 2019).

The education system has a vital role in preparing the younger generation to excel in an era of rapid societal and technological transformations, and to tackle challenging environmental, social, and economic issues (Education Council, 2019). Education is instrumental in fostering the intellectual, physical, social, emotional, ethical, spiritual, and aesthetic growth and wellbeing of young Australians. Young people must learn to manage the overflow of information and discern its reliability and authenticity. They require adaptability, resilience, creativity, and the motivation and capability for lifelong learning (Education Council, 2019). It is evident that mathematics education must evolve to equip students with the diverse skills and attributes necessary to thrive in the 21st century and beyond.

Students need real-world problem-solving, internships or apprenticeships in real work settings, and other more authentic learning experiences, to make learning relevant and meaningful (Trilling & Fadel, 2009). Therefore, rather than just mastering skills and applying them, students need to be engaged in solving problems (Bray & Tangney, 2016). The Alice Springs (Mparntwe) Education Declaration (Education Council, 2019) supported these claims, acknowledging that major changes in the world are placing new demands on Australian education. Pound (2006) suggested that children should not just be taught mathematical content knowledge but also use mathematical knowledge to solve problems. Askew (2013) concurred, suggesting that collaborative approaches such as paired or group work, which foster collaborative emergence, may result in more sophisticated and improvised mathematical performance than what individual students could achieve alone.

According to Bray and Tangney (2016), an essential component of students' preparation for life in the 21st Century is teachers having the ability to make informed decisions and to interpret and apply mathematics in a variety of contexts. They further state that it is essential that teachers become 21st Century learners themselves, learning from inquiry, design, and collaborative approaches that build a strong community of professional teachers (Bray & Tangney, 2015). Teachers must continually sharpen their skills of using the power of learning technologies to help deepen understanding and further develop 21st Century skills (Trilling & Fadel, 2009). Furthermore, teachers should encourage students to cultivate a curiosity for mathematics, facilitate the expression of their mathematical thinking, and help them build confidence in their calculations and estimations (Pound, 2006).

Despite the currency of issues surrounding the development of 21st Century skills, Perry and MacDonald (2015) reported that literature regarding the development of 21st Century skills through mathematics in early childhood was very limited, stating that more instances occur in primary school contexts. Researchers such as Hannay and Earl (2012) and Trilling and Fadel (2009) have explored 21st Century learning and how imperative it is for future student success but have not explored the connection to classroom practice, particularly within the Early Years environment.

2.3 Effective Mathematics Teaching

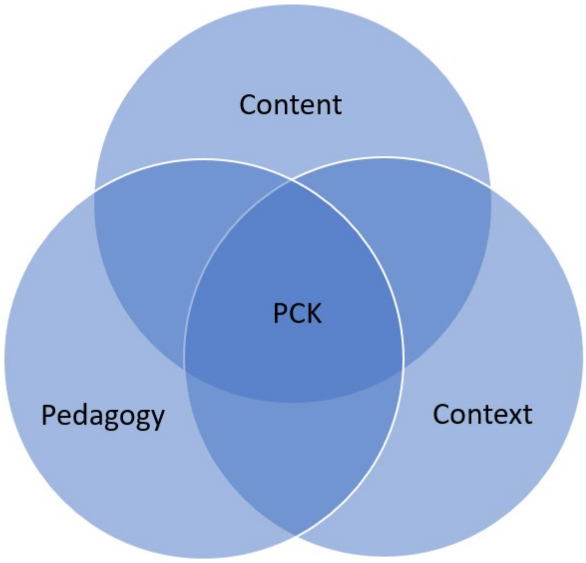
2.3.1 Pedagogical Content Knowledge

The teaching of mathematics is complex, as it requires teachers to have a deep mathematical knowledge of what they are teaching and an understanding of how students’ mathematical learning develops and progresses across year levels (National Council of Teachers of Mathematics [NCTM], 2014). Hattie (2010) drew on earlier work in the field that claimed that one of the greatest influences on student learning is quality teacher instruction. Similarly, the work of Clarke and Clarke (2004) suggested that highly effective mathematics teachers have knowledge and awareness of conceptual connections between the areas of the mathematics curriculum. It is widely accepted that teachers of mathematics require an appropriate balance of both content and pedagogical content knowledge (Muir, et al., 2017). Shulman (1987), in his seminal work, claimed that Pedagogical Content Knowledge [PCK] is the union of knowledge of content, context and pedagogy and is crucial at all levels of mathematics and with all groups of students (Figure 2.2).

Shulman's (1987) conceptualisation remains pertinent and insightful in understanding the dynamic interplay between content, context, and pedagogy in the field of education. Shulman’s work on PCK has strongly influenced later developments by scholars who assume that such knowledge exists and impacts effective teaching and students’ learning (Loewenberg Ball et al., 2008). Shulman (1986) described PCK as understanding what facilitates or hinders the learning of specific subjects involves recognising the ideas and preconceived notions that students of various ages and backgrounds bring to the table when they engage with commonly taught topics and lessons. Often, these preconceptions are misconceptions. In such cases, teachers require knowledge of the most effective strategies for restructuring learners’ understanding.

Figure 2.2

Shulman's (1986) Domains of Pedagogical Content Knowledge



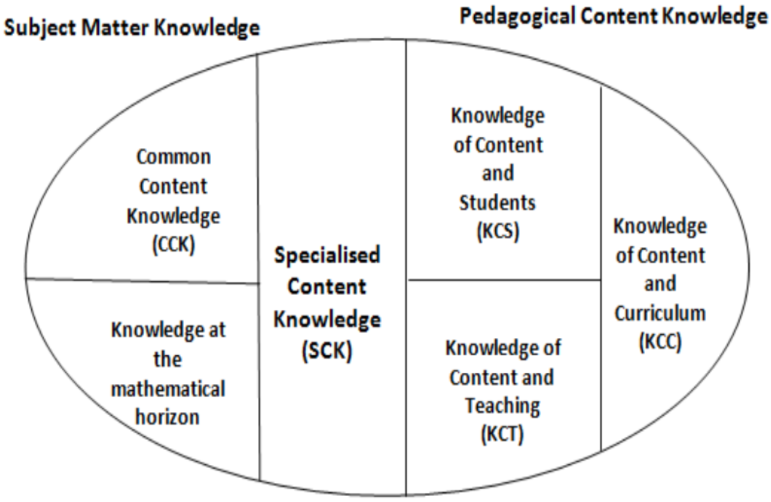
Reproduced with permission from "What teachers need to know to teach mathematics: An argument for reconceptualised model" by D. Hurrell, 2013, *The Australian Journal of Teacher Education*, *38*, p.55.

2.3.2 Mathematical Knowledge for Teaching

Effective teachers possess a distinct understanding of students' mathematical ideas and thinking, and a teacher's mathematical knowledge influences the quality of their instruction (Beswick, 2012; Loewenberg Ball et al., 2008). Loewenberg Ball et al. (2008) defined teaching knowledge as "knowledge in and for teaching rather than on teachers themselves." (p. 394). Loewenberg Ball et al. (2008) elaborated and further defined Shulman's (1986) PCK model into a framework for Mathematical Knowledge for Teaching [MKT], divided into two categories known as subject matter knowledge and pedagogical content knowledge, as shown in Figure 2.3. It is documented that effective teachers must know more than the content of the subject they are teaching; they need to have specialised content knowledge [SCK] (Loewenberg Ball et al., 2008). According to Livy et al. (2019) effective mathematics teachers use and require different types of mathematical content knowledge [MCK] for teaching mathematics. Teachers must know the subject they are teaching and in addition "teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students" (Loewenberg Ball et al., 2008, p. 404).

Figure 2.3

Domains of Mathematical Knowledge for Teaching



Reproduced from "Content Knowledge for Teaching: What Makes It Special?" by D. Loewenberg Ball et al., 2008, *Journal of Education*, *59*(5), p .403.

In 2006, the Australian Association of Mathematics Teachers' [AAMT] Council released a series of 'Standards' for excellence that were a consensus view by the profession, for the profession, that described the knowledge, skills and attributes required for good teaching of mathematics. These 'Standards' provided targets for teachers of mathematics to aspire and work towards. The 'Standards' were aligned with Shulman's PCK model (1987). The 'Standards' stated, "…excellent teachers of mathematics have rich knowledge of how students learn mathematics. They understand current theories relevant to the learning of mathematics" (AAMT, 2006, p. 2). Knaus' (2017) study found that it is imperative that teachers have knowledge of mathematical content and pedagogical practices required to engage children in learning mathematical concepts to ensure better developmental outcomes.

2.3.3 Western Australian Curriculum: Mathematics

In Australia, decisions about what mathematics to teach are primarily influenced by the Australian Curriculum: Mathematics (ACARA, 2010) and supplementary support material developed by state and territory education authorities. Since 2013, the School Curriculum and Standards Authority in Western Australia [SCSA] have utilised the Western Australian Curriculum: Mathematics [WAC:M] to refine their teaching methodologies. In 2017 SCSA amended the curriculum in mathematics, clarifying some content strands, and identified the mathematics curriculum as version 8.1 (SCSA, 2018). The number of amendments to the

ACARA (2010) K-10 curriculum were minimal. Version 8.1 is currently used in schools in Western Australia (SCSA, 2018).

The WAC:M has three strands, Number and Algebra, Measurement and Geometry, and Statistics and Probability, laying the groundwork for more advanced topics. It emphasises the importance of problem-solving and critical thinking, encouraging students to analyse problems, employ logical reasoning, and devise solutions, skills that are beneficial in real-world contexts (SCSA, 2018). Additionally, the WAC:M stresses the need for fluency in mathematical procedures and understanding the rationale behind them, ensuring that students not only perform calculations but also comprehend their validity. The WAC:M has promoted professional discourse and learning, encouraging teachers to consider and adopt pedagogical changes recommended by the WAC:M. A key aspect of the WAC:M implementation is the Proficiency Strands, which outline essential instructional strategies for planning and teaching (SCSA, 2018).

The WAC:M provides a structured framework of content and achievement standards from Pre-primary to Year 10, which schools use to develop student learning programs, assess student progress, and communicate outcomes to parents. The curriculum is essential for imparting fundamental mathematical skills and knowledge, fostering a foundation for continuous education and practical application in various life situations (SCSA, 2018). The WAC:M emphasises the recognition of patterns, relationships, and problem-solving, thereby nurturing critical thinking skills that students can apply beyond the academic setting (SCSA, 2018). The Mathematics Curriculum is constructivist by nature, as it helps teachers to help students become self-motivated and confident learners through active participation in authentic and challenging experiences (ACARA, 2010).

Despite the comprehensive scope of the WAC:M, there are challenges in aligning it with the Early Years Learning Framework (EYLF). Perry et al., (2015) argued that the EYLF and the Australian Curriculum were developed at different times by different groups with minimal consultation between them. The same is true for the WAC:M and the EYLF, as both documents were produced at different times by separate groups with little consultation between them. Connor (2011) acknowledged that while the Australian Curriculum recognises the EYLF's role in establishing foundational learning, the differences in structure, content, and philosophy between the two documents make it difficult to see clear connections. However, some efforts have been made to bridge these gaps. Connor (2011) used the general capabilities in the Australian Curriculum across all subject areas to link with the learning outcomes in the EYLF. Perry et al. (2012) have made these connections explicit in mathematics, further extended through the introduction of reflective continua to support teachers in prior-to-school settings and schools in developing young children's powerful mathematical ideas (Perry & Dockett, 2013). The EYLF and the WAC:M are different in structure and in some of their emphases because they focus on phases in the learning lives of children and young people. However, the two sets of documents are complementary and can provide an articulated pathway of learning from prior-to-school, into school and beyond (Connor, 2011).

2.3.4 Pedagogical Practices and Principles

Fundamental to Shulman's PCK Model (1987), is the belief that mathematics learning is most effective when the students are engaged in higher order thinking through working on tasks that are appropriately challenging for them (Anthony & Walshaw, 2009). An effective mathematics teacher engages students through individual and collaborative experiences in meaningful learning that promotes their ability to mathematically reason and make sense of their mathematical ideas (NCTM, 2014). The main stimulus for mathematical learning is through the teacher providing and posing mathematical tasks that engage the students in thinking for themselves about mathematics (Anthony & Walshaw, 2009).

Further to teachers posing mathematically rich tasks, the level at which tasks are pitched needs to be considered. It is crucial that the tasks are posed within the zone of proximal development [ZPD], which Vygotsky (1978) described as the "distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined by problem-solving under adult guidance or in collaboration with more capable peers" (p. 86). Sullivan et al. (2012) supported Vygotsky's (1978) claim, stating that tasks should be within the range in which students can engage and, with support, achieve success. Tasks should also go beyond and extend the current level of students' mathematical knowledge.

One set of pedagogical approaches, that was developed by Clarke and Clarke (2004), arose from teachers who had been identified as particularly effective. The set of pedagogical practices are grouped under ten headings and 25 specific pedagogical actions as shown in Table 2.1.

Table 2.1

A Set of Pedagogical Practices

Pedagogical practice	Explanation
Mathematical focus	Focus on important mathematical ideas. Make the mathematical focus clear to the children.
Features of task	Structure purposeful tasks that enable different possibilities, strategies, and products to emerge Choose tasks that engage children and maintain involvement.
Materials, tools and representation	Use a range of materials/representations/contexts for the same concept.
Adaptions/connections/links	Make connections to mathematical ideas from previous lessons or experiences.
Organisational style(s), teaching approaches	Use a range of question types to probe and challenge children’s thinking and reasoning. Hold back from telling children everything. Encourage children to explain their mathematical thinking/ideas. Encourage children to listen and evaluate others’ mathematical thinking/ideas and help with methods and understanding. Listen attentively to individual children.
Expectations	Have high but realistic mathematical expectations of all children. Promote and value effort, persistence, and concentration.
Learning community and classroom interaction	Engage and focus children’s mathematical thinking through an introductory, whole group activity. Choose from a variety of individual and group structures and teacher roles within the major part of the lesson.
Reflection	Draw out key mathematical ideas during and/or towards the end of the lesson. After the lesson, reflect on children’s responses and learning, together with activities and lesson content.
Assessment Methods	Collect data by observation and/or listening to children, taking notes as appropriate.

Personal attributes of the teacher	Use a variety of assessment methods. Modify planning as a result of assessment. Believe that mathematics learning can and should be enjoyable. Are confident in their own knowledge of mathematics at the level they are teaching. Show pride and pleasure in individuals' success.

Reproduced with permission from "Mathematics teaching in grades K-2: Painting a picture of challenging, supportive, and effective classrooms," by D. M. Clarke and B. A. Clarke, 2004, *Perspectives on the Teaching of Mathematics*, *66*, p.68.

Sullivan et al. (2012) argued that an important component of understanding teaching and improving mathematical learning is to "…identify the types of tasks that prompt engagement, thinking, the making of cognitive connections, and the associated teacher actions that support the use of such tasks, including addressing the needs of individual learners" (p. 12). A second set of pedagogical actions associated with mathematics teaching was developed by Sullivan (2011), who drew on Clarke and Clarke's (2004) work, and recommendations by Hattie and Timperley (2007). These are shown in Table 2.2

Table 2.2

Six Principles for Effective Teaching of Mathematics

Principles	Effective Teaching of Mathematics
Principle 1. Articulating Goals	Identify big ideas that underpin the concepts you are seeking to teach and communicate to students that these are the goals of the teaching including explaining how you hope they will learn.
Principle 2. Making Connections	Build on what the students know mathematically and experientially including creating and connecting students with stories that both contextualise and establish a rationale for the learning.
Principle 3 Fostering Engagement.	Engage students by utilising a variety of rich and challenging tasks that allow students time and opportunities to make decisions, and which use a variety of forms of representation.
Principle 4. Differentiating Challenges	Interact with students while they engage in the experiences; encourage students to interact with each other including asking and answering questions and specifically planning to support students who need it; and challenge those who are ready.
Principle 5. Structuring Lessons	Adopt pedagogies that foster communication and mutual responsibilities by encouraging students to work in small groups and using reporting to the class by students as a learning opportunity.

Principle 6. Promoting Fluency and Transfer	Fluency is important, and it can be developed in two ways: by short everyday practice of mental calculation or number manipulation; and by practice, reinforcement and prompting transfer of learnt skills.

Adapted with permission from "Teaching mathematics: Using research informed strategies.," by P. Sullivan, 2011, *Australian Education Review.* Australian Council for Educational Research, pp. 25-29.

2.3.5 Mathematical Lesson Structure

The Organisation for Economic Co-operation and Development [OECD] (2018) proposed global aspirations for education by the year 2023. To achieve this, the OECD outlined a shared vision. The vision stated that "we are committed to helping every learner develop as a whole person, fulfil his or her potential and help shape a shared future built on the well-being of individuals, communities, and the planet" (OECD, 2018, p. 3). Gervasoni et al. (2021) stated that mathematical learners need to be prepared for 2030 and beyond, therefore they need to become creative, confident, collaborative problem solvers who are ethical and can communicate their mathematical ideas through arguments and models and who persist when faced with new and complex challenges. Students of today must develop proficiency in 21st Century skills. Teachers need to structure their mathematical lessons so that they represent and model this future, that students thrive and flourish (Gervasoni et al., 2021).

Another key aspect of the OECD (2018) shared vision was about preparing students for uncertain and rapidly changing times, suggesting that through the education system the capabilities for creative and independent thinkers is fostered. Developing creative and critical thinking skills has emerged as a fundamental requirement for individuals, not only to enhance their career opportunities but also to improve their personal and societal life (Vincent-Lancrin, 2023). Students entering school must learn to discard the belief in limitless resources available for exploitation (OECD, 2018). Instead, students should prioritise common prosperity, sustainability, and well-being. Students need to be responsible and empowered, emphasising collaboration over division, and sustainability over short-term gains. In an increasingly volatile, uncertain, complex, and ambiguous world, education plays a crucial role in determining whether individuals embrace or succumb to the challenges they face (OECD, 2018). In many nations, there is a consensus within civil society that educational institutions should foster creativity and independent thinking among students (Vincent-Lancrin, 2023). Furthermore, in an era marked by rapid scientific advancement and complex societal issues, curricula should evolve, possibly in transformative ways (OECD, 2018).

Aligned with the OECD (2018) work, Sullivan (2020) observed that structuring mathematical lessons with a teacher-centred approach, where the teacher begins by explaining the task and students subsequently practise, poses a dual risk. Firstly, the students will become dependent on the teacher for learning. Secondly, the students will see mathematics as something they are "told" or "given" to them, as opposed to engaging students in thinking for themselves about mathematics (Sullivan, 2020). These insights correlate with the earlier work of Anthony and Walshaw (2009), reinforcing the importance of fostering independent mathematical thinking among students.

A fundamental assumption is that students learn better and actively engage in tasks they are working on when they are challenged (Sullivan et al., 2016). To initiate student learning, to activate student thinking and to develop an orientation to persistence, teachers need to pose challenging mathematical tasks that prompt problem-solving (Stein et al., 2008; Sullivan et al., 2016). Evidence indicated that students embraced struggle and persisted when engaged in mathematics lessons with challenging tasks. Moreover, many students enjoyed the process of being challenged (Russo & Hopkins, 2017). A mathematics lesson designed to allow students to use their own thinking to solve problems and emphasise student-centred structured inquiry, could follow a 'Launch, Explore and Summarise' format (Stein et al., 2008; Sullivan et al., 2019).

According to the research by Sullivan et al., (2019), an effective lesson structure is one where students are presented with tasks but not given explicit methods to solve them. Students often find problem-solving more engaging and motivating when they are allowed to explore and discover solutions on their own. This intrinsic motivation is essential for sustained interest in mathematics (Smith & Stein, 2018). Effective problem-solving tasks require students to encounter obstacles, prompting them to think, experiment, attempt and fail, and apply their knowledge in new ways to overcome challenges (Liljedahl, 2021). The teacher's role in this structure is to differentiate tasks based on the students' needs, providing additional support to those who require it and offering extended tasks to those who finish quickly. Furthermore, the teacher observes and selects responses to the tasks during the lesson to facilitate a classroom dialogue that emphasises students' explorations and mathematical thinking. This approach underscores the importance of student autonomy, differentiated instruction, and the facilitation of classroom dialogue focused on students' explorations and mathematical thinking. It highlights the role of the teacher as a guide and observer, rather than a direct instructor, fostering a more student-centred learning environment (Sullivan et al., 2019).

It was argued by Sullivan et al., (2019) that it is preferable for students to figure out how to solve problems independently rather than simply apply a taught method. Posing a task without explaining a solution pathway, allows the students to engage in thinking about how to solve the problem (Lampert, 1990; Sullivan et al., 2019). Students at this stage are encouraged to reflect on and activate prior knowledge. Students then need to decide what they need to do. It was argued that a self-invented strategy to a solution is more likely to be remembered and able to be transferred to different situations (Sullivan et al., 2019).

2.3.6 Mathematical Disposition

It has been documented that quality early educational experiences are fundamental to children’s later achievements (e.g., Fleer & Raban, 2005; McCain & Mustard, 1999; Siraj-Blatchford, 2009). Quality educational experiences include developing children’s positive attitudes, beliefs, and practices (Clements & Sarama, 2014). This is supported by The Early Years Learning Framework for Australia [EYLF] which stated, “literacy and numeracy capabilities are important aspects of communication and are vital for successful learning across the curriculum” (AGDE, 2022, p. 57).

If children engage in meaningful mathematical learning experiences, they are more likely to develop positive dispositions encompassing enthusiasm, curiosity, perseverance, and confidence. These dispositions enable them to approach mathematical challenges with eagerness and resilience (Clements & Sarama, 2014). Knaus (2017) supported this assertion, affirming that children who participate in such experiences during early childhood tend to develop positive attitudes towards mathematics in primary school and are more likely to construct deeper mathematical understandings. This aligns with the definition provided by the AGDE (2022), which stated that "being numerate is the capacity, confidence, and disposition to use mathematics in daily life" (p. 57), emphasising the importance of both confidence and disposition alongside mathematical skills and understanding.

Boaler (2016) claimed that it is crucial to encourage students to believe in themselves and ensure that students are aware that there is no such thing as a ‘mathematics person’, that everyone with hard work and persistence can achieve the highest levels in mathematics. While many applaud Boaler's (2016) emphasis on promoting a growth mindset and inclusivity in mathematics education (Dweck, 2017, Hattie, 2023), Kim and Quinn (2013) argued that the reality of individual differences in mathematical aptitude cannot be entirely dismissed. While hard work and persistence are undoubtedly essential for success in mathematics, inherent cognitive factors may still play a significant role in determining an individual's proficiency in the subject (Kim & Quinn, 2013). Thus, while Boaler's (2016) message of empowerment and equity resonates with many teachers, it also sparks debates regarding the interplay between effort, ability, and achievement in mathematics.

Negative beliefs about mathematics, coupled with a fixed mindset that many students hold about mathematics, may lead to deleterious effects in mathematical learning (Boaler & Williams, 2020). Knaus (2017) stated that teachers who have negative perceptions about mathematics, as well as a lack of knowledge of curriculum content and pedagogy are the major factors limiting students' mathematics experiences in the Early Years. Knaus (2017) noted the importance of investigating teachers' perceptions of teaching mathematics, but equally as important is how those perceptions and practices align with 21st Century skills, as this is crucial for future success in a rapidly changing technological world.

2.3.7 Mathematical Engagement

Attard (2021) stated, "engagement with mathematics during the primary years of schooling is crucial if students are to develop an appreciation for and understanding of the value of mathematics learning" (p. 4). Engagement is necessary for learning. If students experience decreased engagement, this can have a negative effect on students' opportunities for the future in terms of tertiary courses requiring elevated levels of mathematics, and the capacity for students to understand life experiences through a mathematical perspective (Attard, 2013; Finn & Zimmer, 2012; Fredricks et al., 2004).

Mathematical engagement occurs when the student is actively involved, is appreciating, and enjoying mathematics, and experiencing cognitive challenge (Attard, 2021). For true engagement to occur, students need to be engaged on three levels: affective, cognitive, and operative (Attard, 2021; Skinner, 2016). Engaging cognitively means the learner is reflectively involved in deep understanding of mathematical concepts and applications. Engaging operatively means the learner is actively participating in group discussions, practical, relevant activities, and homework tasks. Engaging affectively is when the student genuinely values that the learning will be useful outside the classroom and enjoys being actively involved in the experience (Attard, 2013).

Attard (2021) concluded that teachers have the most impact on student engagement levels in mathematics. The teacher's influence on student engagement can be broken into two separate but interconnected levels. The first level is the teacher's pedagogical repertoire, the day-to-day practices of teaching. The second level is positive pedagogical relationships. Student engagement requires the development of positive pedagogical relationships, in other words an understanding of students as learners. (Attard, 2021). To achieve a positive relationship, teachers need to understand how their students learn and when and where they need assistance (Attard, 2021). Student engagement relies on fostering positive pedagogical relationships, which involves understanding students as learners (Sullivan, 2011). This is crucial because such relationships enhance students' motivation, participation, and overall learning outcomes (Sullivan, 2011). A positive pedagogical relationship exists when the student's prior knowledge and background is recognised; the teacher has strong PCK, enthusiasm and enjoyment for mathematics; the teacher understands each student's learning needs and mathematical abilities; and student feedback is timely, purposeful, and constructive (Attard, 2021).

While Attard (2021) emphasised the significant impact of teachers on student engagement levels in mathematics, other factors beyond teacher influence also play crucial roles in student engagement (Fredricks et al., 2004). For example, socio-economic factors, family support, and peer relationships can significantly impact a student's engagement with mathematics (Fredricks et al., 2004). Student engagement is multifaceted and the various factors that contribute to it, offering a perspective that extends beyond the direct influence of teachers.

2.4 Young Children's Early Years Mathematical Learning

2.4.1 Early Years Mathematical Learning

The National Quality Framework for Australia [NQF] (ACECQA, 2011) introduced a new quality standard known as The National Quality Standard for Australia [NQS] (ACECQA, 2011). The NQF mandated the implementation of a recognised learning framework in early childhood education and care [ECEC] settings across Australia. In Australia, the adopted framework was The EYLF (AGDE, 2022). Both the NQS and the EYLF recognised that high-quality teaching takes many forms and is most effective when attuned to and responsive to the needs of children, families, and communities (AGDE, 2022). Nonetheless, the NQS mandate and the implementation of the EYLF required some teachers to reconceptualise their professional practice.

The NQS and the EYLF emphasise a holistic approach to early childhood education, focusing on the importance of play-based learning, child-centred pedagogy, and responsive teaching practices. For some teachers, particularly those who may have been accustomed to more traditional or directive teaching styles, embracing the principles of the NQS and EYLF required a significant shift in mindset and practice. This reconceptualisation entails moving away from teacher-centred approaches towards creating rich, stimulating learning environments where children are actively engaged in meaningful experiences and where teachers serve as facilitators of learning rather than disseminators of knowledge (Hannay & Earl, 2012).

Previous studies have concluded that many teachers wrestle with this notion of changing their teaching practice (Hannay & Earl, 2012). In Australian education, mathematics is recognised as a core curriculum area in teaching and learning. However, what should be taught in the Early Years has been a matter of controversy (Knaus, 2017). Seo and Ginsburg (2004) claimed that early childhood teachers need to begin with an understanding of children's mathematical interests, motivations, and competence.

Knaus (2017) found that young children have knowledge of complex mathematical ideas. This aligns with the findings from Perry and Dockett (2005) who insisted that young children are knowledgeable about mathematics and called on teachers in the school settings "to recognise the mathematical power of young children and to nurture this power to the full" (p. 36). Pound (2006) furthered the argument to nurture young children's mathematics by writing that preschool and kindergarten children are more capable in mathematics than is often realised. This position was further supported by Clements and Sarama (2014) who asserted that young children have the potential to learn mathematics that is complex, deep, and broad. This assertion is tempered by the knowledge that this potential is often unrealised (Clements & Sarama, 2014). To realise that potential, all young children need to be encouraged to work like mathematicians, and teachers need to provide activities that are contextual and that are important to the children at that moment in time (Clauscen, 2013).

For teachers to get children to work like mathematicians it is essential that they have a high level of mathematical knowledge for teaching and effective pedagogies to make a positive influence on students' mathematical learning (Knaus, 2017). Seo and Ginsburg (2004) suggested that early childhood teachers need to learn new forms of pedagogy, new mathematical content, and new psychological insights. However, Guskey (2002) suggested that "…teachers are reluctant to adopt new practices or procedures unless they feel sure they can make them work" (p. 386). If teachers are to lead student improvement, then new practices and procedures need to be effective and sustained.

Although researchers over the years, such as Knaus (2017) and Sullivan (2020), have advocated for effective mathematics teaching and learning practices and pedagogies, there is limited research aligning these practices and pedagogies with 21st Century skills. This is an issue, as researchers claim that studies have shown that what children learn in the early years is a predictor of mathematics achievement and success in later years (Knaus, 2017; Zippert & Rittle-Johnson, 2018). Additionally, there is a concern around the limited evidence surrounding effective mathematics pedagogy and aligning it with 21st Century skills in a classroom setting. It is this alignment of how Early Years teachers support mathematical practice with the development of 21st Century mathematical skills the research presented here sought to address.

2.4.2 Mathematical Pedagogical Practices for the Early Years

Sharing Piaget's (1964) constructivist view that children are active participants in constructing their own knowledge, Bruner (1966) emphasised the role of representation in the learning process using three modes of representation: enactive, iconic, and symbolic. In the enactive representation mode, children manipulate real materials to depict relationships. This hands-on approach allows them to understand concepts through direct physical interaction. The iconic representation mode involves children working with drawings or pictures to represent relationships, helping them transition from tangible to visual understanding. Finally, the symbolic representation mode involves the use of abstract symbols to describe relationships, allowing children to engage with more complex and abstract concepts.

These modes are crucial for understanding how children develop and learn, especially in their early years. Each mode builds on the previous one, with enactive experiences providing the foundation for iconic representation, which in turn supports the development of symbolic understanding. Bruner (1966) emphasised that effective teaching should incorporate all three modes, enabling children to progress from concrete experiences to abstract thinking. By applying these modes, teachers can better support the cognitive development of young learners.

As Bruner (1966) asserted, children need opportunities to work with objects in the physical world before they are ready to engage with pictures and other representations. Following this, they can move on to working with symbols. According to Reys et al. (2021) young children possess a substantial intuitive understanding of mathematics when they start school, derived from their family environment, play, and social experiences. Bruner's (1966) emphasis on hands-on experiences aligns with Reys et al. (2021) observation that children's intuitive mathematical understanding is rooted in their early interactions with the world around them. In other words, the concrete experiences advocated by Bruner (1966) contribute to the intuitive mathematical sense that Reys et al. (2021) described. The progression from physical interactions to symbolic understanding, as proposed by Bruner (1966), lays the groundwork for the intuitive mathematical abilities that Reys et al. (2021) highlighted in young learners.

In the Early Years, teachers must recognise that children's mathematical thinking might surpass their ability to learn in a formal classroom setting. Understanding this can help teachers create more effective learning environments that cater to the developmental stages of young children.

2.5 Early Years School Environment

2.5.1 Early Years Environment

It is important to note that the following section is primarily grounded in the analysis of policy documents, rather than empirical research studies, and should be interpreted with this context in mind. Early Years learning environments in schools should encourage conversations between teachers, children and families and promote opportunities for sustained shared thinking and collaborative learning (AGDE, 2022). The AGDE (2022) stated that "Learning Environments include physical, temporal, social and intellectual elements" (p. 23). Learning environments should be inclusive and welcoming and respect and affirm identities to enrich the lives of children and families (AGDE, 2022). This fosters a sense of belonging and acceptance, promotes positive social interactions, and supports the emotional well-being of all individuals involved in the learning community. Clements et al. (2023) supported this, claiming that teachers can enhance a learning environment to be inclusive by fostering relationships with families and encouraging their active participation in the learning process. This collaborative approach not only strengthens the learning environment but also ensures a more holistic and inclusive educational experience for the students.

The AGDE (2022) explained that partnerships "are based on the foundations of respecting each other's perspectives, expectations and values, and building on strength of each other's knowledge and skills" (p. 14). When teachers collaborate in genuine partnerships with families, other professionals, and teachers in schools working in the best interests of children, learning outcomes are most likely to be achieved. Children construct their own understanding as they observe and participate in everyday life and through a range of settings, such as at home, childcare, school, while shopping or even at the park (Knaus, 2013). When parents and teachers provide mathematical experiences and opportunities that are relevant and meaningful in the context of their everyday life, children's early mathematical learning can be enhanced (AGDE, 2022; Knaus, 2013).

2.5.2 Building a Strong Early Years Environment with All Key Stakeholders

In schools, teaching and learning are complex matters and all educational stakeholders need to build strong partnerships to ensure student success (Balch & Balch, 2019). Building a strong team and a positive climate in early childhood education paves the way for the school and its teachers to thrive, and a thriving school means a positive experience for families and children (Education Council, 2019). In early childhood education, collaboration is essential, not just between the school and families, but also among staff and school leadership (Balch & Balch 2019). The Alice Springs (Mparntwe) Education Declaration contended that it is essential for all members of the education community to work in partnership with parents, to support a child's progress through early learning and school (Education Council, 2019.) An educational team that works together harnesses the power of collaboration to confront and overcome challenges in systematic and sustainable ways (Balch & Balch, 2019). The Alice Springs (Mparntwe) Declaration has two distinct but interconnected goals:

> Goal 1: The Australian education system promotes excellence and equity
> Goal 2: All young Australians become:
> confident and creative individuals,
> successful lifelong learners, and
> active and informed members of the community.
> (Education Council, 2019, p. 4)

To achieve these two educational goals for young Australians, teachers and leaders are required to work in partnership (Education Council, 2019).

2.5.3 The Importance of School and Home Partnerships

The importance of a school-home partnership cannot be overstated, as it plays a crucial role in supporting children's holistic development and academic success (ADGE, 2022). Collaborative relationships between teachers and parents create a cohesive support system that reinforces learning and fosters positive outcomes for children. Research has consistently shown that parental involvement in education is linked to higher academic achievement, improved behaviour, and enhanced social-emotional well-being for students (Fan & Chen, 2001; Kim & Hill, 2015). Moreover, cultural responsiveness within these partnerships is essential for honouring the diverse backgrounds and identities of students and their families. When schools engage in culturally responsive teaching practices, such as acknowledging and valuing diverse perspectives, languages, and traditions, families feel more welcomed and included in their children's education (Jeynes, 2010; Lee & Bowen, 2006). By fostering a sense of belonging and cultural affirmation, schools can strengthen their relationships with families and create a more supportive and inclusive learning environment for all students.

Teachers' practices and the relationships they form with children and families have a significant effect on children's involvement and success in learning (AGDE, 2022). When early childhood teachers work in partnership with families, it is well documented that learning outcomes are more likely to be achieved (AGDE, 2022). As emphasised by the AGDE (2022), "Children thrive when they, their families, and their educators work together in partnership to support their learning, development, and wellbeing" (p. 9). By fostering a genuine partnership between teachers and families, the foundation for children's flourishing learning experiences is laid. The importance of valuing knowledge, building trust, empathy, and shared decision-making amongst teachers and families should be emphasised (AGDE, 2022). This collaborative approach supports children's understanding and well-being, emphasising the role of early childhood environments in fostering mathematical competence.

2.5.4 The Importance of Parents as the First Educators

According to the EYLF, children are connected to family, community, culture, and place from birth (AGDE, 2022). The EYLF further stated children's "earliest development and learning takes place through these relationships, particularly within families, who are children's first and most influential educators" (AGDE, 2022, p. 7). The Alice Springs (Mparntwe) Education Declaration (Education Council, 2019) supported this claim stating that the first and most important educational influence in a child's life is parents/carers. Parents/carers have a role that is critical in a child's early development, including social, emotional, intellectual, spiritual, and physical wellbeing. Children experience belonging, being and becoming in many ways, due to the diversity in family life (AGDE, 2022). Children "bring their diverse experiences, perspectives, expectations, knowledge and skills to their learning" (AGDE, 2022). The Alice Springs (Mparntwe) Education Declaration (Education Council, 2019) asserted that families infuse attitudes and values that aid young children to access and participate in education and training and contribute to local and global communities. It is vital that parents play an active role in their child's mathematical education (Attard, 2021; Martin et al., 2012). Parental support, such as assisting with homework, promotes student engagement in their work academically and this includes mathematics (Roche et al., 2021). As parents may suffer mathematics anxiety, it is essential that parents experience contemporary teaching practices that their child is experiencing at school (Attard, 2021).

2.5.5 Early Childhood Teachers and the Key Role They Provide in Laying the Foundations for Future Success

Early childhood teachers hold a vital role in laying the foundations for future success by providing young children with the necessary skills, knowledge, and dispositions for lifelong learning. Teachers’ work encompasses nurturing the holistic development of children, focusing on their physical, social, emotional, and cognitive domains (Ishimine et al., 2010). The early years of a child’s life are a time of rapid growth and development when children experience significant cognitive development and form their language, social, emotional, and physical skills (Education Council, 2019). “The key to children’s earliest learning and development is the quality and depth of interaction they experience; between adult and child and between child and child” (Education Council, 2019, p. 7). According to the Alice Springs (Mparntwe) Declaration it is during the early years that the foundation for learning throughout school and beyond is developed (Education Council, 2019).

Early childhood teachers play a significant role in laying the foundations for future success. However, Heckman (2018), suggested that the influence of teachers in early childhood education is overstated or limited compared to other factors. Factors such as family background, socioeconomic status, and community resources have a more significant impact on children's long-term outcomes than the specific practices of early childhood teachers. Educational attainment and life outcomes are strongly correlated with family income and parental education levels (Heckman, 2018). While early childhood teachers undoubtedly play a crucial role in children's development, Heckman's (2018) research emphasised the critical role of family background, socioeconomic status, and community resources in shaping children's outcomes. This suggests that these factors have a more substantial influence than the specific practices of early childhood teachers alone. The EYLF supported this assertion, affirming when respectful and caring relationships with children and families are established by teachers, all parties can work together to ensure learning experiences and the curriculum are contextually relevant to children (ADGE, 2022). In making professional judgments, teachers intentionally weave together:

- professional knowledge and skills
- contextual knowledge of each child, their families, and communities
- understanding that relationships with children and families are critical to creating safe and trusting spaces
- awareness of how their beliefs and values impact children’s learning and wellbeing
- knowledge and understanding of Aboriginal and Torre Strait Islander perspectives

- awareness of how their beliefs and values impact on children's learning
- use all components in the planning cycle

(AGDE, 2022, p.12)

These elements collectively shape teachers' professional judgments and guide their active role in facilitating children's learning within the EYLF. Early childhood teachers are tasked with creating welcoming environments where families and children are respected and actively encouraged to work alongside teachers about curriculum decisions to ensure learning experiences are meaningful (AGDE, 2022). By laying the groundwork, Early Childhood teachers support children's future academic achievement, social-emotional well-being, and overall success in life. As such, teachers are recognised as key stakeholders in this investigation.

2.5.6 The Importance of an Effective Leadership Team

A leader's personality, including their style of leadership and their disposition, influences the way their behaviour is evaluated and perceived by others (Waniganayake et al., 2012). Leadership is more about the actions of the individual as opposed to their work role (Waniganayake et al., 2012). Murphy (1994) stated that leaders do not lead from the apex of a pyramid but rather from the centre of human relationships. Murphy's research (1994) continues to resonate as the Education Council (2019) suggested that education leaders have the duty to establish and maintain high-quality learning environments. Their role encompasses creating conducive conditions for effective teaching and learning. This involves fostering a culture of elevated expectations, collaboration, and professional development.

Waniganayake et al. (2012), defined a leader as a person who inspires a group of people to achieve a common goal. Effective leadership plays a critical role in supporting and fostering quality teaching and learning (Education Council, 2019). Grootenboer (2018) claimed that in a school, leadership positions may be formal or informal and all of these are central to the effective functioning of a school. Early childhood leaders play a pivotal role in implementing quality educational programs (Waniganayake et al., 2012). Leaders can enhance their work within the school setting, through an awareness of the needs and interests of the children and their families in their school (Waniganayake et al., 2012). The focus of this research is to examine the relationship between the components of the Early Years School environment and the development of 21st Century mathematical skills in young learners.

2.6 Conclusion

Education is seen as the ticket to a brighter economic future (Holzapfel, 2018). How well children are educated, and whether they learn the skills needed to participate and thrive in the global economy, will determine the future health, wealth, and welfare of everyone (Trilling & Fadel, 2009). Mathematics is not only imperative as a core subject during schooling, but it is also a gatekeeper to successful participation in the workforce, especially with respect to high-prestige professions (Blömeke, et al., 2011). It is imperative to have policies and practices in place to ensure students are provided with ample opportunities to learn and develop 21st Century mathematical skills. At present there is a paucity of research around 21st Century skills for mathematics in education, particularly in the early years context as defined as children from birth to eight years old of age. A majority of the current research focuses on 21st Century skills for Technology, and 21st Century skills related to the Secondary school setting. It was from the literature review that the research question "how does the Early Years school environment prepare young children with regards to 21st Century mathematical skills?" emerged. This study investigated the teaching of mathematics in early childhood education in preparing young students with 21st Century skills in mathematics and how the elements of the Early Years environment aligned in supporting these.

Chapter Three: Research Plan

3.1 Introduction

The literature review suggests an integrated, step-by-step approach to early mathematics education, anchored in a constructivist approach and aligned with children's developmental stages is ideal. Further, the literature review highlighted the importance of hands-on experiences, visual aids, and symbolic understanding in building a robust mathematical foundation in early childhood. According to McDonough (2003) teachers do not merely deliver the curriculum, they define, develop, and interpret it. The depth of learning experienced by young people is profoundly influenced by teachers' beliefs, thoughts, and actions.

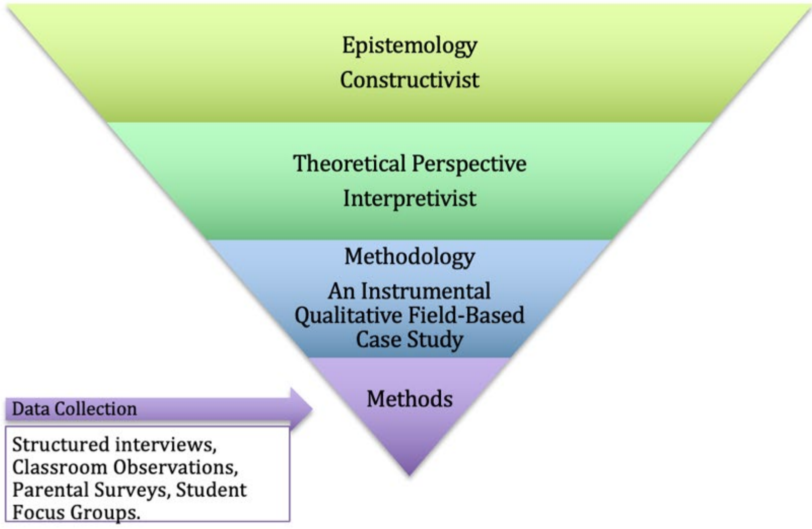
This chapter will identify the research epistemology pertinent to this study and explain and justify why this epistemological stance was adopted for this research. A description of the methodology and a rationale for the design choice will also be described. The design process will describe in detail the data collection methods, the participant selection, the analysis techniques adopted, and the necessary ethical considerations. In data analysis, quantifying holds significant importance, particularly in identifying patterns and deviations within data. This process is essential for making analytic or idiographic generalisations. Sandelowski (2001) emphasised the integral role of numbers within qualitative data, asserting that proficiency with numbers is crucial for conducting high-quality qualitative research. The research instruments used will be discussed, including a detailing of their background and development.

3.2 Theoretical Framework

Neuman and Neuman (2013) asserted that a theoretical framework is "a general theoretical system with assumptions, concepts and specific social theories" (p. 85). There are four crucial elements for the researcher to consider when constructing a theoretical framework for a study, and these inform each other: epistemology; theoretical perspective; methodology; and method (Crotty, 1998). When beginning the research process with epistemology; theoretical perspective; methodology; and methods, in mind, the researcher can justify the methodologies and methods employed in their research (Crotty, 1998). Following the elements outlined by Crotty (1998) allows a penetrating analysis of the research progression, brings attention to the theoretical assumptions that underpin it, and determines the status of the findings (Crotty, 1998). Figure 3.1 illustrates the theoretical framework applied to this research using the elements identified by Crotty (1998).

Figure 3.1

Overview of the Research Design



3.2.1 Epistemology

A constructivist epistemology was used to guide this study. Constructivism is the theory that learners create concepts that are relevant and meaningful, and build on existing knowledge. Epistemology is concerned with the creation of knowledge and how we know what we know (Neuman & Neuman, 2013). Epistemology focuses on identifying the most valid methods for discovering knowledge and understanding (Neuman & Neuman, 2013). A constructivist epistemology focuses on "the meaning-making activity of the individual mind" (Crotty, 1998, p. 58). It is the constructivist epistemology that guides the researcher in the choices of methods in ontologically and epistemologically fundamental ways (Guba & Lincoln, 1994). In constructivist epistemology, the researcher is an orchestrator and facilitator of the inquiry process (Guba & Lincoln, 1994).

Constructivism is more than just being actively involved and immersed in experiences, it is crucial that it is embedded in a social context, as learners construct new knowledge through social interactions (Vygotsky, 1986). A social constructivism position is employed in this research, further to the constructivist epistemological stance of learners making meaning through actively being involved in their experiences. Hedges (2000) stated that social constructivist theories are based on a complex model of teaching and learning in which child-initiated and teacher-initiated learning experiences and interactions are essential. Hedges

(2000) further claimed the constructivist view supports the idea that children be considered active re-organisers of their own experiences, and that the teacher also plays an essential role. As the intent of this study was to understand and interpret a particular social construct, a constructivist epistemology was considered highly suitable.

3.2.2 Theoretical Perspective

The theoretical perspective is the philosophical stance that informs the methodology (Crotty, 1998). The theoretical perspective considered for this research was interpretivism. Schwandt (1994) stated that interpretivism "was conceived in reaction to the effort to develop a natural science of the social" (p. 125). Interpretivism is socially constructed and embraces subjectivity, acknowledging multi-realities, and meanings (Rapley, 2018). Interpretivism argues that truth and knowledge are subjective, as well as culturally and historically situated, based on people's experiences and their understanding of them (Ryan, 2018). Researchers can never be separated from their own values and beliefs, so these will inevitably inform the way in which they collect, interpret, and analyse data (Ryan, 2018). According to Rapley (2018) the researcher is part of the world that is being studied and the data that is being collected. As this study was theoretically influenced by the work of Piaget's (1964) constructivism theory and Vygotsky's (1986) sociocultural theory, interpretivism aligned appropriately with the epistemological position taken by the researcher.

3.3 Methodology

Within the interpretivist perspective, qualitative methodologies are common practice. Qualitative research allows the researcher to provide an interpretation of the observed experiences and actions of individuals and groups in different contexts (Johnson & Rasulova, 2017). Qualitative data permits studies to be more inclusive, collaborative, and involve partnerships and co-production (Johnson & Rasulova, 2017). Therefore, the most appropriate methodology to investigate and examine the proposed research questions for this study was determined to be an instrumental qualitative field-based case study approach.

Lauckner et al. (2012) suggested, "that case studies explicitly seek out the multiple perspectives of those involved in the case, aiming to gather collectively agreed upon and diverse notions of what occurred" (p. 5). This research is building on pre-existing theories based on Clarke's (1997), Hill et al.'s (2008) and Knaus' (2017) work, all of which researched aspects of mathematics teaching and learning in the Early Years environment through case studies. Cresswell and Cresswell (2013) stated that case studies facilitate the use of multiple sources of evidence as a way of triangulating the evidence, on which the study is particularly focusing. Interpretive case studies contain rich, thick, descriptions that give a detailed account of the phenomenon under study (Merriam, 1988). These descriptions are used "to develop conceptual categories or to illustrate, support, or challenge theoretical assumptions held prior to the data gathering" (Merriam, 1988, pp. 27-28).

An instrumental qualitative field-based case study approach is a research methodology that involves in-depth exploration and analysis of a specific case or cases within their natural context or setting. In an instrumental case study, the focus is on understanding the case itself rather than using it as a representative example of a larger population (Guba & Lincoln, 1994). An instrumental qualitative field-based case study approach was chosen because the purposes of the study demanded an in-depth understanding of the situation and its meaning for those involved in an Early Years school setting. In this study, through the methods of semi-structured interviews, classroom observations, student focus groups, parental surveys, and Principal interview, the data were triangulated to highlight all participants' perceptions and knowledge of 21st Century mathematical skills.

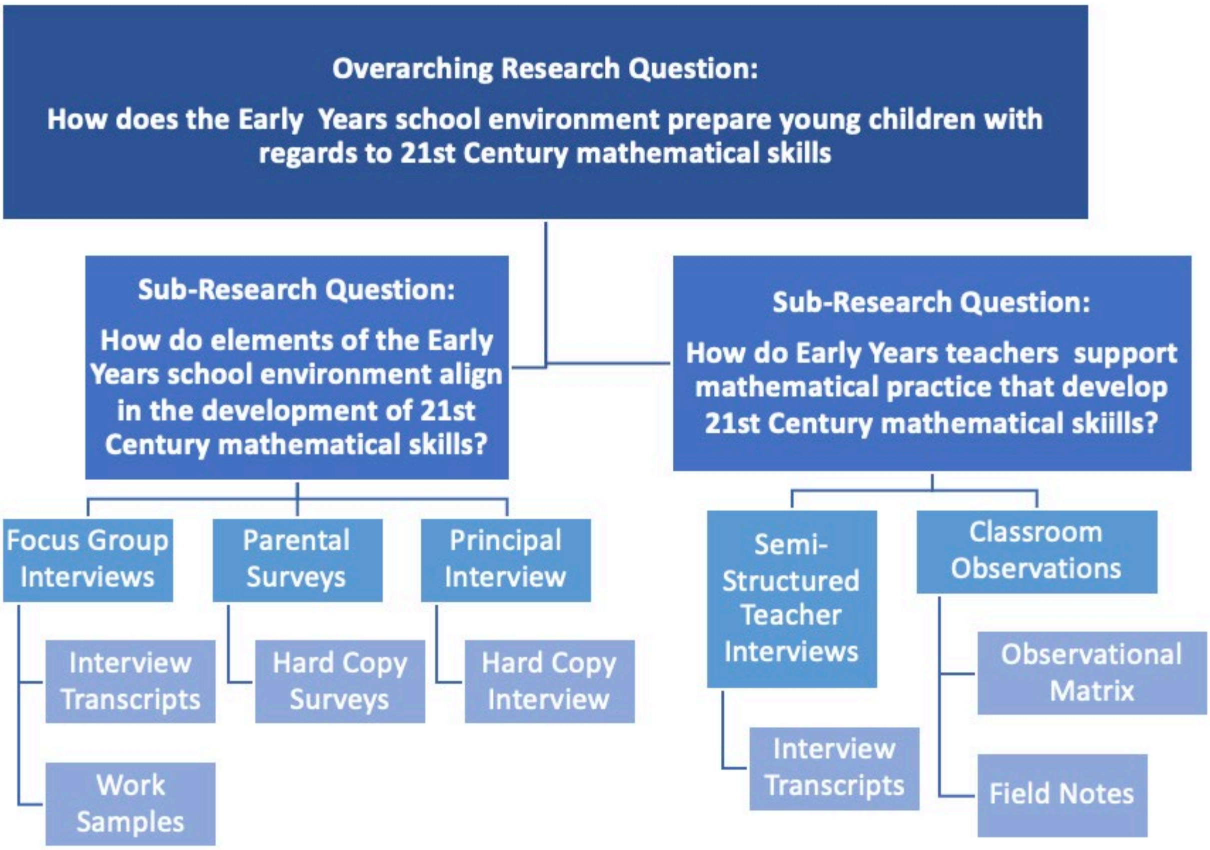
3.4 Methods

3.4.1 Framework

The framework underpinning the methods of data collection is outlined in Figure 3.2. This framework was developed to respond to all the research questions and was aimed at collecting rich, detailed data from all the stakeholders in the study.

Figure 3.2

Framework Underpinning the Data Collection

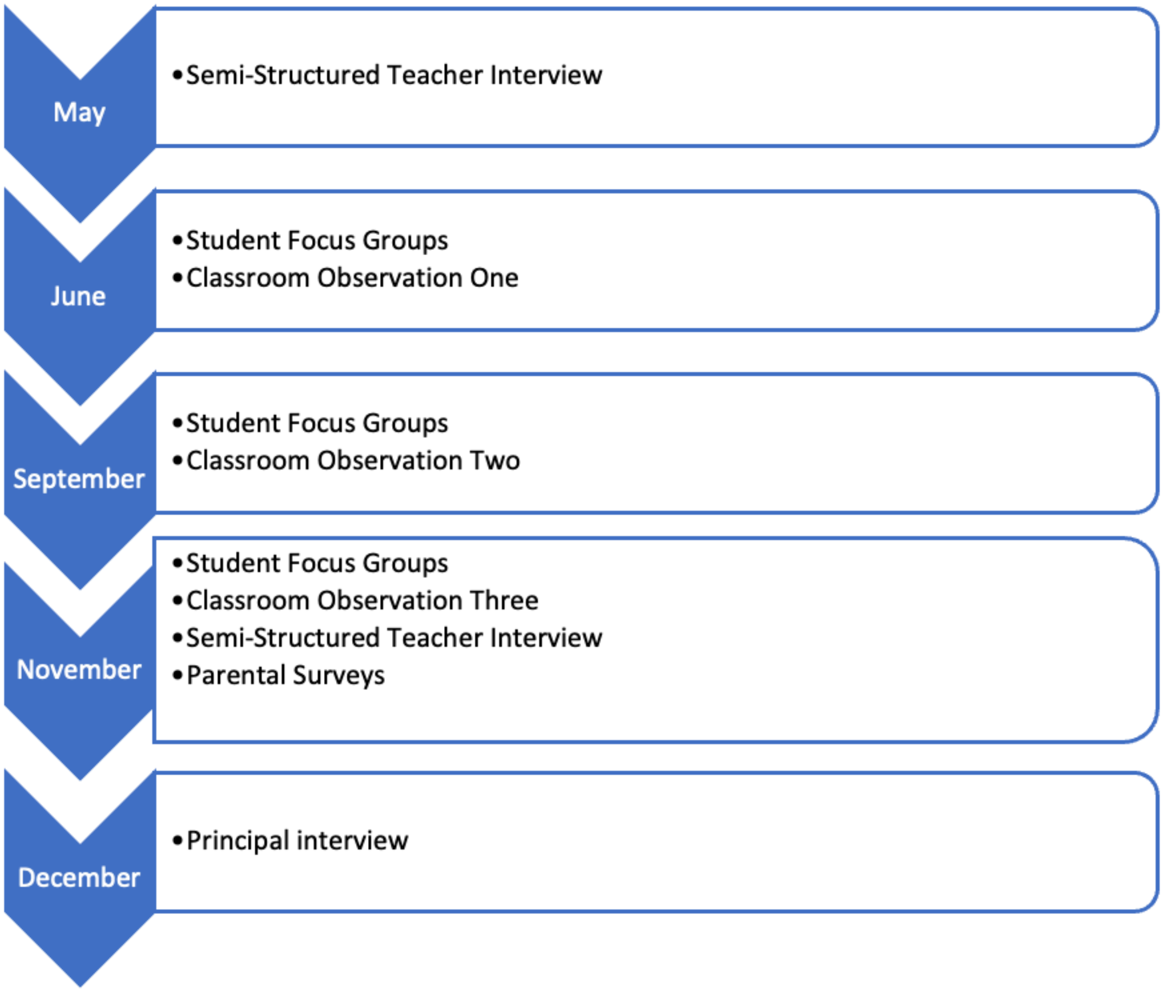


3.4.2 Data Collection Methods

Four types of data collection methods were used to respond to the research questions framing this investigation; interviews completed by the Early Years teacher and the Principal; classroom observations of one classroom, two sets of student focus group interviews, each conducted three times, with twelve students in total each time from the participating class, and seven parental surveys. This collection of data reinforces Creswell et al.’s (2013) assertion that data collection is usually voluminous in case studies, due to the numerous sources of information. The data collection for this study occurred over one school year, from February to December. The timeline below in Figure 3.3 illustrates the way in which the data were collected.

Figure 3.3

Data Collection Timeline



3.4.2.1 Semi-Structured Interviews

Semi-structured interviews are sufficiently structured to address the topics that are specific to the phenomenon of the study as well as leaving additional space for participants to offer new insights (Galletta & Cross, 2013). It was for this reason semi-structured interviews were chosen, as they allow the researcher to have a guide of the types of questions that are key to understanding the knowledge and perception of the participants in relation to 21st Century mathematical skills. Semi-structured interviews also allow the researcher to ask additional questions that may arise due to the participants' responses. Galletta and Cross (2013) stated that semi-structured interviews provide a repertoire of possibilities. This study intended to be open to all possibilities around the participants' perceptions and knowledge of 21st Century mathematical skills.

The selected Early Years teacher participated in two interviews. One interview occurred at the beginning of the data collection period and the second one occurred at the end of the data collection period. The Principal also participated in an interview, at the end of the data collection year. Interviews were chosen as a collection tool as they can provide reliable, comparable, qualitative data and allow the participants the freedom to express their views in their own terms (Galletta & Cross, 2013). To facilitate this, the researcher developed and used a paper-based set of interview questions. The interview was a list of questions and topics that needed to be covered during the conversation, in a particular order. The researcher followed the list of questions but when it was deemed appropriate, was able to follow topical trajectories in the conversation that may have strayed from the guide. The semi-structured interviews included questions based on the participants’ perceptions and knowledge of 21st Century learning and the type of 21st Century mathematical skills occurring in the class and the school. The interview that was completed at the end of the data collection period with the Early Years teacher also included questions that focused on whether the teacher’s perceptions and awareness of 21st Century skills had changed and if the frequency of the mathematical experiences for the students had aligned more with 21st Century skills. These data are outlined in a matrix provided in Appendix A.

The first semi-structured interview with the Early Years teacher was audio-recorded and later transcribed for analysis. The second Early Years teacher interview and the single Principal interview were intended for the end of the data collection year; however, this time of the year was an extremely busy time of year and timetabling the interviews proved to be problematic. Therefore, on the request of both participants, the interviews were sent in paper form and answers were recorded in written form by the participants and then returned to the researcher. The interviews were constructed with the intent of guarding against leading questions and as a control for researcher bias.

The interview questions were created by the researcher and were based on a National Science Technology Engineering and Mathematics [STEM] Research Project, Principals as STEM Leaders [PASL] (Falloon et al., 2021), in which both one of the Supervisors and Researcher had previously been involved. The PASL questionnaire provided the researcher in the current project with a foundation of the key questions to ask the participants and the researcher adapted the PASL questions to suit the context of this study.

3.4.2.2 Student Focus Groups

Focus groups are a useful method that allows for the generation of meaning not only with individuals but with a group of individuals (Galletta & Cross, 2013). Focus groups provide multiple angles of vision and assist with clarifying and amplifying meaning (Galletta & Cross, 2013). Two student focus groups met three times; at the beginning, midpoint and at the end of the data collection period. Each focus group consisted of six students (a total of 12 students each time) from the participating Early Years classroom. In an informal group setting on school grounds, the students were asked questions about their perceptions of their classroom mathematics learning and experiences. The participating students were selected at random for each focus group. This occurred as it was less disruptive for the teacher and it allowed for different combinations of students, and therefore different group dynamics. The focus group questions, like the teacher interviews, were semi-structured in nature. The semi-structured format allowed the researcher to have a guide of the types of questions to ask to ensure an in-depth perception and knowledge of the students' understanding of mathematics. This format allowed the researcher to ask additional questions raised through student responses.

3.4.2.3 Classroom Observations

Qualitative observations are a form of data collection that allow 'in the field' research (Galletta & Cross, 2013). The researcher observed three mathematics lessons conducted by the participating Early Years teacher at spaced intervals, one in June, another in August, and the final one in November. Each of the classroom observations was the duration of the dedicated mathematics lesson, approximately 45 minutes to 60 minutes in length. The classroom observations were used to build a picture of the Early Years teacher's 21st Century pedagogies. The 21st Century Skills Matrix Research Instrument (Appendix A) was used as the tool for recording the observed 21st Century skills. This tool was adapted from two data sources, firstly the Partnership for 21st Century learning framework (Battelle for Kids, 2019), and secondly the conceptual framework 'Components of the Role of the Teacher Using Innovative Approaches to Teaching and Learning and Related Beliefs' developed by Clarke in his 1993 case study. The researcher used a combination of both sources to create the studies matrix. This matrix encompassed a series of questions that suited the current study and consisted of key information from the two previously stated studies. The researcher also created field notes from each of the observed lessons to allow for a richer data set and to assist with data triangulation.

3.4.2.4 Parental Survey

Surveys are especially useful in acquiring data on respondents' perceptions, attitudes, and beliefs that drive their behaviour (Spekle & Widener, 2018). Parental surveys were chosen as a method of data collection, with the intent to investigate the parents' perception of 21st Century skills and mathematical skills needed for 2030. The parental surveys provided an additional data collection method, of the perceptions of the Early Years school environment, allowing for an in-depth perception through all key stakeholders (Principal, parents, teachers, and students). All of the parents of the participating students in the Early Years class were invited to participate in the survey. The researcher, via the Early Years teacher, sent home a letter attached to the parental survey explaining the purpose of the research. Seven out of the twelve parents responded to the survey.

3.5 Research Participants

The participating primary school was a small (120 enrolments, inner city, mid-range socio-economic) independent primary school. The school identified a need for research into 21st Century mathematical skills, which was the impetus for including them in the investigation. One Early Years teacher was willing to participate in the study and was contacted via email after arrangement through the school Principal. The selected Early Years teacher gave consent via a consent form that contained a letter outlining the research aims and intent of the study. The participant had been teaching for more than ten years and at the time of the study was teaching a Pre-primary class.

All twelve students from the participating Early Years teacher's classroom, aged five to six years, were invited to partake in student focus groups and classroom observations. Each student received an information sheet (Appendix E) detailing participation, along with a consent form (Appendix D) featuring a happy/sad face to indicate assent or dissent. Additionally, at the start of each focus group session, students were asked if they were still willing to participate, ensuring ongoing consent. All twelve of the students' parents were also invited to participate in a parental survey. All participants were invited to participate through the selected Early Years teacher via consent forms. The Early Years teacher at the beginning of the data collection year sent home parental consent forms (Appendix B), child classroom observation consent forms (Appendix C) and child focus groups consent forms (Appendix D). Attached to these consent forms was a letter detailing information about the intended study and its aims (Appendix E). In Term Four of the data collection year paper copies of parental surveys (Appendix F) were sent home. The parents had four weeks to complete the surveys and return them to the participating Early Years teacher. The researcher collected the return paper copies at the last focus group visit.

3.6 Data Analysis

The research data analysis utilised a thematic approach, concentrating on the identification, analysis, and interpretation of patterns within the qualitative data. To ensure a coherent analysis, the researcher aligned the theoretical perspective and overarching research design with the data generation methods. According to Flick (2013), this alignment is crucial for preparing the data for analysis, which Bailey (2018) described as a quest for meaning within the data.

Each dataset underwent a meticulous process involving transcription, coding, and individual analysis, followed by triangulation to discern key themes and similarities. This methodological choice, deeply rooted in the theoretical framework and comprehensive research design, directed the analytical process. The methods employed were specifically chosen to capture the subtleties and intricacies inherent in the qualitative field-based case study research design, which allows for an extensive examination of the themes present in each dataset.

Triangulation was pivotal in establishing a solid foundation for the interpretation and analysis of the data, which is at the heart of qualitative research. Flick (2013) emphasised that without interpretation, researchers are unable to derive meaning from their data. Therefore, qualitative researchers are tasked with rendering the data meaningful through a deliberate process of interpretation. The subsequent sections will detail the analytical procedures applied to each dataset.

3.6.1 Semi-Structured Teacher Interviews

The process of asking questions of interviewees and interpreting answers is the beginning of the data analysis phase (Flick, 2013). The interviews were designed around four key category themes; Pedagogy, Planning, Classroom Environment, and Teacher Comfort. These key category themes were based on a National Science Technology Engineering and Mathematics [STEM] Research Project, Principals as STEM Leaders [PASL] (Falloon et al., 2021). In alignment with the chosen theoretical perspective and research design, the analysis process involving the questioning of interviewees and interpretation of their responses serves as a crucial step in uncovering insights related to the four key category themes. The pre and post interviews were combined for analysis and were thematically coded. Bailey (2018) stated coding is the process, initially of reading a significant amount of raw data. It is then assigning a code to most of the lines of text, by grouping the data based on relevant characteristics, and eliminating the irrelevant text, until the remaining portions are organised to be useful for generating analytic insights (Bailey, 2018).

In this study each key category theme was broken into sub-themes. The researcher identified the sub-themes by breaking each question into a key idea; if any questions had the same key ideas, these were grouped together. The Early Years teacher's statement about each sub-theme was then distilled into key points. The two interviews were then compared by the researcher to see if there were any changes in the key points about each theme from the pre to post interviews. An example is shown in Figure 3.4, where a similar question has been asked pre and post interview and the similarity in response has been highlighted. These results are shown in Table 4.1 in Chapter Four.

Figure 3.4

Teacher Interview Analysis Sample

Pre Teacher-Semi-Structured Interview Questions (Transcribed) *Question 2: Can you tell me how you structure your mathematics lessons?*	**Post Teacher Semi-Structured Interview** *Question 2. Has your structure of your mathematics lesson changed over the year? If so, why? And how?*
Response: Um, so it depends on the year group that I have. So, um in the past if I have had a really energetic group then I have had a quick mat section. Introduce a new concept and we will do with my EA two activities and then swap over. So about 20 minutes 15/20 minutes each activity. I have started having a little warm up on the mat. A quick game- something like I might write the number five on the whiteboard, and they have to prove to me that the number is five. Or a quick card game that I teach them, and we do the same warm up every session for a week. Following the warm, we sit in a circle, and I introduce the new concept. Then we go off and do an activity, either two separate activities and swap over or everybody does the same activity. Then we come back and reflect on the activity. Um, and that is it.	Response: Not really – either two small groups and swap over activities or we all work on the same activity together.

3.6.2 Student Focus Groups

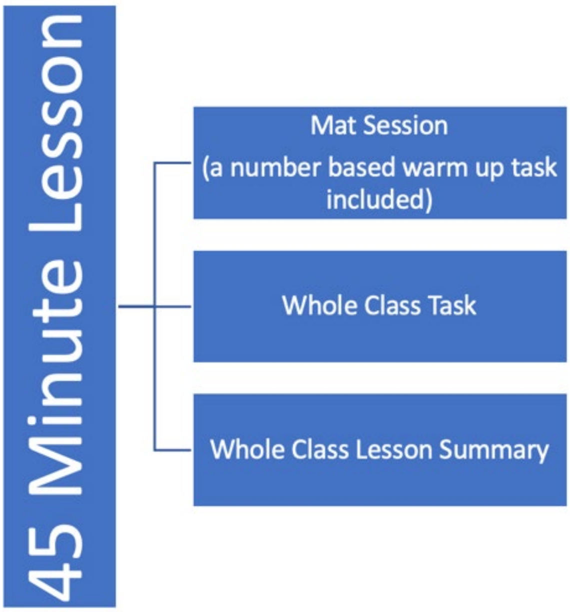
The participating year group had three student focus group interviews. The focus group interviews were recorded and transcribed at a later date, and work samples were collected. The transcripts were compared by question and the researcher looked for key themes. The results of this coding process are presented in Table 4.5. The work samples were also compared to look for key themes and changes in the drawings from observation one to observation two.

3.6.3 Classroom Observations

During the initial interview with the Early Years teacher, it was disclosed that the mathematics lessons for the chosen research class were structured in three phases, as indicated in Figure 3.5. This information assisted the researcher to develop a classroom observation matrix which is provided in Appendix G. The researcher used the code, MS for Mat Session, T for Task, and S for Summary. One of the three classroom observations had an additional component prior to the lesson summary, the Early Years teacher conducted a closure mat session, which included an additional task, therefore on the Matrix for this observed lesson the researcher used the code ST for Summary Task.

Figure 3.5

Mathematical Lesson Structure



Each of the lesson observations was coded using the Observation Matrix, provided in Appendix G. The coding process was undertaken by identifying the four 21st Century skills of Communication, Critical Thinking and Problem-Solving, Mathematical Creativity and Innovation, and Collaboration and noting these using the coding key alongside each occurrence. A sample of the coding is provided in Figure 3.6 and a full matrix sample is provided in Appendix A.

Figure 3.6

Matrix Coding Sample

Practice	**1. Communication**	
Related belief	Communication of mathematical ideas help students clarify and solidify their understanding of mathematics	Lots of questions by teacher during the task.
Evidence	Students participate in rich mathematical conversations **(4)**	MS√ T√ ST√ S√
	The teacher facilitates a balance of oral and written mathematical communication **(3)**	T√ ST√ S√
	The teacher facilitates dialogue of mathematical argumentation **(4)**	MS√ T√ ST√ S√

In addition to using the matrix, field notes were taken by the researcher while observing the classes. These field notes were recorded under the lesson structure components; mat session, whole class, lesson summary and were coded using the key themes; mathematical lesson focus, open questions, closed questions, student focused, teacher focused. Each of the key themes was highlighted, a sample is provided in Figure 3.7. The field notes were then organised under each key theme to allow the researcher to analyse if there were any similarities or key findings across the three observed lessons, as shown in the results chapter in Table 4.4.

Figure: 3.7

A Sample of Field Notes and Coding

Lesson 1:
Mathematical Focus: Statistics and Probability

Key:

Open questions
Teacher focused
Student focused

Part 1 of the lesson: Mat Session
Number & Algebra Focus
The teacher wrote 2+3=5 on the board
The teacher gave out post-it notes for the students to record their responses,
The teacher asked the students to prove if the equation was correct by either drawing or writing

Teacher gave time for all students to complete the post-it notes (wait time)

Introduction to main task
Teacher asked the students what does Probability mean?
The teacher gave examples of each term;

Impossible: it is impossible that an elephant will walk through the front door
Unlikely: means probably, not likely, what is something that is unlikely?
Likely: means should happen
Certain: what does certain mean? Definitely going to happen

Lots of conversations, listening and questioning from the teacher

Part 2 of the lesson: Whole Class Task
The teacher asked the students to draw at least two pictures, one for each.
Extending prompt: a challenge you can do 4 if you like.
They got to draw using texta's on a pink card.
Free play after the task.

Students were seated at a table as a group, but it was an individual task.

Table 1: five students and an Educational Assistant.
Table 2: four students and a practicum student.
The teacher roams the room.

Part 3 of the lesson: Summary
The teacher asked let's look at the poster and they had a discussion about it. The discussion was student led

3.6.4 Parental Surveys

All twelve of the selected class's parents/caregivers were sent a paper copy of a five-question survey. Out of those twelve parents/caregivers seven completed and returned their surveys. The surveys were then combined for analysis and for each of the five questions common themes were established and the frequency of each theme was recorded, the results are provided in Table 4.7.

3.6.5 Principal Interview

The selected school's principal was interviewed at the end of the data collection year. The original intention was that the semi-structured interview would be conducted in person. However, due to calendar constraints the interview was emailed to the Principal and returned via email to the researcher. The interview was correlated into two categories, 21st Century mathematical skills at the school, and mathematics teaching at the school. Each category was then broken into three key areas, Leadership, School, and teachers and the responses were then constructed into key responses.

3.6.6 *Triangulation of Data*

The concept of triangulation suggests that examining a subject from various perspectives enhances the precision and validity of the research findings (Neuman & Neuman, 2013). This multifaceted approach helps to corroborate the data, ensuring that the conclusions drawn are more reliable and reflective of the true nature of the research subject. Notably, this study employed triangulation of data by gathering and analysing perspectives from parents, a school principal, teachers, and students. Social research builds on the principle that we learn more by observing from multiple perspectives than by looking from only one single perspective (Neuman & Neuman, 2013). Tornero (2017) supported this claim stating that multiple approaches and various types of data, allow for a more complete picture of the phenomenon under study. For this reason, five types of data collection were used (interviews, surveys, focus group interviews, observations, and field notes). This study also collected data from multiple perspectives to ensure multiple views and to increase reliability and validity. The data collected from the semi-structured teacher interviews and the classroom observations provided multiple perspectives in relation to the research question, how do Early Years teachers support mathematical practice that develops 21st Century mathematical skills? The data collected from the student focus groups, the parental surveys and the Principal interview provided multiple data sets to the research question, how do the elements of the Early Years school environment align in the development of 21st Century mathematical skills?

3.7 Ethical Considerations

Trustworthiness is often defined as the degree of confidence in the data, interpretation, and methods used to ensure the quality of a study. Judgments made through adopting an interpretivist standpoint can be scrutinised for trustworthiness through the lenses of credibility, transferability, confirmability, and dependability (Guba & Lincoln, 1994). These criteria collectively form a robust framework for assessing the quality and integrity of qualitative research, ensuring that findings are meaningful, applicable, unbiased, and reliable. By applying these criteria, researchers can enhance the trustworthiness and validity of their interpretive analyses, contributing to a more rigorous and credible body of qualitative research (Nazar et al., 2022). A major component of eliminating research bias in qualitative research is trustworthiness of the data. A lengthy engagement with the participant was established to allow for trust to emerge.

This study endeavoured to adopt the standpoint of trustworthiness, through ensuring credibility was met through the methodology using a triangulation method of data collection. Credibility is established when the researcher has confidence in the truth of the findings with regards to the subjects of research and the context where it was conducted (Johnson & Rasulova, 2017). Credibility assesses whether the findings accurately represent plausible information derived from participants' original data and correctly interpret their viewpoints (Nazar et al., 2022). Additionally, credibility is likened to internal validity in quantitative research, as both aim to establish the truthfulness of the research outcomes (Nazar et al., 2022).

Transferability, as stated by Johnson and Rasulova (2017), based on Guba and Lincoln’s criteria, is maintained when the researcher can provide detailed descriptive information that allows the reader to transfer meaning to his/her context. Transferability is the ability to transfer findings of a case study to other settings and the fact that this study is based on previous studies and builds on existing theories will allow for this. Extensive field notes and a detailed literature review will also contribute to the transferability.

According to Johnson and Rasulova (2017), “confirmability is about ensuring that the research process and findings are not biased; hence, it refers to both the researcher and the interpretation” (p. 268). Finally, consistency in the entire research process is the key for achieving dependability (Johnson & Rasulova, 2017). This research is reflective, and this allows for an overall view with interpretations that establish new insights, establishing confirmability of the study.

Dependability in qualitative research refers to the degree of trustworthiness and reliability across various aspects, including methodological design choices, sampling methods, data generation processes, and data analysis techniques (Nazar et al., 2022). Johnson and Rasulova (2017) wrote that “dependability is about being able to trace sources that the data comes from and about documenting the data, methods and decisions made during the fieldwork” (p. 268). An audit trail has been included in the study design to allow for dependability.

Researchers (Guba & Lincoln, 2007; Johnson & Rasulova, 2017) argued that the criteria for rigour around impactful research arises from a positivist paradigm, that truth is valid for a certain time and certain context. Guba and Lincoln (2007) further stated that these criteria for rigour are an incomplete set for application to interpretivism. To ensure trustworthiness using interpretivism the criteria should include the process of inquiry, informing the reader about what has and has not happened before, during, and after the inquiry (Guba & Lincoln, 2007). To address this issue Guba and Lincoln (2007) proposed the term authenticity to distinguish the revised principles from positivistic ones. Guba and Lincoln's five element criteria are based on the constructivist tradition that truth is a matter of consensus among constructs that are sophisticated and informed, subjective meanings matter, and that knowledge is relative to time and place (Johnson & Rasulova, 2017). Guba and Lincoln (2007) wrote that authenticity enables questions to be asked about how the interpretations are made and how the process has evolved, therefore authenticity is an extension of the trustworthiness criteria. In this study, external members, the researchers' supervisors were asked to critique the analysis process and review and discuss the coding process to allow for authenticity to be addressed.

Ethical research involving humans is concerned with ensuring that ethical values and principles always govern research (Habibis, 2006). The research proposal was deemed as being ethically acceptable when submitted to the Human Research Ethics Committee [HREC] of the University of Notre Dame Australia, Fremantle (reference 019169F). The researcher ensured that all participants (Principal, Early Years teacher, parents, and students) received structured and age-appropriate information sheets and informed consent was obtained. These information and consent forms were made available in a format that suited each of the stakeholders. Regarding students, the researcher sought their consent after the parents/carers had provided their consent for their children to be involved. In addition, the researcher was mindful of the fact that ethical procedures needed to be culturally and contextually sensitive.

Confidentiality was maintained as neither the teacher, nor the school, were identified. Pseudonyms were used throughout the transcribed interviews for the Principal, the teacher and the students. The researcher ensured that the observations and focus groups were conducted in a discrete manner by removing the students into a vacant room. All students from the participating Early Years classroom agreed to participate in all three of the classroom observations. Should any student choose not to join the focus groups, their participation was not mandatory. Confidentiality will continue to be addressed, as all data will be securely stored as required in the ethics application.

3.8 Conclusion

This chapter has presented the research rationale detailing the methodology and theoretical perspective underpinning the study. The selection of a constructivism epistemology, the theoretical perspective of interpretivism and the methodology of an instrumental qualitative field-based case study were justified in the chapter as appropriate in response to the research questions. The research participants were outlined, and the ethical considerations of the research process were assured. The data collection methods process of analysis were then detailed in the chapter. The following chapter presents the results from the analysed data sets.

Chapter Four: Presentation of Research Results

4.1 Introduction

The aim of this study was to gain a deeper understanding of young children's 21st Century mathematical skills and the role of the Early Childhood environment in developing these skills. To respond to the research aim, data were collected using teacher interviews, classroom observations that included an observational matrix and field notes, student focus groups that included student work samples, parental surveys, and a principal interview. Following collection, each data set was analysed by the researcher who identified key themes. This chapter presents the findings from each data set, namely; semi-structured teacher interviews, classroom observations, student focus groups, parental survey, and the principal interview.

4.2 Teacher Interviews

Two interviews were compared to see if there were any changes in the key points about each theme from the pre to post interviews, the results are shown in Table 4.1.

Table 4.1

Teacher Interviews

Category Themes	Key Points	Pre/Post Interview	
Pedagogy			
Lesson structure	Warm up-always a number focus. Mat session-explicit teaching. Two groups-hands on, practical application. Whole class summary	Same over the year	
Perception and belief of 21st Century Learning	Collaboration, social skills, communication skills are vital and must be taught and modelled through all areas of learning.	Perceived Change	More mindful, more aware about bringing 21st Century learning into the math's lessons
4Cs in mathematics learning	Problem solving math's problems in creative ways, working as a team and critically assessing things	Same over the year	
Planning			
21st century skills in mathematics instruction	Modelling creative and different ways of solving problems or showing mathematical thinking. Critically assessing answers – using different sources, cross checking references, double checking answers	Same over the year	

Assessing 21st century skills	21st century skills are not formally Assessed	Same over the year
Classroom Environment		
Mathematical Materials	Concrete materials/manipulatives. Art into maths through drawing, painting and creating Sensory materials like playdough, chalk, whiteboards etc. for practising numbers Made resources such as number lines, number cards. The children make their own resources	Same over the year
Grouping	Mostly group tasks Depending on the task groups are selected by ability needs or just whoever works well together	Same over the year
Aspects which enhance or limit the success of mathematics teaching	Limit: Access to technology The noise can be disruptive Enhance: Opportunities for mixed-age learning where the Year Ones buddy up with the PPs and teach them concepts	Same over the year
Teacher Comfort		
Teacher Enjoyment	Enjoys teaching mathematics	Same over the year
Teacher comfort	Comfortable, although extending children working well beyond PP level tricky	Same over the year
Parental Relationship	It's good. They're really supportive	Same over the year

The data collected from pre and post teacher interviews (As presented in Table 4.1) revealed insights into the pedagogy, classroom environment, and teacher comfort in teaching mathematics, and integrating 21st Century skills. The integration of 21st Century skills into mathematics teaching was a key focus, with the teacher becoming more mindful of this aspect over the year. Despite some challenges, the teacher's enjoyment of teaching mathematics, and the supportive parental relationship contributed to a positive teaching and learning environment.

In terms of pedagogy (as presented in Table 4,1), the lesson structure remained consistent throughout the year, starting with a warm-up focused on numbers, followed by explicit teaching during mat sessions, hands-on practical applications in two groups, and concluding with a whole-class summary. The teacher's perception and belief in the importance of 21st Century learning did not change, with a continuous emphasis on the necessity of collaboration, social skills, and communication skills being taught and modelled across all learning areas. However, the teacher did become more mindful and aware of incorporating 21st Century skills into their mathematics lessons. The 4Cs (communication, collaboration, critical thinking, and creativity) were consistently integrated into mathematics learning, with a focus on problem-solving in creative ways, teamwork, and critical assessment. Planning and assessment of 21st Century skills in mathematics instruction also remained unchanged, with the teacher modelling creative problem-solving and critical assessment. As Table 4.1 illustrates, whilst the teacher modelled skills of problem-solving and critical assessment, these were not formally assessed.

Regarding the classroom environment, the use of mathematical materials such as concrete manipulatives remained the same over the year. These materials helped students in practising numbers and creating their own resources. Grouping of students continued to be task-dependent, either by ability needs or compatibility. The lessons primarily involved group tasks. Factors enhancing or limiting the success of mathematics teaching were also consistent. Limited access to technology and disruptive noise were noted as limiting factors, while mixed-age learning opportunities, where Year Ones buddied up with Pre-Primary students, were seen as enhancing the learning experience.

The teacher's enjoyment and comfort in teaching mathematics remained unchanged throughout the year indicating a stable and positive attitude towards mathematics. The relationship with parents was also consistently supportive, though extending students beyond the Pre-Primary level presented some challenges.

Overall, the analysis highlighted a stable teaching environment with a consistent pedagogical approach, use of resources, and grouping strategies. The teacher demonstrated increased mindfulness of incorporating 21st Century skills into mathematics lessons, although formal assessment of these skills was not evident. Enhancements such as mixed-age learning continued to positively impact the teaching experience, despite some limitations like technology access and noise disruption.

4.3 Classroom Observations

The three classroom observations were combined and coded. The coding process was undertaken by identifying the four 21st Century skills of Communication, Critical Thinking and Problem-Solving, Mathematical Creativity and Innovation, and Collaboration and noting these alongside each occurrence. The use of numbers in qualitative research is common when it complements other forms of data analysis. Maxwell (2010) suggested using numbers to determine frequency is legitimate in qualitative methodologies. In this research, each of the 21st Century skills was broken into observable sub-themes and the frequency of each was recorded as a number in the observations, as indicated in Table 4.2. If a sub-theme appeared multiple times in a single observation or across different observations, each occurrence was counted individually. The total frequency reflects the sum of these occurrences, irrespective of how many students demonstrated the sub-theme at any one time.

Table 4.2

Frequency of the 21st Century Mathematical Skills and Sub-themes Observed During Classroom Observations

Communication	Observation One	Observation Two	Observation Three	Total Frequency
Students participate in rich mathematical conversations	3	3	4	10
The teacher facilitates a balance of oral and written mathematical communication	3	3	3	9
The teacher facilitates dialogue of mathematical argumentation	3	3	4	10
Critical Thinking & Problem Solving	Observation One	Observation Two	Observation Three	
The teacher promotes and challenges students to think and reason	3	3	4	10
The teacher encourages students to reflect on their thinking and learning	3	3	2	8
Tasks engage students and maintain their involvement	3	3	4	10
The teacher promotes a positive disposition	3	3	4	10
Open ended tasks, the teacher enables different possibilities, strategies, and products to emerge	1	1	2	4
Closed tasks	0	2	0	2
Students are encouraged to persevere	1	3	4	8

Students discuss mathematical reasoning	3	3	4	10
Students use trial and error	1	3	2	6
Mathematical Creativity and Innovation				
	Observation One	Observation Two	Observation Three	
The teacher facilitates opportunities to imagine, and mental imaginary encouraged	3	1	3	7
The teacher provides a place and space to work creatively	3	3	4	10
Students try a variety of heuristics	2	3	0	5
The teacher builds a connection to other domains and subject areas.	2	2	0	4
Students create a large range of ideas for solutions	1	2	4	7
Collaboration				
	Observation One	Observation Two	Observation Three	
Students complete joint tasks	1	2	0	3
Students work independently	0	1	2	3
The teacher questions the students	3	3	4	10
Students question the teacher	3	3	4	10
Students question their peers	3	3	2	8

The data presented in Table 4.2 reflects the frequency of various 21st Century mathematical skills and sub-themes observed during the classroom observations.

Communication:

- Rich Mathematical Conversations: The consistent frequency across observations (3, 3, 4) indicates a stable environment for student participation in discussions, with a slight increase in the third observation.
- Balance of Oral and Written Communication: The uniformity (3, 3, 3) suggests that the teacher consistently facilitates both oral and written communication.
- Mathematical Argumentation: Similar to rich conversations, there is an increase in the third observation, showing progressive engagement in argumentation.

Critical Thinking & Problem Solving:

- Promoting Challenge and Reasoning: The frequency pattern (3, 3, 4) mirrors that of rich conversations, indicating a parallel emphasis on critical thinking.
- Reflecting on Thinking and Learning: A slight decrease in the third observation (3, 3, 2) may suggest a need for more encouragement in self-reflection.
- Engagement in Tasks: The pattern (3, 3, 4) shows that tasks are effectively engaging students, with increased involvement in the final observation.
- Positive Disposition: The consistent increase (3, 3, 4) demonstrates the teacher's success in fostering a positive attitude towards mathematics.
- Open vs. Closed Tasks: The higher frequency of open-ended tasks (1, 1, 2) compared to closed tasks (0, 2, 0) indicates a preference for tasks that allow for multiple strategies and solutions.
- Perseverance and Trial and Error: These sub-themes show variability, with perseverance showing a significant increase in the third observation (1, 3, 4), suggesting that students are being encouraged to persist despite challenges.

Mathematical Creativity and Innovation:

- Imagine and Mental Imagery: The frequency (3, 1, 3) suggest that the teacher provides ample opportunities for students to imagine and create mental images with a notable dip in the second observation for imagination.
- Creative Place and Space: (3, 3, 4) for this sub-theme suggest that the teacher provides ample opportunities for creativity,

Collaboration:

- Joint Tasks and Independent Work: Both sub-themes have low frequencies (1, 2, 0) for group tasks and independent tasks (0, 1, 2), indicating that there may be room for improvement in promoting collaborative and independent work.
- Questioning: High frequencies (3, 3, 4) for questioning by the teacher, students to question the teacher, and among peers show a strong culture of inquiry in the classroom.

The overall trend suggests a strong emphasis on communication, critical thinking, and problem-solving, with a particular focus on fostering a positive disposition towards mathematics. While there is evidence of creativity and innovation, opportunities for students to engage in heuristic approaches and for the teacher to connect to other subject areas and domains were less frequent. The high level of questioning indicates an interactive and reflective learning environment. The slight increase in several sub-themes during the third observation could point to a cumulative effect of the teaching strategies employed. However, the data also highlight areas such as self-reflection and group work that could benefit from additional focus to further enhance the development of 21st Century skills in the mathematical domain. Out of the twenty-two sub-themes, nine of the sub-themes have the highest frequency of 10. The top nine sub-themes are provided in Table 4.3.

Table 4.3

The Sub-themes of the Observed 21st Century Mathematical Skills with the Highest Frequency

Sub-Theme	Total Frequency
Students participate in rich mathematical conversations	10
The teacher facilitates dialogue of mathematical argumentation	10
The teacher promotes and challenges students to think and reason	10
Tasks engage students and maintain their involvement	10
The teacher promotes a positive disposition	10
Students discuss mathematical reasoning	10
The teacher provides a place and space to work creatively	10
The teacher questions the students	10
The students question the teacher	10

During the classroom observations in the moment field notes were collected and analysed. Each of the observational field notes was highlighted under the themes; open questions, teacher focused, and student focused. These results were then simplified into dot points and combined under each theme. The results are provided in Table 4.4.

Table 4.4

Field Notes

Mathematical lesson focused	**Lesson 1 Statistics and Probability**	**Lesson 2 Measurement Time**	**Lesson 3 Measurement Length**
	Exploration and drawing of images related to probability	**On a blank clock putting favourite time and a picture to match**	**Exploring and measuring different animal pictures**
Open questions	• Prove it, draw and write. • Teacher asked the students what does Probability mean?	• You want to be strategic. What does strategic mean? • Does it matter who wins?	• Is this where we start? Why? • What is a rodent? Lots of discussion. • Where do I start? Why? lots of non-examples. • How do I measure the tiger properly? What happens if I left gaps? • Where do you start at the tip, the very end and where would I finish? • Is it accurate, is it going to give me an accurate measure?
Teacher focused	• Teacher gave time for all students to complete the post-it notes (wait time) • The teacher gave examples of each term; • Impossible: it is impossible that an elephant will walk through the front door. • Unlikely: means probably, not likely, what is something that is unlikely? • Likely: means should happen. • Certain: what does certain mean?	• Counters if you need. • Extension cards, 8,9 and 10. • Enabling cards 1,2,3,4,5 and 6. • Teacher handed out clocks. • Can you make 1 o'clock? You don't need to call out, just hold up your clock. • Remember the minute hand on the 12. • Put your hand up if you can see the card that says 1 o'clock. • Show me 4 o'clock.	• Story 'Big and Little'. • I was so proud; everyone was trying so hard to measure tip to tip with no gaps. • Teacher used sophisticated language during questioning. • We don't have the real animals and they are not life size, but remember we did this before and we had some trouble with measuring accurately? • If we are measuring how long is he?

	Definitely going to happen.	• **Has anyone seen a horseshoe before?** • **Rock, paper, scissors for who goes first?** • **Player 1 chooses a card and writes that number on their whiteboard.**	
Student focused	• **Post-it notes to record responses.** • **Lots of conversations, listening and questioning from the teacher.** • **Draw at least two pictures, you can do 4 if you like. One for each, at least 2.** • **They got to draw using textas on a pink card.** • **Free play after the task.** • **At a table as a group, but an individual task. Table 1 five students and an Educational Assistant.** • **Table 2 four students and a practicum student. Teacher roams the room.** • **Look at the poster and have a discussion, discussion was student led.**	• **Go to your own private space.** • **Counting on 23-9, knew how to count on but not how to write the number.** • **A horseshoe game with partners.** • **Circle self-check.** • **Limited instructions.** • **On the blank clock write your favourite time and draw a picture to match, then paint.**	• **Talk to partner.** • **Task in partners.** • **With clipboards exploring length-very unstructured in terms of instructions.**

The results from Table 4.4 show that lesson three had the greatest number of open questions. Lesson two had the most teacher focused points, and lesson one had the most student focused points. It is important to note that although all three lessons had a significant number of teacher focused points, most of these points focussed on vocabulary, introducing the mathematical topic or providing scaffolding prompts. In conclusion, the lessons were well-structured with clear content foci and employed diverse pedagogical strategies to cater to different learning styles and objectives. The teacher's role was pivotal in guiding the students through the learning process, while the students' active participation indicates a high level of engagement and interest in the subject matter. Outlined below are three observed mathematics lessons, each with a distinct content focus and pedagogical approach, aimed at fostering an engaging and effective learning environment.

Lesson 1: Statistics and Probability

- Content Focus: This lesson introduces students to the concept of probability through exploration and image drawing.
- Pedagogical Approach: The teacher employs open-ended questions to encourage critical thinking and understanding of probability. The use of examples to explain terms like "impossible," "unlikely," "likely," and "certain" helps students grasp abstract concepts in a concrete way.

Lesson 2: Measurement Time

- Content Focus: Students learn to measure time by setting clocks to represent different times.
- Pedagogical Approach: The lesson uses hands-on activities with clocks and counters to reinforce the learning of time measurement. The teacher's strategic questioning and the use of games like "rock, paper, scissors" add an element of fun and competition.

Lesson 3: Measurement Length

- Content Focus: The focus is on measuring length, as students explore and measure animal pictures.
- Pedagogical Approach: The lesson emphasises accuracy in measurement, with the teacher encouraging students to measure "tip to tip with no gaps". The use of real-world examples, like measuring a tiger, makes the lesson relatable and engaging.

In the observed lessons, student engagement was consistently high, as evidenced by their involvement in various activities such as drawing, playing games, and engaging in discussions. The teacher played a crucial role in supporting this engagement by providing ample wait time for responses, using examples for clarity, and employing sophisticated language to extend students’ thinking. The learning environment was notably supportive, characterised by the presence of an educational assistant and a practicum student, which ensured individual attention for the students.

Regarding the content and pedagogical focus, each lesson had a distinct aim. The first lesson was designed to foster a conceptual understanding of probability, encouraging students to engage in discussion and express their understanding creatively. The second lesson shifted the focus to procedural knowledge, with students practising the setting of time on clocks, thereby reinforcing their skills in measuring time. The third lesson was centred on the practical skill of accurate measurement, highlighting the significance and application in real-world contexts. Overall, the lessons were structured to address different learning objectives and styles, ensuring a comprehensive educational experience.

4.4 Student Focus Groups

During the first focus group sessions the students seemed to be unsure of the term mathematics and were reluctant to answer some of the questions. Students’ responses to the question “what do you think of when I say the word maths?” consisted of “I don’t know” or “I am not sure” or “ummm I think it means, I don’t even know”. When asked, “what is the best thing about maths at school?”, one response was “you learn your letters and it is really important” another response was “you learn to write”. One student also proceeded to reply to the questions with “poo” and “poo and wee”, which led to the focus group getting very distracted. Therefore, for the second and third focus group meetings the researcher asked the students to draw a picture of them learning maths at school. The questions asked over the three student focused groups have been recorded in Table 4.5, as well as the student responses and how many times those responses were noted. The two focus groups for each session have been combined as the students in each group changed each session.

Table 4.5

Student Focus Group Questions

Questions	Session 1	Session 2	Session 3
	Focus Group Session (This includes both randomly selected focus groups.)		
1. What is the best thing about maths in this school?	• Number charades: 1 • Making Stuff: 1	• Charades: 1 • Learning to read and write: 1	
2. Do you do maths lessons in groups, with your friends or mainly alone?	• Groups: 6 • By yourself: 1	• By myself: 1 • Groups: 2 • Alone: 1	• By yourself: 1 • Groups: 1 • Both: 1
3. What sorts of things do you do in your maths lessons?	• We made a volcano: 1 • Writing: 1 • Poo: 1 • Poo and Wee: 1 • Learning how to do new numbers: 1 • Learning your letters: 1 • Not sure: 2 • Learn things: 1 • Writing numbers: 1 • Colours: 1	• Counting: 2 • I don't know: 1 • Numbers: 1 • Writing teen numbers: 1	• Time: 2 • Counting: 2 • Shapes:1 • Multiplication: 1
4. Do you have a favourite game/task/activity? What is it?	• Number charades: 1 • Poo and Wee: 1 • No: 2 • Pokemon:1 • Number game: 1	• I don't know: 1 • Lego Game: 2 • Pokémon:1	• Pokémon Cards: 1
5. Do you think your teacher enjoys teaching maths?	• Yes: 5	• Not answered	• Yes: 1
6. Who helps you learn maths?	• Teacher: 3 • Mum and dad when it is literacy: 1 • Myself: 1 • My mum: 1	• No-one: 1 • The teacher: 1	• My mum: 2 • My dad: 2 • The Teacher: 2
7. What do you do if you are stuck/you don't know what to do in the maths lesson?	• Ask the teacher: 2 • I just work, I don't get stuck: 1	• You try and then ask a friend: 1 • You start and then you keep going: 1	• Ask the teacher: 2
8. Do you think you learn maths best with others or by yourself?	• Others: 3 • By yourself: 2 • Both: 1	• Others: 1	• Not answered

9.	Do you use lots of materials when you are learning maths at school? If so what sorts of materials?	• Not answered	• Not answered	• Not really: 1
10.	Do you like doing activities that are hard or easy in maths at school?	• Easy: 3 • Easy and hard: 3 • Hard: 4	• Pretty hard: 1 • Hard: 1 • None, I don't like doing maths	• Hard: 3 • Easy: 4
11.	Do you do any maths things at home? If so, what sorts of things do you do?	• Yes, play: 3 • I do: 1 • Yes, times: 1 • Games: 2 • Counting game: 1 • Pokémon: 1 • No: 1	• Yes, Reading Eggs • Yes, I write numbers: 1 • I don't know: 1 • No: 2 • Yes, counting	• Yes, counting: 2 • Yes, I have a math's book: 1 • Yes, I have a number chart • I draw, shapes

The results in Table 4.5 indicated that students enjoyed interactive and game-like activities in mathematics, such as number charades, which were the most appreciated aspects of their mathematics classes. Both group and individual work were common, with a variety of activities that included creative projects and basic skills development. The teacher was seen as enjoying teaching mathematics and was the main source of help, supplemented by parental support. Students showed a preference for either group work or individual study, and when faced with difficulties, they usually sought help from the teacher. The use of materials in learning mathematics was not clearly documented, and students had mixed feelings about whether they preferred easy or challenging activities. At home, mathematics-related activities often involved games and counting exercises.

The data indicated that some students might not have had a clear understanding of what constituted mathematics, as seen in responses that were unrelated to the subject. This could have been due to a variety of factors, including the age of the students, the way mathematics was presented in the curriculum, or the methods used to teach it. However, over time, there was a noticeable improvement in the students' understanding of mathematics, as later sessions showed more specific references to mathematical concepts such as time, counting, multiplication, and money. Initially, students' work samples depicted general activities, but later samples focused on numbers and equations, indicating a deeper engagement with mathematical concepts (Figure 4.1).

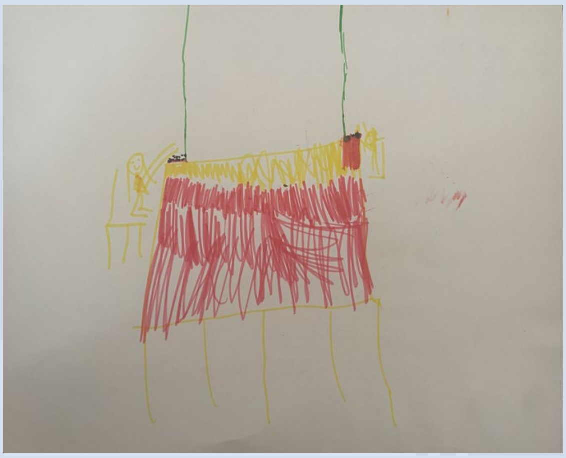
Despite this progress, students' preferences regarding the difficulty level of mathematics activities did not significantly shift towards challenging tasks. This finding stood out given the increased complexity observed in their work samples. Responses to mathematics activities at home remained consistent across all sessions. Taking these insights into account, it can be stated that even though students' understanding of mathematics was progressively improving, their preference regarding the subject's difficulty and their participation in mathematics-related activities outside of school showed little to no change. The analysis suggested the importance of interactive learning and the roles of teachers and parents in supporting students' mathematical development.

Figure 4.1

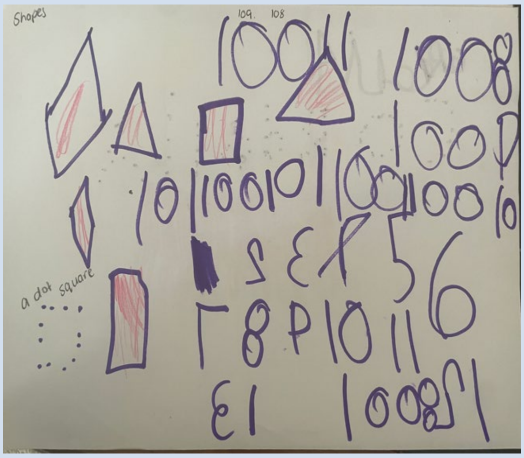
Student Work Samples

Work Sample 1

Session 2

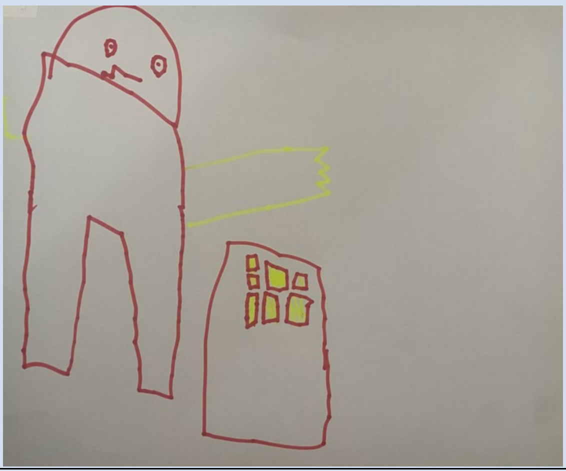


Session 3



Work Sample 2

Session 2



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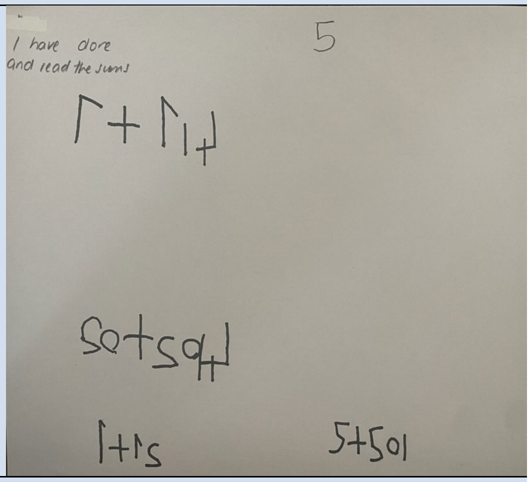
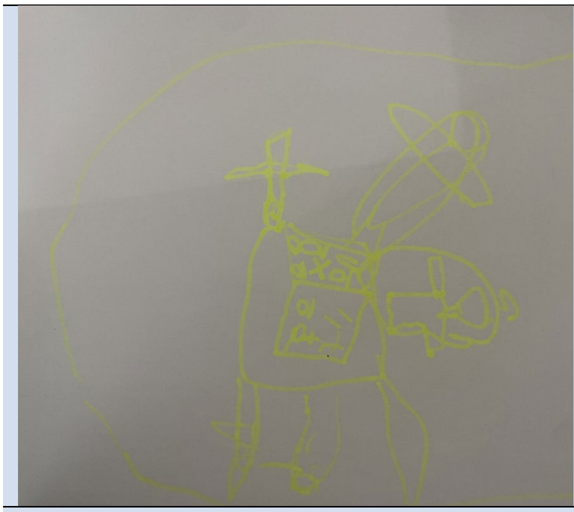
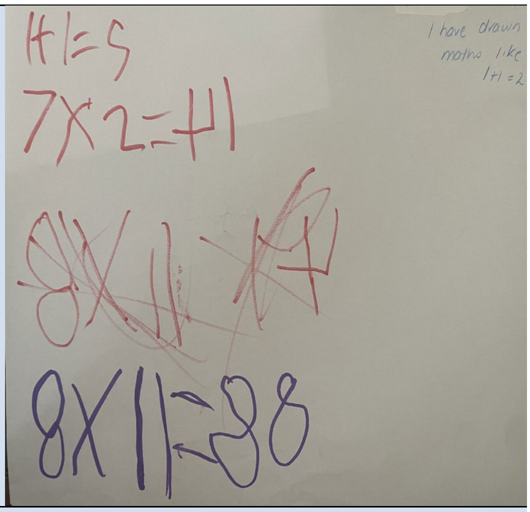


Work Sample 3

Session 2

Session 3

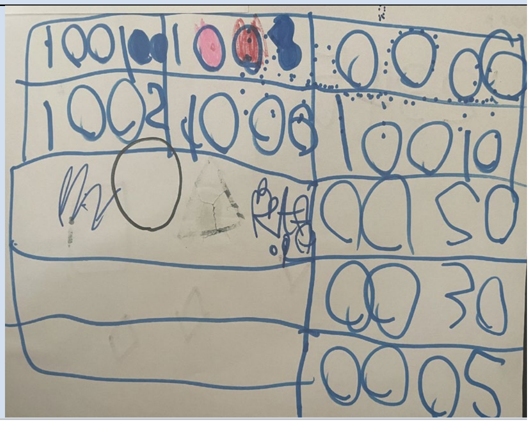




Work Sample 4

Session 2

Session 3



4.5 Parental Surveys

All twelve of the selected class’s parents/caregivers were sent a paper copy of a five-question survey. Out of those twelve parents/caregivers seven completed and returned their surveys. The surveys were then combined for analysis and for each of the five questions common themes were established, indicated in Table 4.6.

Table 4.6

5 Focus Questions and the Key Themes from Parental Survey

Question Focus	Yes, Response n=7	Key Theme from the Responses
Do you play mathematical activities and games at home?	6	Counting: 4 Cooking: 3 Board games: 2 Water play: 1 Imaginary play: 1
What 21st Century skills in general do you believe your child will need for living and working 2023 and beyond?	7	Empathy & compassion: 3 Specific maths skills: 3 People skills: 3 Creativity & Innovation: 3 Research skills: 2 Relationship skills: 2 Technology skills: 2 Adaptability: 2 Problem solving skills: 1
What mathematics do you believe your child will need for living and working 2030 and beyond?	7	Calculations skills/formulas: 6 Rational numbers: 2 Same as above: 2 Impossible to know: 1 Problem solving skills: 1
What in the program at school helps your child develop mathematics for their future?	5	Patience & risk taking: 2 Play based: 2 Calculus and complex quadratic equations: 2 Curiosity and innovation: 1
What would you like to see included in the program at this school to help your child develop mathematics for their future beyond school?	7	Nothing additional to what the school is currently doing: 3 Greater emphasis on theoretical maths: 2 Practical: 1 Money related activities: 1

The survey results from Table 4.6 reveal that a significant majority of parents, specifically six out of seven, actively engage in mathematical activities with their children at home. These activities were distinctly hands-on and play-based, which was evident from the variety of games and tasks mentioned. The activities mentioned were counting, cooking, board games, water play, and imaginary play.

- Counting, the most frequently cited activity, is a foundational skill and was noted by four parents.
- Cooking, mentioned by three parents, is another practical activity that naturally incorporates mathematics through measuring ingredients and understanding proportions.

- Board games, which two parents noted, involve numeracy skills, strategic planning, and probability, making them both engaging and educational.
- Water play and imaginary play (play experiences), each mentioned by one parent, also contributes to a child's mathematical development by introducing concepts such as volume and abstract problem-solving.

These findings suggested that the parents at home blend mathematics learning with play, thereby fostering an environment where children can see the relevance of mathematics in everyday life and enjoy the process of learning.

Inspection from the results in Table 4.6, show that parents tend to believe that higher order, 21st Century skills in general are important for 2030 and beyond. Regarding specific mathematical skills for 2030 and beyond, the parents noted that very traditional and specific skills are needed. This suggests that they believe there is a significant difference between the general skills their child will need for the future and the mathematical skills their child will need for the future. Further, when asked what skills help their child to learn mathematics now at school the highest results were patience and risk taking, suggesting a focus on higher order skills. One parent asked, "what in the program at this school helps your child develop the mathematics for their future beyond school?" stated "a focus on play, experimentation, and curiosity. The ability to make mistakes without judgment". Both the parents and the students stated that at home the mathematical activities consisted of counting, games and play experiences. These similarities are shown in Table 4.7.

Table 4.7

Mathematics at Home, the Responses from Students and Parents

Parents	**Student**
Counting n:4 Cooking n:3 Board games n:3 Play experiences n:2	Counting n:4 Times Tables n:1 Games n:3 Play experiences n:3 Number chart n:1 Writing numbers n:3

4.6 Principal Interview

The key themes and responses that emerged through the Principal's interview have been summarised in Table 4.8.

Table 4.8

Principal's Interview Results

21st Century Mathematical Skills at the School	
Leadership- The Principal	
Key Theme	Key Responses
Beliefs and understanding	• 21st Century Skills important across the whole curriculum • The belief that 21st Century skills are critical life skills that prepare students for their future
Environment-The School	
Key Theme	Key Responses
Reforms to address the key issues	• Standardised assessments have been introduced • Allocation of an annual budget for Professional Learning, ICT and concrete resources • Individual Education Plans for students at risk
Key focusses at the school moving forward	• Professional Learning for staff • Introduce a targeted support program for mathematics • Use a standardised assessment to its full capacity
Educators-The Early Childhood Teachers	
Key Theme	Key Responses
Planning	• In the overall planning and their pedagogies
Assessment	• No formal assessing of 21st century skills • Assess learning behaviours, some are 21st century skills
Mathematics Teaching at the school	
Leadership-The Principal	
Key Theme	Key Responses
	• The belief that there is a misunderstanding in the community that mathematics learning is not creative, and that is a solitary pursuit with right or wrong answers
Environment-The School	
Key Theme	Key Responses
Barriers	• Not always possible to provide the individual support to a child in a busy classroom • The ICT is not always reliable
Affordances	• We have a strong belief in 'hands on learning' and have the concrete resources and consumables to support this • We work as a team to try and support students that need support or extension • We arrange where possible for students to have access to the mathematics curriculum above their

	year grade if they require subject acceleration in mathematics • The collaborative environment supports students working together and supporting each other • Students are encouraged to seek support and ask questions • Students are supported and encouraged to 'have a go' to have a positive mindset and understand that making mistakes is part of learning • The inquiry based learning pedagogy at our school supports the development of 21st century skills
Policies	• School Improvement Plan • Curriculum plan • Assessment And Reporting Plan • Learning Area Statements for Mathematics
Teachers-The Early Childhood Teachers	
Key Theme	Key Responses
Teacher collaboration	• They discuss this with each other and at times ask for advice from others • We encourage the staff strong in this learning area to share their insights • The ECE teachers work with a play-based and inquiry learning pedagogy which is very much about teaching the 21st century skills to students • I am confident our teachers understand the importance of teaching 21st century skills

The results in Table 4.8 show that the Principal believes that 21st Century mathematical skills are important and are critical life skills that prepare students for their future. The Principal stated in the interview "learning 21st Century skills prepare students for their future in a fast-paced changing world. These critical life skills support students to be critical and creative-thinkers, communicators, and collaborators". The Principal further stated in the interview "the Early Childhood teachers work with a play-based and inquiry learning pedagogy, which is very much about teaching the 21st Century skills to students". The Principal further concluded in the interview "I am confident our teachers understand the importance of teaching 21st Century skills".

The findings in Table 4.8 highlight a contrast between the Principal's belief and the parental community's practices regarding mathematics learning. The Principal expressed, "I believe that there is still a misunderstanding in the community that mathematical learning is not creative, and that it is a solitary pursuit with answers that are either right or wrong." This statement reflects a perception that mathematical learning lacks creativity and is an individual endeavour focused solely on correct or incorrect answers. However, the parental engagement in hands-on, play-based mathematical activities at home, as shown in the survey results, suggests that parents are fostering a more dynamic and interactive approach to mathematics learning with their children. However, when the parents were asked "What mathematics do you believe your child will need for living and working 2030 and beyond?", the highest response from parents was calculations, skills, and formulas shown in Table 4.6. When asked "what would you like to see included in the program at this school to help your child develop the mathematics for their future beyond school?" one parent stated, "a greater emphasis on developing the theoretical maths concepts".

The Principal's perspective differed, emphasising the need for mathematics education to be recognised in the broader community as a means of imparting 21st Century skills to students. The Principal concluded the interview stating that "I would like to see mathematics learning acknowledged within the wider community as teaching students many of the 21st Century skills". This underscores the claim that while the parents tended to prioritise traditional mathematical skills such as calculations and formulas, the Principal advocated for a more holistic approach that includes 21st Century skills.

4.7 Conclusion

The aim of this study was to explore young children's 21st Century mathematical skills and the role of the Early Childhood environment in fostering these skills. Once analysed, the findings, drawn from teacher interviews, classroom observations, student focus groups, parental surveys, and a principal interview, reveal several key themes. The findings demonstrate the Principal and the teacher emphasised the importance of 21st Century mathematical skills, describing them as critical life skills that prepare students for a rapidly changing world. The Principal noted a potential community misunderstanding about the nature of mathematical learning, viewing it as non-creative and solitary. However, parental practices at home contradicted this perception, demonstrating a dynamic and interactive approach to mathematics learning. The results also highlight a contrast between the Principal's belief in the importance of 21st Century skills and some community perceptions of mathematics learning. Despite this, parental engagement in dynamic, hands-on mathematical activities at home suggests a supportive environment for children's mathematical development.

The study's findings suggest a strong emphasis on fostering a positive disposition towards mathematics through communication, critical thinking, and problem-solving in the classroom. There is evidence of creativity and innovation, although opportunities for heuristic approaches could be more consistent. The high level of questioning in the classroom indicates an interactive and reflective learning environment. However, areas such as self-reflection and group work could benefit from additional focus to further enhance the development of 21st Century skills. As this chapter presented the findings as they emerged from the data analysis process, The following chapter discussed will discuss these findings within the context of the literature and to address the research questions.

Chapter Five: Discussion of Results

5.1 Introduction

This research aimed to explore how the Early Years environment prepares students with the 21st Century mathematical skills of critical thinking, problem-solving, innovation, collaboration, creativity, and communication. This chapter discusses the findings presented in Chapter Four and examines them in relation to the literature presented in Chapter Two. The analytical discussion will respond to the overarching research question and sub-questions that have framed this investigation:

Overarching research question

How does the Early Years school environment prepare young children with regards to 21st Century mathematical skills?

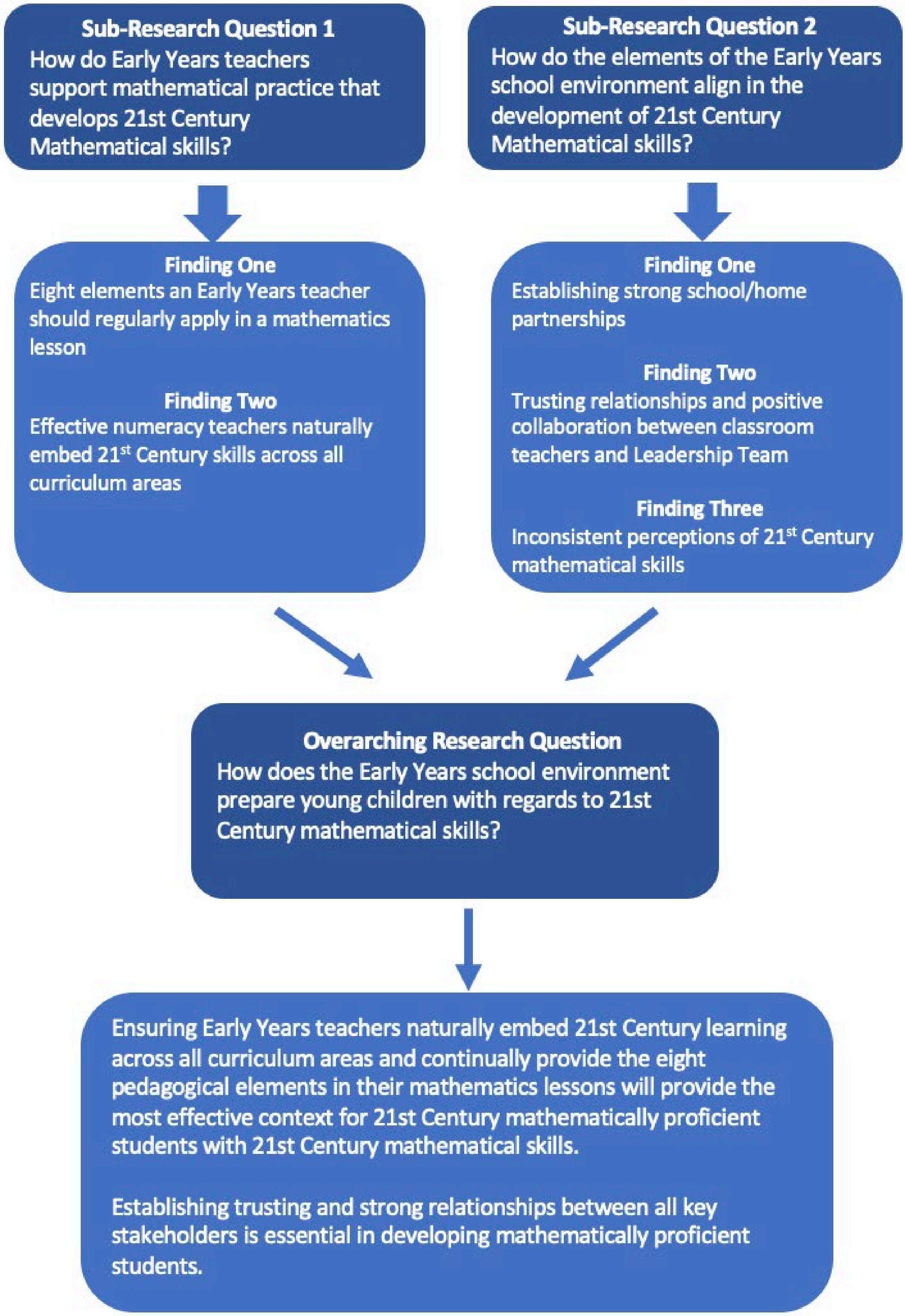
Research Sub-Questions

- How do Early Years teachers support mathematical practice that develop 21st Century mathematical skills?
- How do the elements of the Early Years school environment align in the development of 21st Century mathematical skills?

As previously stated, the impetus for the research was a recognised need by the participant school to consider Early Years teacher perspectives of 21st Century mathematical skills. The findings are based on the data obtained from one teacher only and this means that the results cannot be generalised. The aim was to investigate links between those perspectives of the teacher and the early years environment as well as the role of all key stakeholders in developing 21st Century mathematical skills in the students. In this chapter, five findings are discussed in response to the two research sub-questions that emerged from the review of literature. The structure of the discussion is shown in Figure 5.1.

Figure 5.1

Structure of Discussion



5.2 How do Early Years Teachers Support Mathematical Practice that Develops 21st Century Mathematical Skills?

Chapter Four outlined the key findings that emerged during the data analysis process in response to the first research sub-question: How do Early Years teachers support mathematical practice that develops 21st Century mathematical skills? Two key findings have emerged for discussion: Eight elements an Early Years teacher should regularly apply in a mathematics lesson and Effective numeracy teachers naturally embed 21st Century skills across all curriculum areas. These two findings are discussed in the following section 5.2.1 and 5.2.2.

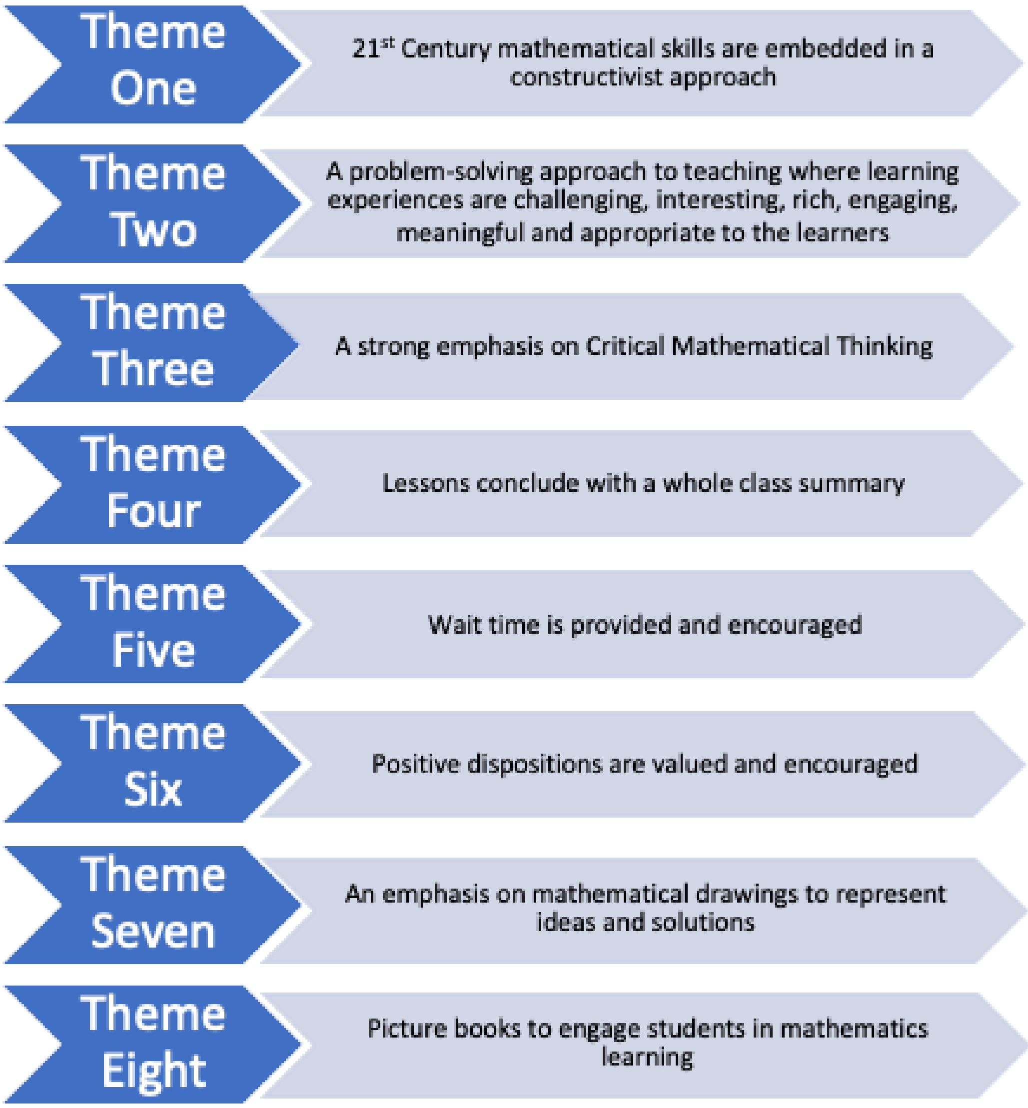
5.2.1 Eight Elements an Early Years Teacher Should Regularly Apply in a Mathematics Lesson

Early Years teachers play a crucial role in laying the foundation for mathematical skills that align with the demands of the 21st Century. The review of literature presented in this chapter together with the findings suggest that among various influencing factors, teachers have the most substantial impact. Furthermore, the Early Years teacher is identified as having the most profound influence (Clements et al., 2023). So, early mathematics teachers need to use the most effective pedagogical strategies (Clements et al., 2023).

The results suggest that certain key practices should be regularly applied within a mathematics lesson to support the development of proficient students with 21st Century mathematical skills. These suggested practices are discussed within eight elements that emerged from this research and that are supported by the contributions of several researchers (Clarke, 2004; Mallamaci, 2018; Monteleone et al., 2023; Sullivan 2012). The eight elements are illustrated in Figure 5.2. The research suggests the regular inclusion of certain components in mathematics lessons, by the Early Years teacher, can potentially foster a conductive environment for learning. This approach may help students develop mathematical abilities that could be beneficial for their future.

Figure 5.2

Eight Elements an Early Years Teacher Should Regularly Apply in a Mathematics Lesson to Support the Development of Proficient Students with 21st Century Mathematical Skills



5.2.1.1 Element One: 21st Century Mathematical Skills Are Embedded in A Constructivist Approach

This research found that Early Years teachers embedding a constructivist approach to teaching in their mathematics lesson is a key component in developing mathematically proficient students. O'Shea and Leavy (2013) supported that a constructivist approach emphasises inquiry in mathematics, as learning in a constructivist environment requires active engagement, inquiry, problem-solving, and collaboration with others. A constructivist approach is a powerful teaching philosophy (Mallamaci, 2018) which allows students to engage with mathematical ideas in meaningful contexts, making the learning experiences more relevant and applicable to their daily lives.

Student-centered learning and the constructivist approach share a fundamental philosophy that places learners at the centre of the educational experience, actively engaging them in the process of constructing their own knowledge. In essence, the student-centered approach aligns seamlessly with the constructivist perspective, creating a symbiotic relationship that enhances the depth and effectiveness of the learning experiences (Mallamaci, 2018).

A constructivist approach to teaching was evident in all three observed lessons. These lessons focused on the students being at the centre of the learning process. By providing a constructivist approach the Early Years teacher afforded students the opportunity to create their own meaning, rather than the 'traditional' methods of repetition and rote learning being present (Mallamaci, 2018). The findings presented in Chapter Four (Table 4.4) support the teacher's commitment to prioritising student-centered approaches in their teaching instruction. This emphasis on a student-centred approach was notably observed during the first classroom observation, where eight key points were identified in the student focused section of the observational checklist. All the observed lessons identified student-centered approaches, therefore validating that the Early Years teacher embedded and valued the constructivist teaching philosophy in their classroom (See Chapter Four, Section 4.3 *Classroom Observations*). These observations highlight the teacher's emphasis on students constructing mathematical understanding through hands-on exploration allowed the learners to be actively involved in the process of constructing their own knowledge.

Integrating 21st Century mathematical skills into a constructivist approach in the Early Years enhances the quality of learning experiences, promotes a deeper understanding of mathematical concepts, and equips young students with the skills needed for success in the 21st Century world (Bell, 2016). Integrating 21st Century mathematical skills through a constructivist approach in Early Years education is an efficient strategy for equipping young learners for the challenges of 2030 and beyond. A constructivist approach seeks to nurture a profound comprehension of mathematical concepts and equip students with the capabilities needed to navigate the complexities of the contemporary world (Tornero, 2017).

5.2.1.2 Element Two: A Problem-Solving Approach to Teaching Where Learning Experiences Are Challenging, Interesting, Rich, Engaging, Meaningful and Appropriate to The Learners

This research revealed that while a constructivist approach remains crucial in Early Years mathematics instruction, its implementation must extend beyond mere student involvement. Findings illustrated that learning experiences need to be challenging, interesting, rich, engaging, meaningful, and appropriate to the learners (See Chapter Four, Section 4.3 *Classroom Observations*). It is imperative to provide students with both individual and collaborative learning opportunities that are not only challenging but also intriguing, rich, engaging, meaningful, and age appropriate as recommended by Monteleone et al. (2023). Research has shown that teaching mathematics from an early age through a problem-solving approach is more powerful than teaching mathematical content first and then introducing problem-solving methods and techniques (Hattie, 2023).

The Early Years teacher in this research frequently provided tasks that engaged the young students. These tasks that challenged and maintained the students' involvement, as well as open-ended tasks, that enabled a range of problem-solving strategies and multiple solutions to emerge, are shown in Table 4.2. The Early Years teacher in all three classroom observations frequently promoted and challenged the young students to think and reason. The Early Years teacher presenting open-ended tasks to the students facilitated the emergence of various possibilities, strategies, and products to emerge.

In Table 4.5 students stated that they preferred to do tasks that were hard as opposed to easy, and this contribution is supported by the research of Russo and Hopkins (2017) that found many students reported that they enjoyed being challenged by the tasks. Clarke and Clarke (2004), as shown in Table 2.1, stated that effective teachers structure purposeful tasks that enable different possibilities, strategies, and products to emerge as they choose tasks that engage children and maintain involvement. Similarly, Sullivan's (2012) Six Principles for Effective Teaching of Mathematics (Table 2.2, Principle 3) indicated that effective teachers engage students by utilising a variety of rich and challenging tasks that allow students the time and opportunities to make decisions, and which use a variety of forms of representation.

The Early Years teacher in the three observed lessons provided a range of group structures, including partner work, small group work, and whole class work. Clarke and Clarke (2004) asserted that an effective pedagogical approach is when teachers choose from a variety of individual and group structures, as well as assuming various teacher roles throughout a significant portion of the instructional session (Table 2.1). Effective mathematics teachers expect students to work together to solve problems, whilst listening and understanding one another (Fedock, 2014). While mathematics involves numerous facts and procedures, emphasising conceptual understanding through a problem-solving approach is crucial, where reasoning and strategic thinking are important.

Addressing relational understanding allows for full mathematical development and learning. Relational understanding entails comprehending the reasons behind mathematical concepts, not just the procedures themselves. Thus, advocating for a pedagogical approach that promotes relational understanding over instrumental understanding (rules without reason) is essential (Skemp, 1976). Prioritising relational understanding benefits all students by enabling them to apply facts and procedures more adeptly, enhance their problem-solving strategies, and foster a deeper appreciation of the essence of mathematics. Findings from this research affirm existing research that promotes a problem-solving approach as essential to developing mathematically proficient students (Liljedahl, 2016; Sullivan, 2020). Within this approach lies the importance of the learning experiences being challenging, interesting, rich, engaging, meaningful and appropriate to the learners.

5.2.1.3 Element Three: A Strong Emphasis on Critical Mathematical Thinking

A mathematical thinking classroom, is a place where individuals engage in both independent and collective thought processes, A classroom centred on mathematical thinking is a space where students participate in shared learning experiences, and collaboratively build knowledge and understanding through active involvement and discourse (Liljedahl, 2016). To establish a mathematically thinking classroom, teachers need to give students something to think about (Liljedahl, 2016). A crucial element of effective learning experiences is the teacher facilitating and encouraging students to justify and explain their strategies and solutions. Therefore, embedded in a problem-solving approach is critical mathematical thinking.

Research suggests that critical mathematical thinking needs to be more closely considered in mathematics, with strong foundations formed early in a child's schooling (Monteleone et al., 2023). The Early Years teacher in this study in developing 21st Century mathematically proficient students, promoted and encouraged the students throughout all sections of the observed mathematical lessons to reason, justify and explain their thinking. This contribution is supported by Clarke and Clarke's (2004) findings (Table 2.1). Clarke and Clarke (2004) asserted that an effective pedagogical approach occurs when teachers actively focus and direct students to mathematical thinking throughout all sections of the mathematics lesson, as well as using a range of question types to probe and challenge students' thinking and reasoning. Metacognition and metacognitive questions, such as those probing "how?” and “why?" empower students to reflect on and think about their responses (MacDonald, 2018).

Clements et al. (2023) claimed that effective mathematics teachers engage students in mathematical discussion and facilitate open-ended questioning. Effective teachers ask their students, “why?” and “how do you know that?” (Clements et al., 2023). In the classroom observations detailed in Field Notes (Table 4.4) the Early Years teacher frequently and actively asked the students, “why?” and “how do you know that?”. The Early Years teacher used open-ended questions throughout the entire mathematics lessons and engaged students in mathematical discussions.

The Early Years teacher, by utilising questioning as a pedagogical approach, aided in discerning and expanding students’ thinking. According to Clarke and Clarke (2004), an effective pedagogical approach involves teachers actively listening to individual students and encouraging them to articulate their mathematical thoughts and ideas. Clements et al. (2023) emphasised the importance of teachers providing multiple opportunities for students to interact with each other through strategies like "think-pair-share" and engaging in productive discussions with the teacher. The Early Years teacher in this study facilitated such interactions, implementing the "think-pair-share" strategy in observed classroom lesson three, as detailed in Chapter 4 (Table 4.5). Additionally, the results in Table 4.3, demonstrate that the Early Years teacher consistently created opportunities for students to engage in meaningful mathematical conversations, argumentation, and questioning, fostering dynamic interactions between the teacher and students, as well as among the students themselves.

A mathematically thinking classroom goes beyond merely providing a suitable environment for thought; it actively encourages and facilitates individual and collective thinking (Liljedahl, 2016). Within this space, individuals engage in collaborative learning, constructing knowledge, and understanding through interactive activities and discussions. The Early Years teacher did not only foster thinking but also expected it, both implicitly and explicitly.

Developing critical mathematical thinking is crucial not only for success in mathematics but also for its application in various real-world situations. Critical mathematical thinking enables individuals to make informed decisions, solve problems in diverse contexts, and understand the broader implications of mathematical concepts (Monteleone et al., 2023). The Early Years teacher supported the development of 21st Century mathematical skills by prioritising critical thinking through a problem-solving approach. This approach equipped the students with the necessary tools to navigate the complexities of the modern world, aligning with the goal of fostering mathematical proficiency.

5.2.1.4 Element Four: Lessons Conclude with a Whole Class Summary

The Early Years teacher concluded each of the observed lessons with a whole class summary. This is consistent with what the Early Years teacher stated during the first semi-structured interview and is represented in Figure 3.5. This finding provided clear evidence that the Early Years teacher aligned their practice with their statements.

The Early Years teacher facilitated the young students in establishing connections between their solutions and those of their peers, while also emphasising the key mathematical concepts of the lesson. The Early Years teacher guided the students to recognise that various approaches exist for solving a range of problems. Rather than conducting mathematical discussions through isolated presentations of diverse problem-solving methods, the objective was to have students produce work that complemented each other, fostering the collaborative development of powerful mathematical ideas. During the lessons observed the Early Years teacher provided the opportunity for students to state their learning, alongside providing affirming comments such as “I was so proud, everyone was trying so hard to measure tip to tip with no gaps”. Responses such as this provided specific feedback to the students’ learning in the moment.

Sullivan et al. (2016) stated that a summary serves as a crucial tool to reinforce key concepts covered during the lesson, helping students consolidate their understanding. Summarising the main ideas and strategies used in solving problems enables students to reflect on their learning and connect new information with their existing knowledge (Sullivan et al., 2016). Moreover, a whole class summary provides an opportunity for the teacher to address any misconceptions that may have arisen, fostering a clearer understanding among students (Sullivan et al., 2016). Clarke and Clarke (2004), as shown in Table 2.1, stated that an effective pedagogical approach is when teachers draw out key mathematical ideas during and/or towards the end of the lesson.

The aim of this section of the lesson is to consolidate the learning, offering a synthesis of the mathematical ideas (Smith & Stein, 2018). Clements et al. (2023) concluded that effective mathematics teachers assist and encourage students to summarise critical ideas at the end of each lesson. The teacher's responsibility is to utilise students' solutions, ideas, and strategies to underscore the significant mathematical concepts within the task. Although the end of lesson summary is led by the students, it represents a teaching opportunity rather than merely a time for sharing (Smith & Stein, 2018). Through the questions that the Early Years teacher asked during the lesson summary and the ways in which the Early Years teacher pressed the students to clarify what they had done and why, the young students were assisted to make connections with the mathematical ideas that were the target of the lesson.

5.2.1.5 Element Five: Wait Time is Provided and Encouraged

This research found that a key characteristic of the Early Years teacher's practice was to provide wait time for students after posing a question or introducing a mathematical concept. Rather than the Early Years teacher allowing one or two students to dominate the whole class discussions at the beginning and end of the lessons, the teacher provided wait time. Wait time afforded students the opportunity to think deeply and to create something that they felt comfortable to share, in turn encouraging more students to be willing and able to share. Wait time in mathematics lessons has been recognised as a crucial pedagogical strategy with far-reaching implications for student learning (Smith & Stein, 2018; Stahl, 1994). Wait time, defined as the period of silence between a teacher's question and a student's response, has been shown to positively impact student engagement, cognitive processing, and the quality of responses (Smith & Stein, 2018).

Research by Smith and Stein (2018) stated the importance of providing adequate wait time in facilitating deeper mathematical thinking among young children. Wait time allows students to reflect on the mathematical content, formulate thoughtful responses, and engage in collaborative sense-making (Stahl, 1994). Additionally, wait time has been linked to increased participation and confidence among students, fostering a supportive learning environment (Stahl, 1994). As teachers play a pivotal role in orchestrating classroom discourse, understanding and implementing effective wait time practices can significantly contribute to the enhancement of early years mathematics education (Stahl, 1994).

The results in Chapter Four demonstrate that the participating teacher posed tasks with limited demonstration and instructions. Table 4.4 includes findings where it states in the field notes recording such as 'wait time' and 'limited instructions'. Sullivan et al. (2019) stated that posing a task without explaining a solution pathway allows the students to engage in thinking about how to solve the problem. As previously discussed, good questions challenge students to think and the deeper the questions the more students need to think and the greater the opportunity to develop conceptual understanding (MacDonald, 2018).

5.2.1.6 Element Six: Positive Dispositions are Valued and Encouraged

This research found that mathematical dispositions influence teachers’ practice, just as much as they influence students’ learning of mathematics. Developing positive engagement in learning strengthens students’ understanding of fundamental concepts and the creative thinking and inquiry processes that are necessary for lifelong learners (ADGE, 2022). The Early Years teacher possessed a positive disposition to teaching mathematics. A key finding from this research indicated that the students were aware of the teacher’s positive disposition. In the focus group sessions, the young students were asked “do you think your teacher enjoys teaching maths?” All of the student participants responded “yes”, as illustrated in Chapter Four, Table 4.5. Boaler and Williams (2020) stated that it is crucial for students' learning outcomes that teachers maintain a positive disposition towards mathematics. Furthermore, Boaler and Williams (2020) suggested a teacher's attitude can significantly impact students' perceptions of the subject and influence their motivation and engagement.

When teachers approach mathematics with enthusiasm, encouragement, and confidence, an environment conducive to learning is created. A positive disposition not only fosters a supportive atmosphere but also helps students build confidence in their own mathematical abilities. The Early Years teacher conveyed a genuine appreciation of mathematics and in turn inspired curiosity and a willingness to explore mathematical concepts, making the learning process more enjoyable and effective for students. In such a positive learning environment, students are likely to develop a stronger foundation in mathematics and a more enduring interest in the subject (Boaler & Williams, 2020).

Table 4.2 shows that in all three observed lessons the teacher provided tasks that engaged the students and maintained their involvement. In addition, the teacher modelled and promoted a positive disposition. Mathematical learning and providing young students with rich learning experiences in their mathematics lessons is important, but the research highlighted that it is more effective if students are engaged and willing to take risks and have a positive disposition.

5.2.1.7 Element Seven: An Emphasis on Mathematical Drawings to Represent Ideas and Solutions

It was evident from this research that Early Years teachers can play a pivotal role in fostering children's mathematical development, and emphasising mathematical drawings to represent ideas and solutions is important for several reasons (Clements & Sarama, 2007). The Early Years teacher used the drawings produced by the students to highlight connections among different mathematical representations. Drawing is well established as a tool that assists in bridging the gap between mathematical concepts and the real world (Way & Bobis, 2017). When children draw representations of everyday objects, situations, or problems, they can see the practical applications of mathematical ideas, making the learning experience more meaningful. Recent research shows that young children's drawings in mathematical tasks indicate a strong awareness of patterns and structures, which is essential in developing conceptual understanding across all areas of mathematics (Way & Bobis, 2017). The participating teacher in the observed lessons one and two tasked the students with answering a problem with a drawing. Drawings allow students the opportunity to construct and represent mathematical ideas in meaningful ways (MacDonald, 2018).

Incorporating mathematical drawings in Early Years classrooms is essential for promoting a deep understanding of mathematical concepts, fostering effective communication, and laying the foundation for future success in more advanced mathematical skills. This was evidently recognised by the participating teacher. The teacher supported a holistic approach to mathematical learning that engaged the children's cognitive, spatial, and creative abilities.

5.2.1.8 Element Eight: Picture Books to Engage Students in Mathematics Learning

The use of picture books aligns with the constructivist perspective, as students construct their understanding by actively engaging with the visual and narrative elements of the texts (Van den Heuvel-Panhuizen, 2003). Stories engage students' interest and curiosity and help them to understand abstract concepts (Knaus, 2013). The participating teacher in this research engaged students in mathematics through stories. Table 4.4 illustrates that the participating teacher read the story "Big and Little". Picture books allow mathematical concepts to be relevant as they provide students with narrative context problems and solutions (MacDonald, 2018). Incorporating picture books into mathematics learning is important as it offers a multifaceted approach to engage students and enhance their understanding of mathematical concepts.

Picture books are not only pedagogically appropriate in the Early Years as they captivate students' attention, but they also provide visual representations to clarify abstract mathematical ideas. The combination of narrative and imagery supports the contextualisation of mathematical concepts, making them more relatable and accessible. Through storytelling and visual aids, picture books create a bridge between everyday experiences and mathematical principles, fostering a deeper comprehension of mathematical ideas. Additionally, these books often incorporate problem-solving scenarios, encouraging critical thinking and application of mathematical skills in real-life situations.

The use of picture books in mathematics learning promotes a positive and enjoyable learning environment and contributes to the development of a well-rounded understanding of mathematical concepts from an early age. By incorporating this practice, the Early Years teacher created a rich and stimulating learning environment that nurtured 21st Century mathematical skills and prepared the students for a future that demands critical thinking, problem-solving, and adaptability (Van den Heuvel-Panhuizen, 2003).

5.2.2 Effective Numeracy Teachers Naturally Embed 21st Century Skills Across All Curriculum Areas

While the primary emphasis in this section has centred on the eight practices implemented by the Early Years teacher during mathematics lessons, it is recognised that another key component contributed to the students' opportunities to learn mathematics and develop positive mathematics identities. A further finding emerged in which the researcher established eight essential elements that an Early Years teacher should regularly provide in a mathematics lesson to develop mathematically proficient students with 21st Century Skills. A key insight that emerged from the study was that effective Early Years teachers naturally embed 21st Century skills across all learning areas. This inclusion was not about teacher confidence or intentional planning and assessment of 21st Century mathematical skills but about 'good teaching'. The literature suggests that the integration of 21st Century skills into mathematics education is a complex process that extends beyond the area of confidence, intentional planning, and assessment (Darling-Hammond, 2017). While effective mathematics teachers may not always explicitly set out to embed 21st Century skills, the principles of 'good teaching' inherently encompass the development of these skills (Darling-Hammond, 2017). Good teaching involves fostering critical thinking, problem-solving, communication, and collaboration, which are all key components of 21st Century skills.

During the first interview the Early Years teacher in this research was not confident in articulating how 21st Century mathematical skills were intentionally planned for in their mathematics lessons, stating that since the research project began "I am more mindful, more aware about bringing 21st Century learning into the maths lessons" (See Chapter Four, Section 4.1). Presented in Table 4.1, the four identified mathematical skills of communication, critical thinking and problem-solving, mathematical creativity, and innovation and collaboration were apparent and observed in all three lessons. The teacher also stated during the first interview, "I model creative and different ways of solving problems or showing mathematical thinking" (See Chapter Four, Section 4.1). This uncertainty suggests that the teacher's practice aligns with 21st

Century mathematical learning and that they embed 21st Century mathematical skills in their mathematical planning and mathematical lessons. However, it was also evident that this occurred unintentionally or was a natural inclusion rather than being an explicit intention. Darling-Hammond (2017) echoed this sentiment, asserting that although integrating 21st Century mathematical skills is important, it is the emphasis on high-quality teaching practices that truly fosters these skills within the realm of mathematics.

5.3 Research Sub-Question One: How Do the Elements of The Early Years School Environment Align in the Development of 21st Century Mathematical Skills?

Chapter Four outlined the key findings that emerged during the data analysis process in response to the first research sub-question: How do the elements of the Early Years school environment align in the development of 21st Century mathematical skills? Three key findings have emerged for discussion: establishing strong school-home partnerships, trusting relationships and positive collaboration between classroom teachers and the Principal, and inconsistent perceptions of 21st Century mathematical skills.

5.3.1 Establishing Strong School/Home Partnerships

Families are often seen as the initial teachers of mathematics for students (MacDonald, 2018). Parents play a role in shaping early mathematical understanding through informal experiences and more structured activities. Teachers can enhance this process by building partnerships with families and promoting their involvement (Clements et al., 2023). Research has validated the benefits of encouraging parents to play mathematical games at home or promote mathematical learning through everyday experiences (Clements et al., 2023; MacDonald, 2018).

In the current research, both the parents and students responded that they did mathematics at home through games, counting, and play experiences. In addition, parents said they did mathematics at home through cooking, as shown in the results in Chapter Four, Table 4.8. This research found that the students believed that both the parents and teacher played a valuable role in their mathematics learning. When the students were asked “who helps you learn maths?” there was an equal response rate of mum and dad and teacher (Chapter Four, Table 4.5).

Children’s development in mathematics can also be motivated by contexts outside educational settings, such as, in the children’s home environment during their interactions with family members. The home-learning environment and engagement of primary carers helps to reinforce the value of learning (Sonnemann & Hunter, 2023). One way of schools establishing the understanding that families play a key role is by actively building strong relationships between families and teachers to support students' mathematical education (MacDonald, 2018). This finding was supported by the Early Years teacher in this study. The Early Years teacher, when asked about the relationship with the students' parents, stated "It's good. They're really supportive" (Chapter Four, Table 4.1). "When strong, collaborative partnerships have been built among families and early childhood teachers, there is an immense potential for the educators to influence the families' thinking around their children's mathematics" (Perry & Gervasoni, 2012, p. 21). The alignment of parents, teachers, and leadership in the Early Years school environment plays a pivotal role in fostering the development of 21st Century mathematical skills among students.

Parental involvement has been identified as a key factor in shaping a child's academic success, including mathematical proficiency (Sui-Chu & Willms, 1996). When parents actively engage in their children's mathematical learning experiences, a supportive home environment is created that complements formal classroom instruction. According to Fredricks (2011) enhancing engagement in learning is achievable when teachers listen to children and create learning experiences which build connections with children's lives. Undoubtedly, engagement plays a crucial role in establishing a supportive environment that facilitates effective learning. In essence, a collaborative effort among parents, teachers, and leadership ensures a cohesive and holistic approach to nurturing the mathematical skills essential for success in the rapidly evolving landscape of the 21st Century.

5.3.2 Positive Collaboration Between Classroom Teachers and The Leadership Team

Teachers, as primary facilitators of mathematics education, contribute significantly to skill development by employing pedagogical strategies that emphasise critical thinking, problem-solving, and collaborative learning (National Council of Teachers of Mathematics, 2014). Moreover, effective school leadership is crucial in providing a vision that promotes the integration of modern teaching methods and technological tools into the mathematics curriculum, aligning it with the demands of the 21st Century (Leithwood et al., 2008). The school in the current research included policies such as a School Improvement Plan, a Curriculum Plan, and Learning Area Statements for Mathematics. The Principal indicated a commitment to overcoming challenges and providing a supportive learning environment, stating "we work as a team to try and support students and the collaborative environment supports students working together and supporting each other" (See Chapter 4, Section 4.9

Principal Interview). A collaborative approach encourages a culture of continuous improvement (Leithwood et al., 2008).

This research found that the teachers and the leadership team worked collaboratively to consider 21st Century skills in the schools overall planning of the curriculum and the pedagogies in which were used across the school. The school had a strong inquiry-based learning pedagogy which supported the development of 21st Century skills. The Principal was confident that the Early Years teachers understood the importance of teaching 21st Century skills and their ability to work within a play-based and inquiry learning pedagogy to teach these skills (See Chapter 4, Section 4.9 Principal Interview).

Collaboration fosters an environment where innovative teaching methods and creative approaches to mathematics education can thrive. This is essential for developing critical thinking and problem-solving skills, which are integral components of 21st Century mathematics education. In summary, positive collaboration between classroom teachers and the leadership team is essential for creating a dynamic and effective mathematics education that equips students with the 21st Century skills needed for success in an increasingly complex and technology-driven world.

5.3.3 Inconsistent Perceptions of 21st Century Mathematical Skills

The Principal's emphasis on 21st Century skills aligns with a forward-looking and student-centric approach to education, (Table 4.9). This perspective aligns with literature that highlights the necessity of equipping students with skills such as critical thinking, creativity, and collaboration to navigate an increasingly complex and dynamic world (Trilling & Fadel, 2009). However, the study found that parents often hold onto traditional models of academic success, viewing mathematics as a solitary pursuit with right or wrong answers rather than a creative and collaborative discipline (Section 4.5 Parental Surveys). This dichotomous understanding of mathematics is consistent with historical perceptions that emphasise rote learning and procedural proficiency over conceptual understanding and problem-solving skills (Boaler & Williams, 2020).

Research indicates that promoting a holistic view of mathematics education, which includes its creative and collaborative aspects, can lead to better learning outcomes and greater student engagement (Boaler & Williams, 2020; National Council of Teachers of Mathematics, 2014). Therefore, aligning the efforts of parents, teachers, and educational leaders is essential for fostering a supportive ecosystem that nurtures students' mathematical abilities and prepares them for the challenges of the 21st Century. Such collaboration has the potential to shift the educational paradigm towards a more inclusive and effective approach to developing mathematical skills in early childhood education (Dweck, 2017; Ginsburg et al., 2008).

5.4 Responding to the Overarching Research Question

The overarching research question was 'How does the Early Years environment prepare young children with 21st Century mathematical skills?' To respond to this research question, all of the findings were analysed and considered in relation to the literature and consolidated, resulting in two key contributions of this research;

1. Ensuring Early Years teachers naturally embed 21st Century learning across all curriculum areas and continually provide the eight pedagogical elements in their mathematics lessons will provide the most effective context for 21st Century mathematically proficient students.
2. Establishing trusting and strong relationships between all key stakeholders is essential in developing mathematically proficient students with 21st Century mathematical skills.

5.4.1. Ensuring Early Years Teachers Naturally Embed 21st Century Learning Across All Curriculum Areas and Continually Provide the Eight Pedagogical Elements in Their Mathematics Lessons Will Provide the Most Effective Context For 21st Century Mathematically Proficient Students

Effective mathematics teachers prepare students with 21st Century skills without even realising they are doing it, as it is simply good practice. The Early Years teacher in this research stated "collaboration, social skills, communication skills are vital and must be taught and modelled through all areas of learning" (Chapter Four, Table 4.1). The Principal confirmed this notion stating "21st Century Skills are important across the whole curriculum" (Chapter Four, Table 4.9). Lilijedahl (2016) supported this view of the participants, stating that an effective mathematics classroom, one where students are mathematical thinkers, needs to intersect with all the aspects of teaching and learning, both within mathematics education and across all learning areas.

The modern Early Years 21st Century classroom should be a place where students have the freedom to make decisions about their learning. A child-centred view is the recognition that all children are able and interested in engaging in important mathematics. If teachers do not teach mathematics, then they are not teaching the "whole child" (Clements et al., 2023). Gervasoni et al. (2021) stated that mathematical learners need to be prepared for 2030 and beyond, therefore they need to become creative, confident, collaborative problem solvers who are ethical and can communicate their mathematical ideas through arguments and models and who persist when faced with new and complex challenges. The investigation results reveal that the Early Years teacher has structured mathematical lessons to represent and model the future. This approach ensures that all students are given the opportunity to thrive and flourish. The findings also highlighted an on-going need to challenge teachers' beliefs and to consider new pedagogical practices if they are to meet the demands of early childhood mathematics education, and cater for the future mathematics learning needs of young children.

5.4.2 Establishing Trusting and Strong Relationships Between All Key Stakeholders is Essential in Developing Mathematically Proficient Students With 21st Century Mathematical Skills

Social capital in education significantly boosts students' academic success (Erkan, 2011). It fosters well-being and self-esteem, leading to higher academic achievements (Zajda, 2023). A child's development is largely shaped by their family's social capital, including trust, networks, and norms, which influence their opportunities and educational outcomes (Erkan, 2011). Understanding this can guide teachers, parents, and school leaders in strategising for improved educational success. Inclusive environments, where students feel accepted and safe, are foundational to this process (Zajda, 2023).

Recent research has discussed the importance of social capital and cultural capital. Clements et al. (2023) proposed the concept of a culturally responsive classroom. This approach encourages teaching strategies and classroom environments that acknowledge and respect the cultural and linguistic diversity of students. By gaining insights into the students, their families, and the communities they belong to, teachers can tailor classroom settings to reflect the unique experiences of the learners (Clements et al., 2023). In alignment with the importance of culturally responsive teaching practices, the EYLF (AGDE, 2022) emphasised the significance of cultural competence and respectful relationships. By acknowledging and respecting diverse cultural backgrounds, the EYLF promotes an inclusive and culturally responsive early childhood education environment. The framework encourages teachers to engage in reflective practices that consider the cultural contexts of the children and families they work with, fostering a learning environment that is respectful of diverse perspectives and experiences (AGDE, 2022).

Establishing trusting and strong relationships between all key stakeholders, including teachers, parents, and leadership teams, is considered fundamental. Through effective communication and shared decision-making, teachers can gain valuable insights into the cultural backgrounds, and the values of the families they serve, contributing to a more meaningful and culturally responsive early childhood education experience (AGDE, 2022). Culturally responsive teaching practices help families engage with their child's mathematics education (MacDonald, 2018). This is a new field, and such practices are at least partially research-based and need additional study to establish research-validated pedagogies.

5.5 Conclusion

The objective of this chapter was to conduct a detailed discussion of the research results in conjunction with the existing research literature. The chapter also sought to frame this discussion in response to the research questions that guided this investigation.

Overarching Research Question:

> How does the Early Years school environment prepare young children with regards to 21st Century mathematical skills?

Research Sub-Questions:

- How do Early Years teachers support mathematical practice that develop 21st Century mathematical skills?
- How do the elements of the Early Years school environment align in the development of 21st Century mathematical skills?

In conclusion, the current research reveals two pivotal findings in response to the overarching research question: ensuring Early Years teachers naturally embed 21st Century learning across all curriculum areas, and continually provide the eight pedagogical elements in their mathematics lessons. Doing this will provide the most effective context for 21st Century mathematically proficient students. Secondly, establishing trusting and strong relationships between all key stakeholders are essential to develop mathematically proficient students with 21st Century mathematical skills. The final chapter will draw on the key contributions of this investigation and outline any perceived limitations. In addition, the researcher will pose implications and recommendations of the research contribution.

Chapter Six: Conclusion and Recommendations

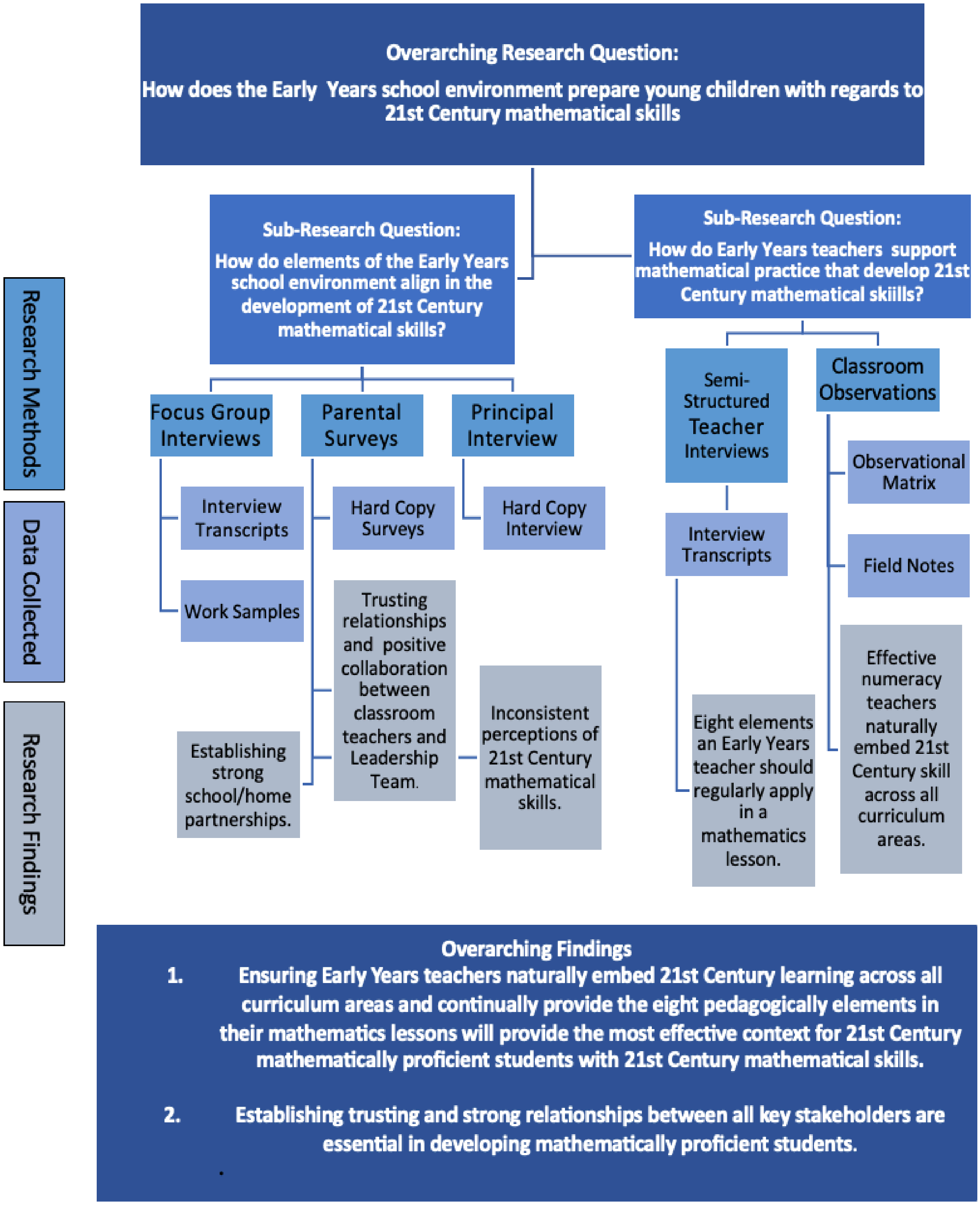
6.1 Findings Presented

The aim of this research was to investigate how the Early Years setting equips young children with 21st Century mathematical skills, including critical thinking, problem-solving, innovation, collaboration, creativity, and communication. The Early Years environment encompassed the perspectives and beliefs of key stakeholders regarding these 21st Century mathematical skills. The key stakeholders of this research were the Principal, an Early Years teacher, the pre-primary parents, and the pre-primary students at a small independent primary school in metropolitan Perth. The study used a qualitative, field-based, case study and was conducted using qualitative data gained from semi-structured interviews, surveys, classroom observations, and student focus groups. The research employed a constructivist epistemology using an interpretive case study theoretical perspective. A framework underpinning the research is shown in Figure 6.1. This case study delved into the understanding of 21st Century skills, effective mathematics teaching, and the overall Early Years school environment. These areas were identified as key themes in the existing literature. The primary goal of this research was to contribute to the broader discourse on Early Years mathematics education by emphasising the significance of integrating 21st Century mathematical skills throughout all areas of the curriculum.

In this chapter, the research questions are addressed, implications will be discussed, and recommendations will be provided for both practical application and future research in the field of Early Years mathematics education.

Figure 6.1

Research Framework



6.2 Research Questions Answered

In Chapter Four of this study the findings pertaining to the research questions were presented. Chapter Five provided a thorough examination and analysis of the findings in the context of the existing literature. This chapter aligns the research aims with relevant literature, allowing for a refined presentation of the connections, inconsistencies, and implications of the results within the extended academic perspective. A summary of the findings in response to the research questions is presented in Table 6.1.

Table 6.1

Overall Findings of Research Questions

Research Question	**Overarching Findings**
How does the Early Years school environment prepare young children with regards to 21st Century mathematical skills?	• Ensuring Early Years teachers naturally embed 21st Century learning across all curriculum areas and continually provide the eight pedagogically elements in their mathematics lessons will provide the most effective context for 21st Century mathematically proficient students with 21st Century mathematical skills. • Establishing trusting and strong relationships between all key stakeholders are essential in developing mathematically proficient students.
Sub-Research Questions	**Key Findings**
1. How do Early Years teachers support mathematical practice that develop 21st Century Mathematical skills?	• Eight elements an Early Years teacher should regularly apply in a mathematics lesson. • Effective Early Years mathematics teachers naturally embed 21st Century skills across all curriculum areas.
2. How do the elements of the Early Years school environment align in the development of 21st Century Mathematical skills	• Establishing strong school/home partnerships. • Trusting relationships and positive collaboration between classroom teachers and Leadership Team. • Inconsistent perceptions of 21st Century mathematical skills.

6.3 Limitations of the Research

Two potential limitations were evident, and these are discussed. The first notable limitation pertains to the restricted scope of school representation in this research. The research was conducted within a singular independent primary school in metropolitan Perth. While the chosen school willingly participated to enhance their practices, the outcomes will not encapsulate the diversity of approaches and challenges across various educational institutions. Future research should strive to incorporate a more extensive range of schools to ensure a more comprehensive understanding of the subject matter.

A second limitation relates to the number of teachers involved in the study. Initially, two teachers were selected and invited to participate. However, one teacher had to withdraw due to personal reasons. As this research investigated one teacher and one class of students the findings cannot be generalised

6.4 Implications

An implication for students’ mathematical learning that emerged from this research in this school, is regarding the school philosophy and the alignment with parental school selection and teachers employed at the school. In the ever-evolving landscape of education, the alignment between a school's philosophy and parental beliefs plays a crucial role in shaping students' learning experiences. This notion becomes particularly pertinent when considering schools that adopt unconventional approaches to teaching, emphasising student efficacy and independence over traditional practices. It is essential to highlight that the selected school adopts an unconventional approach distinct from traditional practices, emphasising student efficacy, and independence. The school's learning statement claims that it is student centred, where learners are offered a curriculum that is dynamic, exciting, rewarding, and responsive to the needs, capabilities, and interest of all students. "Students are at the centre of learning at X School. We know our learners and offer a curriculum that is dynamic, exciting, rewarding, and responsive to our students' needs, capabilities, and interests."

Parents who opt for this school likely endorse its holistic and 'non-traditional' approach to teaching, demonstrating an already established belief in the importance of 21st Century skills for the present and future learning of students. Although the alignment between parental beliefs and a school's philosophy was not explored in this study, it presents an avenue for further investigation. This prompts the question: did parents choose the school specifically for this reason? Surprisingly, the parental surveys in Table 4.7 indicate that parents still value the teaching of traditional mathematical skills. This discrepancy raises the need for additional research, specifically focusing on alternative approach schools—those outside the conventional Catholic, independent, or government school structures—and examining whether this alignment with parental beliefs about mathematical skills holds true across diverse educational settings.

6.5 Recommendations

Building upon the insights gained from this study, seven recommendations are proposed for future practice and future research in the field of Early Years mathematics education and the integration of 21st Century skills. These are presented within two categories; recommendations for practice and recommendations for future research.

6.5.1 Recommendations for Practice

It is recommended that teachers in Early Years mathematics education embrace a holistic approach by integrating 21st Century skills, such as communication, critical thinking and problem-solving, and mathematical creativity and innovation, into their instructional strategies. Three recommendations for practice have been suggested: classroom-based practical recommendations, professional learning based on the eight pedagogical elements, and creating culturally responsive classrooms.

6.5.1.1 Classroom-based Practical Recommendations

For effective classroom-based practices in Early Years mathematics education, teachers are encouraged to adopt a constructivist approach that provides open-ended problem-solving tasks, enabling young learners to apply mathematical concepts in real-life contexts (Mallamaci, 2018). As was made clear in the findings presented in this thesis, the Early Years classroom would benefit from an environment that fosters critical mathematical thinking with a strong emphasis on students' reasoning and justifying their mathematical ideas and solutions throughout the lesson and during the whole-class summary (See Chapter Four, Section 4.3). Intentional wait time should be incorporated and encouraged to allow students the opportunity for thoughtful reflection (Smith & Stein, 2018). Promoting a productive disposition towards mathematics is essential, with Early Years teachers valuing and encouraging students' positive attitudes and perseverance in learning experiences that are challenging, rich, interesting, engaging, meaningful, and appropriate. Additionally, mathematical lessons can be enriched by incorporating picture books and drawings to represent mathematical ideas and solutions (See Chapter Four, Section 4.4). These pedagogical approaches aim to equip young learners with a mathematical skill set that will serve as a foundation for future academic success and a lifelong love of learning.

6.5.1.2 Professional Learning Based on The Eight Pedagogical Practices

It is recommended that Early Years teachers seek out opportunities to develop and enhance their professional knowledge and skills to support quality improvement in their practice (AGDE, 2022). Early Years teachers need to be critically reflective and actively seeking out professional learning opportunities (AGDE, 2022). Professional learning based on the eight pedagogical practices would ensure that Early Years teachers are equipped to significantly impact classroom teacher practices.

6.5.1.3 Creating Culturally Responsive Classrooms

Given the increasing recognition of the importance of cultural responsiveness in education, it is recommended that Early Years teachers provide a culturally responsive classroom (Clements et al., 2023). Future practice should prioritise the integration of teaching strategies that acknowledge and embrace cultural diversity, aiming to enhance the development of 21st Century mathematical skills. Early Years teachers would benefit from focusing on implementing practices that foster inclusivity and equity, ensuring that the learning environment reflects and respects the cultural backgrounds of all students (ADGE, 2022). A culturally responsive classroom seeks to not only recognise the importance of cultural diversity but also to empower Early Years teachers to implement effective strategies that promote equitable learning experiences for all students (Clements, et al, 2023).

6.5.2 Recommendations for Future Research

As we navigate the evolving landscape of Early Years mathematics education and the imperative integration of 21st Century mathematical skills, four recommendations for future research have been suggested: diversification of school settings, expanded teacher participation, longitudinal studies, and instinctive versus developing teachers.

6.5.2.1 Diversification of School Settings

To address the limitation of limited school representation, it is recommended future studies should intentionally select a diverse range of Early Years settings, encompassing various educational models, socioeconomic contexts, and geographic locations. This approach will contribute to a more holistic understanding of the challenges and effective strategies associated with integrating 21st Century mathematical skills.

6.5.2.2 Expanded Teacher Participation

Recognising the impact of teacher participation on the study's outcomes, a recommendation arising relates to future research. Future research should employ strategies to mitigate potential withdrawal, such as establishing contingency plans or selecting additional participants from the outset. A more extensive and varied pool of teachers would enrich the study's findings and enhance the transferability of recommendations to a broader educational context.

6.5.2.3 Longitudinal Studies

To deepen the understanding of the long-term effectiveness of integrating 21st Century mathematical skills in the Early Years, future research might consider adopting a longitudinal approach. Tracking the progress of students and teachers over an extended period may provide valuable insights into the sustained impact of such interventions and contribute to the ongoing development of best practices.

6.5.2.4 Instinctive Versus Developing Teachers

An intriguing dimension for future research involves an investigation into the distinction between instinctive and developing teachers in the Early Years. Specifically, the study could explore whether instinctive teachers, who naturally create conditions for deep conceptual learning, exhibit a more inherent integration of 21st Century skills across all areas compared to developing teachers who may need to be more intentional in their planning. By understanding the unique strengths and challenges associated with each distinction of teacher, future research could offer targeted recommendations for professional development programs tailored to the needs of both instinctive and developing teachers. It may also contribute to the refinement of teacher training initiatives and the optimisation of classroom practices to ensure the holistic development of 21st Century mathematical skills in young learners. Recognising the distinctions between instinctive and developing teachers is one aspect, among others, that is essential for designing effective professional development programs tailored to their specific needs and growth trajectories. Therefore, subsequent research initiatives can further advance our knowledge of Early Years mathematics education and the integration of 21st Century skills, fostering continuous improvement and innovation in educational practices.

6.6 Conclusion

This investigation was premised on exploring how the Early Years school environment prepares young children with regards to 21st Century mathematical skills. Diverse and unique perspectives were acknowledged by including the Principal, an Early Years teacher, pre-primary parents, and pre-primary students as stakeholders in a qualitative, field-based, case study. This research has added depth to the discourse surrounding Early Years mathematics education and has provided practical suggestions for Early Years teachers and schools to enhance the development of 21st Century mathematics skill development. In addition to the implementation of pedagogically sound practices, an integration of diverse perspectives, and a continual commitment to refining and improving practices are important. In these ways mathematically proficient learners may be developed within an evolving and rapidly changing educational landscape. It has also provided recommendations for future studies.

It is evident from this study that fostering 21st Century mathematical skills in the Early Years is a collaborative effort, one that requires a strong and trusting relationship between all key stakeholders (the parents, Early Years teachers, the Leadership Team, and the students). In addition to the implementation of pedagogical practices, an integration of diverse perspectives and a continual commitment to refining and improving practices for developing mathematically proficient young learners in an evolving and rapidly changing educational landscape.

References

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. Educational series 19. International Academy of Education; International Bureau of Education.

Ata Baran, A., & Kabael, T. (2021). An investigation of eighth grade students' mathematical communication competency and affective characteristics. *The Journal of Educational Research, 114*(4), 367–380. https://doi.org/10.1080/00220671.2021.1948382

Attard, C. (2021). Applying a framework for engagement with mathematics in the primary school. *Australian Primary Mathematics Classroom*, *26*(4), 4–9. https://doi.org/10.3316/informit.448676280333774

Attard, C. (2013). "If I had to pick any subject, it wouldn't be maths": Foundations for engagement with mathematics during the middle years. *Mathematics Education Research Journal, 25*(4), *569–587*. https://doi.org/10.1007/s13394-013-0081-8

Askew, M. (2013). Big ideas in primary mathematics: Issues and directions. *Perspectives in Education*, *31*(3), 5-18.

Australian Association of Mathematics Teachers (2006). *Standards for excellence in teaching mathematics in Australian schools*. http://www.aamt.edu.au

Australian Children's Education and Care Quality Authority. (2011). *Guide to the national quality standard*. ACECQA.

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2010). *Australian Curriculum, general capabilities. https://www.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/*

Australian Government Department of Education. (2022). *Belonging, being and becoming: The Early Years Learning Framework for Australia (V2.0).* Australian Government Department of Education for the Ministerial Council.

Bailey, C. A. (2018). *A guide to qualitative field research* (3rd ed.). SAGE Publications, Inc.

Battelle for Kids. (2019). Framework for 21st century learning. *Partnership For 21st Century Learning*. https://static.battelleforkids.org/documents/p21/p21_framework_brief.pdf

Bell, D.V.J. (2016). Twenty-first century education: Transformative education for sustainability and responsible citizenship. *Journal of Teacher Education for Sustainability, 18(*1), 48-56. https://doi.org/10.1515/jtes-2016-0004

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics*, *79*(1), 127-147. https://doi.org/10.1007/s10649-011-9333-2

Binkley, M., Erstad, O., Herman, J., Raizen, S., Ripley, M., Miller-Ricci, M., & Rumble, M. (2012). Defining twenty-first century skills. In *Assessment and teaching of 21st century skills*. (pp. 17-66). Springer.

Balch, T. C., & Balch, B. V. (2019). *Building great school counselor-administrator teams: A systematic approach to supporting students, staff, and the community (Balancing guidance counselor and administrator responsibilities)*. Solution Tree.

Blömeke, S., Suhl, U., & Kaiser, G. (2011). Teacher education effectiveness: Quality and equity of future primary teachers' mathematics and mathematics pedagogical content knowledge. *Journal of Teacher Education*, *62*(2), 154-171. https://doi.org/10.1177/0022487110386798

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* Jossey-Bass.

Boaler, M. J., & Williams, C. (2020). *Mindset mathematics: Visualizing and investigating big ideas. Grade 8.* John Wiley & Sons.

Bray, A., & Tangney, B. (2016). Enhancing student engagement through the affordances of mobile technology: A 21st century learning perspective on realistic mathematics education. *Mathematics Education Research Journal*, *28*(1), 173-197. https://doi.org/10.1007/s13394-015-0158-7

Clarke, D. M. (1993). *Influences on the changing role of the mathematics teacher* (Order No. 9318612). [Doctoral thesis, University of Winconsin – Madison] ProQuest Dissertations & Theses Global. http://ipacez.nd.edu.au/login?url=https://www.proquest.com/dissertations-theses/influences-on-changing-role-mathematics-teacher/docview/304056415/se-2?accountid=41561

Clarke, D. M. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education*, *28*(3), 278-308.

Clarke, D. M., & Clarke, B. A. (2004). Mathematics teaching in grades K-2: Painting a picture of challenging, supportive, and effective classrooms. *Perspectives on the Teaching of Mathematics*, *66*, 67-81.

Clements, D. H., Lizcano, R., & Sarama, J. (2023). Research and pedagogies for early math. *Education Sciences*, *13*(8), 839–. https://doi.org/10.3390/educsci13080839

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). Routledge. https://doi.org/10.4324/9780203520574

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the building blocks project. *Journal for Research in Mathematics Education, 38*(2), 136–163. https://doi.org/10.2307/30034954

Clauscen, H. (2013). Environments for teaching mathematics: Supporting students with special needs. *Educating Young Children, 19*(3), 7-9.

Connor, J. (2011). *Foundations for learning: Relationships between the early years learning framework and the Australian Curriculum an ECA-ACARA paper*. Early Childhood Australia & Australian Curriculum, Assessment and Reporting Authority.

Creswell, J. W., & Creswell, J. W. (2013). *Qualitative inquiry & research design: Choosing among five approaches* (3rd ed.). SAGE Publications.

Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. Allen & Unwin.

Darling-Hammond, L. (2017). Teacher education around the world: What can we learn from international practice? *European Journal of Teacher Education, 40*(3), 291-309.

Dweck, C. S. (2017). *Mindset: The new psychology of success: How we can learn to fulfill our potential*. Skillsoft.

Education Council (2019). *Alice Springs (Mparntwe) Education Declaration*. http://www.educationcouncil.edu.au/site/DefaultSite/filesystem/documents/Reports%20and%20publications/Alice%20Springs%20(Mparntwe)%20Education%20Declaration.pdf

Erkan, A. (2011). Effects of social capital on academic success: A narrative synthesis. *Educational Research and Reviews*, *6*(6), 456-461.

Falloon, G., Stevenson, M., Beswick, K., Fraser, S., & Geiger, V. (2021). Building STEM in Schools: An Australian cross-case analysis. *Educational Technology & Society, 24*(4), 110–122. https://doi.org/10.30191/ETS.202110_24(4).0009

Fedock, B. (2014). Principles to actions: Ensuring success for all. *Mathematics Teaching in the Middle School, 20*(3), 186- 190.

Finn, J. D., & Zimmer, K. S. (2012). Student engagement: What is it? Why does it matter? In *Handbook of research on student engagement* (pp. 97–131). Springer. https://doi.org/10.1007/978-1-4614-2018-7_5

Fleer, M., & Raban, B. (2005). *Literacy and mathematics that counts from birth to five years- A review of the literature.* DEST Early Childhood Learning Resources.

Flick, U. (2013). The SAGE handbook of qualitative data analysis. In *The SAGE handbook of qualitative data analysis.* SAGE Publications. https://doi.org/10.4135/9781446282243

Fredricks, J. A. (2011). Engagement in school and out-of-school contexts: A multidimensional view of engagement. *Theory into Practice*, *50*(4), 327–335. https://doi.org/10.1080/00405841.2011.607401

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59–109. https://doi.org/10.3102/00346543074001059

Galletta, A., & Cross, W. E. (2013). *Mastering the semi-structured interview and beyond: From research design to analysis and publication*. New York University Press. https://doi.org/10.18574/9780814732953

Gelmez Burakgazi, S., Karsantık, Y., Aktan, T., Ayaz, M. A., Büge, B. C., Karataş, F., Ödün, S., Varol Şanlı, Ş., Tarım, B., & Yavaşca, O. (2019). Equipped or not? Investigating pre-service teachers' 21st century skills. *Asia Pacific Journal of Education, 39*(4), 451–468. https://doi.org/10.1080/02188791.2019.1671803

Gervasoni, A. (2018). The impact and challenges of early mathematics intervention in an Australian context. In *Invited lectures from the 13th International Congress on Mathematical Education* (pp. 115-133). Springer International Publishing.

Gervasoni, A., Hunter, R., Bicknell, B., & Sexton, M. (2012). *Powerful pedagogical actions in mathematics education*. (pp. 193-218). Sense Publishers. https://doi.org/10.1007/ 978-94-6091-970-1_10

Gervasoni, A., Roche, A., & Downton, A. (2021). Differentiating instruction for students who fail to thrive in mathematics: The impact of a constructivist-based intervention approach. *Mathematics Teacher Education and Development, 23*(3), 207-233.

Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, 22* (1), 1-24.

Grootenboer, P. (2018). *The practices of school middle leadership: Leading professional learning*. Springer. https://doi.org/10.1007/978-981-13-0768-3

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105-117). Sage.

Guba, E. G., & Lincoln, Y. S. (2007). But is it rigorous? Trustworthiness and authenticity in naturalistic evaluation. *New Directions for Evaluation*, *114*, 15–25. https://doi.org/10.1002/ev.223

Guskey, T. R. (2002). Professional development and teacher change. *Teachers and Teaching, 8(*3), 381-391. https://doi.org/10.1080/135406002100000512

Habibis, D. (2006). Ethics and social research. In M. Walter (Ed.), *Social research methods: An Australian perspective.* (pp. 53 - 82). Oxford University Press.

Hannay, L. M., & Earl, L. (2012). School district triggers for reconstructing professional knowledge. *Journal of Educational Change, 13*(3), 311-326. https://doi.org/10.1007/s10833-012-9185-2

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81– 112.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. Routledge.

Hattie, J. (2010). On being a "critic and conscience of society": The role of the education academic in public debates. *New Zealand Journal of Educational Studies*, *45*(1), 85-96.

Hattie, J. (2023). *Visible learning: The sequel: A synthesis of over 2,100 meta-analyses relating to achievement*. Routledge. https://doi.org/10.4324/9781003380542

Heckman, J. J. (2018). Skill formation and the economics of investing in disadvantaged children. In D. Grusky, & J. Hill (Eds.). *Inequality in the 21st Century* (1st ed., pp. 460–463). Routledge. https://doi.org/10.4324/9780429499821-78

Hedges, H. (2000). Teaching in early childhood: Time to merge constructivist views so learning through play equals teaching through play. *Australian Journal of Early Childhood, 25*(4), 16-21.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400. https://doi.org/10.5951/jresematheduc.39.4.0372

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*(4), 430-511. https://doi.org/10.1080/07370000802177235

Holzapfel, B. (2018). Class of 2013: What do today's kindergartners need to be life-ready? https://educationblog.microsoft.com/editors/barbara-holzapfel/

Hurrell, D. P. (2013). What teachers need to know to teach mathematics: An argument for a reconceptualised model. *The Australian Journal of Teacher Education*, *38*(11), 54–64. https://doi.org/10.14221/ajte.2013v38n11.3

Ishimine, K., Tayler, C., & Bennett, J. (2010). Quality and early childhood education and care: A policy initiative for the 21st century. *International Journal of Child Care and Education Policy (Seoul), 4*(2), 67–80. https://doi.org/10.1007/2288-6729-4-2-67

Jerald, C. D. (2009). Defining a 21st century education. *Center for Public Education, 16*, 1-10.

Jeynes, W. (2010). *Parental involvement and academic success* (1st ed.). Routledge.

Johnson, S., & Rasulova, S. (2017). Qualitative research and the evaluation of development impact: Incorporating authenticity into the assessment of rigour. *Journal of Development Effectiveness*, 9(2), 263–276. https://doi.org/10.1080/19439342.2017.1306577

Kim, S. W. & Hill, N. E. (2015). Including fathers in the picture: A meta-analysis of parental involvement and students' academic achievement. *Journal of Educational Psychology, 107*(4), 919–934. https://doi.org/10.1037/edu0000023

Kim, J. S., & Quinn, D. M. (2013). The effects of summer reading on low-income children's literacy achievement from kindergarten to grade 8: A meta-analysis of classroom and home interventions. *Review of Educational Research, 83*(3), 386–431. https://doi.org/10.3102/0034654313483906

Knaus, M. (2013). *Maths is all around you: Developing mathematical concepts in the early years*. Teaching Solutions.

Knaus, M. (2017). Supporting early mathematics learning in early childhood settings. *Australasian Journal of Early Childhood, 42(*3), 4-13. https://doi.org/10.23965/AJEC.42.3.01

Lamb, S., Maire Q., & Doecke E. (2017) *Key skills for the 21st century: An evidence-based review.* Victoria University. https://education.nsw.gov.au/our-priorities/innovate-for-the-future/education-for-a-changing-world/research-findings/future-frontiers-analytical-report-key-skills-for-the-21st-century/Key-Skills-for-the-21st-Century-Analytical-Report.pdf

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29–63. https://doi.org/10.3102/00028312027001029

Lauckner, H., Paterson, M., & Krupa, T. (2012). Using constructivist case study methodology to understand community development processes: Proposed methodological questions to guide the research process. *The Qualitative Report, 17*(13), 1-22.

Leithwood, K., Harris, A., & Hopkins, D. (2008). Seven strong claims about successful school leadership. *School Leadership & Management, 28*(1), 27-42.

Liljedahl, P. (2016). *Building thinking classrooms: Conditions for problem-solving. Posing and solving mathematical problems: Advances and new perspectives* (pp. 361-386). Springer.

Liljedahl, P. (2021). *Building thinking classrooms in mathematics, grades K-12: 14 teaching practices for enhancing learning*. Corwin.

Lindeman, K. W., & Anderson, E. M. (2015). Using blocks to develop 21st century skills. *YC Young Children, 70*(1), 36.

Livy, S., Herbert, S., & Vale, C. (2019). Developing primary pre-service teachers' mathematical content knowledge: Opportunities and influences. *Mathematics Education Research Journal, 31*(3), 279–299. https://doi.org/10.1007/s13394-018-0252-8

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education*, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554

MacDonald, A. (2018). *Mathematics in early childhood education*. Oxford University Press.

Mallamaci, L. (2018). Constructivism in mathematics. *Vinculum, 55*(2), 20–21.

Martin, A. J., Anderson, J., Bobis, J., Way, J., & Vellar, R. (2012). Switching on and switching off in mathematics: An ecological study of future intent and disengagement among middle school students. *Journal of Educational Psychology, 104(*1), 1–18. https://doi.org/10.1037/a0025988

Maxwell, J. (2010). Using numbers in qualitative research. *Qualitative Inquiry 16*(6) 475–482. https://doi.org/10.1177/1077800410364740

McCain, M., & Mustard, J. (1999). *Early years study: Reversing the real brain drain*. Final report to the Government of Ontario. The Canadian Institute for Advanced Research.

McDonough, A. (2003). Effective teachers of numeracy in the early years and beyond. In B. Clarke, A. Bishop, R. Camerson, H. Forgasz, & W. Seah (Eds.) *Making mathematicians*, 26-41. Brunswick, Victoria: Mathematical Association of Victoria.

Merriam, S. B. (1988). *Case study research in education: A qualitative approach*. Jossey-Bass.

Ministerial Council on Education, Employment, Training and Youth Affairs (2008). *Melbourne Declaration on Educational Goals for Young Australians*. Curriculum Corporation.

Monteleone, C., Miller, J., & Warren, E. (2023). Conceptualising critical mathematical thinking in young students. *Mathematics Education Research Journal*, *35*(2), 339–359. https://doi.org/10.1007/s13394-023-00445-1

Muir, T. A., Fielding-Wells, J., & Chick, H. L. (2017). Developing an understanding of what constitutes mathematics teacher educator PCK: A case study of a collaboration between two teacher educators. *The Australian Journal of Teacher Education (Online)*, *42*(12), 60-79.

Murphy, J. (1994, April 4-8). *Transformational change and the evolving role of the principal: Early empirical evidence* [paper]. Annual Meeting of the American Education Research Association, New Orleans.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

Nazar, Z. J., Nazar, H., Rainkie, D., El-Awaisi, A., ElJaam, M., & Stewart, D. C. (2022). Evidence produced while using qualitative methodologies including research trustworthiness. In

Encyclopedia of evidence in pharmaceutical public health and health services research in pharmacy (pp. 699–712). Springer International Publishing. https://doi.org/10.1007/978-3-030-64477-2_76

Neuman, W. L., & Neuman, L. (2013). *Social research methods.* Pearson Education, Limited.

O'Shea, J., & Leavy, A. M. (2013). Teaching mathematical problem-solving from an emergent constructivist perspective: The experiences of Irish primary teachers. *Journal of Mathematics Teacher Education, 16*(4), 293-318. https://doi.org/10.1007/s10857-013-9235-6

Organisation for Economic Co-operation and Development. (2018). *The future of education and skill education 2030.* OECD. .https://www.oecd.org/education/2030/E2030%20Position%20Paper%20(05.04.2018).pdf

Perry, B., & Dockett, S. (2005). What did you do in maths today? *Australian Journal of Early Childhood, 30*(3), 32–36.

Perry, B., & Dockett, S. (2013). Reflecting on young children's mathematics learning. In L. D. English, & J. T. Mulligan (Eds.) *Reconceptualizing early mathematics learning* (pp. 149–161). Springer. https://doi.org/10.1007/978-94-007-6440-8_8

Perry, B., & Gervasoni, A. (2012). *Let's count educators' handbook.* The Smith Family.

Perry, B., Dockett, S., & Harley, E. (2012). The early years learning framework for Australia and the Australian curriculum: Mathematics - linking educators' practice through pedagogical inquiry questions. In In B. Atweh, M. Goos, R. Jorgenson, & D. Siemon (Eds.) *Engaging the Australian curriculum mathematics: Perspectives from the field* (pp. 153-174). Mathematics Education Research Group of Australasia.

Perry, B., & MacDonald, A. (2015). Educators' expectations and aspirations around young children's mathematical knowledge. *Professional Development in Education, 41*(2), 366–381.

Piaget, J. (1964). Development and learning. *Journal for Research in Science Teaching, 2*, 176-186.

Pound, L. (2006). *Supporting mathematical development in the early years* (2nd ed.). Open University Press.

Rapley, E. (2018). 'Seeing the light'. Personal epiphanies and moving towards interpretivism: A researcher's tale of exploring teacher pedagogic practice. *Ethnography and Education, 13*(2), 185-203. https://doi.org/10.1080/17457823.2017.1315311

Reys, R., Lindquist, M., Lambdin, D. V., Smith, N. L., Rogers, A., Bragg, L. A., Cooke, A., Fanshawe, M., & Gronow, M (2021). *Helping children learn mathematics* (4th Australian ed.). John Wiley & Sons, Incorporated.

Roche, A., Gervasoni, A., & Kalogeropoulos, P. (2021). Factors that promote interest and engagement in learning mathematics for low-achieving primary students across three learning settings. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-021-00402-w

Russo, J., & Hopkins, S. (2017). Student reflections on learning with challenging tasks: 'I think the worksheets were just for practice, and the challenges were for maths'. *Mathematics Education Research Journal, 29*(3), 283–311. https://doi.org/10.1007/s13394-017-0197-3

Ryan, G. (2018). Introduction to positivism, interpretivism and critical theory. *Nurse Researcher*, *25*(4), 14–20. https://doi.org/10.7748/nr.2018.e1466

Sandelowski, M. (2001). Real qualitative researchers do not count: The use of numbers in qualitative research. *Research in Nursing & Health*, *24*(3), 230–240. https://doi.org/10.1002/nur.1025

School Curriculum and Standards Authority. (2018). *Mathematics v8.1*. https://k10outline.scsa.wa.edu.au/home/p-10-curriculum/curriculum browser/mathematics-v8

Schwandt, T.A. (1994). Constructivist, interpretivist approaches to human inquiry. In N. K. Denzin, & Y. S. Lincoln (Eds.) *Handbook of qualitative research* (pp. 118-137). Sage.

Seo, K. H., & Ginsburg, H. P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education.* (pp. 91-104). Lawrence Erlbaum Associates.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4– 14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Siraj-Blatchford, I. (2009). Quality teaching in early years. In A. Anning, J. Cullen, & M. Fleer (Eds.), *Early childhood education: Society and culture* (pp. 147–157). Sage.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*(1), 20-26.

Skinner, E. A. (2016). Engagement and disaffection as central to processes of motivational resilience and development. In In K. R. Wentzel, & D. B. Miele (Eds.) *Handbook of motivation at school* (2nd ed.) (pp. 145–168). https://doi.org/10.4324/9781315773384

Smit, L. S. (2016). *A better understanding of 21st century skills in mathematics education and a view of these skills in current practice*. Utrecht University. https://dspace.library.uu.nl/handle/1874/335464

Smith, M., & Stein, M. K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). National Council of Teachers of Mathematics.

Sonnemann, J., & Hunter, J. (2023). Tackling under-achievement: Why Australia should embed high-quality small-group tuition in schools. https://grattan.edu.au/wp-content/uploads/2023/01/Tackling-under-achievement-Grattan-report.pdf

Spekle, R. F., & Widener, S. K. (2018). Challenging issues in survey research: Discussion and suggestions. *Journal of Management Accounting Research, 30*(2), 3–21. https://doi.org/10.2308/jmar-51860

Stahl, R. J. (1994). Using "think-time" skilfully in the classroom. *Journal of College Science Teaching, 23*(4), 241-245.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*(2), 213-226.

Sui-Chu, E. H., & Willms, J. D. (1996). Effects of parental involvement on eighth-grade achievement. *Sociology of Education, 69*(2), 126-141.

Sullivan, P. (2011). Teaching mathematics: Using research informed strategies. *Australian Education Review.* Australian Council for Educational Research.

Sullivan, P. (2020). *Leading improvement in mathematics teaching and learning*. Australian Council for Educational Research.

Sullivan, P., Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., & Russo, J. (2019). Dilemmas in suggesting mathematics representations to students. *Australian Primary Mathematics Classroom, 24*(4), 36–40. https://doi.org/10.3316/aeipt.227338

Sullivan, P., Borcek, C., Walker, N., & Rennie, M. (2016). Exploring a structure for mathematics lessons that initiate learning by activating cognition on challenging tasks. *Journal of Mathematical Behavior, 41*, 159-170. https://doi.org/10.1016/j.jmathb.2015.12.002

Sullivan, P., Clarke, D., & Clarke, B. (2012). *Teaching with tasks for effective mathematics learning*. Springer.

Tornero, B. (2017). The impact of a socio-constructivist approach on prospective primary teachers' attitudes about teaching and learning in general and mathematics in particular. *Páginas de Educación, 10*(1), 1–35. https://doi.org/10.22235/pe.v10i1.1357

Trilling, B., & Fadel, C. (2009). *21st century skills: Learning for life in our times*. Jossey-Bass.

Trudinger, M. (2017). Future proofing our kids. *The Update.* (5th ed.). Australian Mathematical Sciences Institute.

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9-35

van Laar, E., van Deursen, A. J. A., van Dijk, J. A. G., & de Haan, J. (2020). Determinants of 21st-century skills and 21st-century digital skills for workers: A systematic literature review. *Sage Open, 10*(1), 1–14.

Vincent-Lancrin, S. (2023). Fostering and assessing student critical thinking: From theory to teaching practice. *European Journal of Education*, *58*(3), 354–368. https://doi.org/10.1111/ejed.12569

Voogt, J., & Roblin, N. P. (2012). A comparative analysis of international frameworks for 21st century competences: Implications for national curriculum policies. *Journal of Curriculum Studies, 44*(3), 299-321. https://doi.org/10.1080/00220272.2012.668938

Vygotsky, V. (1978). *Mind in society*. Harvard University Press.

Vygotsky L. (1986). *Thought and language*. The MIT Press.

Waniganayake, M., Cheeseman, S., Fenech, M., Hadley, F., & Shepherd, W. (2012). *Leadership: Contexts and complexities in early childhood education*. Oxford University Press.

Way, J., & Bobis, J. (2017). *The literacy of mathematics.* PETAA Paper 208, 1-12. Primary English Teaching Association Australia.

Zajda, J. (2023). *Globalisation and inclusive schooling: Engaging motivational environments* (1st ed.). Springer Nature Switzerland. https://doi.org/10.1007/978-3-031-24400-1

Zippert, E. L., & Rittle-Johnson, B. (2018). The home math environment: More than numeracy. *Early Childhood Research Quarterly*, *50*(3), 4-15. https://doi.org/10.1016/j.ecresq.2018.07.009

Appendix A

Research Instrument

21st Century Mathematical Skills Matrix

DATE:	TIME:	Lesson Objective:
Practice	**1. Communication**	
Related belief	Communication of mathematical ideas help students clarify and solidify their understanding of mathematics	
Evidence	Students participate in rich mathematical conversations	
	The teacher facilitates a balance of oral and written mathematical communication	
	The teacher facilitates dialogue of mathematical argumentation	
Practice	**2. Critical Thinking & Problem Solving**	
Related belief	Is a mode of thinking about a problem? It is the ability to think clearly and rationally The process of finding solutions to difficult or complex issues. Identify and ask significant questions that clarify various points of view and lead to better solutions.	
Evidence	The teacher promotes and challenges students to think and reason	
	The teacher encourages student to reflect on their thinking and learning	
	Tasks engage students and maintain their involvement	
	The teacher promotes a positive disposition	
	Open ended tasks The teacher enables different possibilities, strategies and products to emerge.	
	Closed tasks	
	Students are encouraged to preserve.	
	Students discuss mathematical reasoning	
	Students use trial and error	
Practice	**3. Mathematical Creativity and Innovation**	
Related belief	Mathematical creativity ensures the growth of the field of mathematics as a whole. It is a divergent process. Innovation is to act on creative ideas to make a tangible and useful contribution to the field in which the innovation will occur	
Evidence	The teacher facilitates opportunities to image and mental imaginary encouraged.	
	The teacher provides a place and space to work creatively i.e. not at a traditional desk, use of IT	
	Students try a variety of heuristics	
	The teacher builds a connection to other domains and subject areas.	
	Students create a large range of ideas for solutions.	
Practice	**4. Collaboration**	
Related belief	Mathematics is a social activity Collaborate with others. Demonstrate ability to work effectively and respectfully with diverse teams. Exercise flexibility and willingness to be helpful in making necessary compromises to accomplish a common goal. Assume shared responsibility for collaborative work, and	

	value the individual contributions made by each team member.	
Evidence	Students complete joint tasks.	
	Students work independently	
	The teacher questions the students	
	Student question the teacher	
	Students question their peers	

Evidence recorded in a tally with a final percentage.

(Adapted from Battelle for Kids, 2019).

Appendix B

Parental Consent Form
Classroom Observations and Student Focus Group

Early Childhood Teachers' Preparation of Young Children with 21st Century Mathematics Skills

- I agree to allow my child to take part in this research project.
- I have read the Information Sheet provided and been given a full explanation of the purpose of this study, the procedures involved and of what is expected of my child.
- I understand that my child will be observed by the researcher during three mathematics lesson that will be conducted by your child's classroom teacher. Each of the classroom observations will be the duration of the dedicated mathematics lesson.
- I understand that my child may be asked to participate in a student focus group which will meet three times, at the beginning, midpoint and at the end of the data collection period. Each focus group will consist of five to seven students from my child's classrooms. My child will be asked questions about their perceptions of their classroom mathematics learning and experiences in an informal group setting on school grounds. Focus groups will all be audio-recorded.
- The researcher has answered all my questions and has explained possible problems that may arise as a result of my child's participation in this study.
- I understand that my child may withdraw or that I may withdraw my child from the research project at any time without the need for an explanation.
- I understand that all information provided by me or my child is treated as confidential and will not be released by the researcher to a third party unless required to do so by law.
- I agree that any research data gathered for the study may be published provided my name or my child's name or other identifying information is not disclosed.
- I understand that research data gathered may be used for future research, but my name or my child's name and other identifying information will be removed.

I consent to my child being involved in the classroom observations	
I consent to my child being involved the student focus group	

(Please tick the above box if you give your consent)

Name of parent/guardian			
Signature of parent/guardian		Date	

- I confirm that I have provided the Information Sheet concerning this research project to the above participant, explained what participating involves and have answered all questions asked of me.

Signature of Researcher		Date	

Appendix C

Child Classroom Observations Consent Forms

Early Childhood Teachers’ Preparation of Young Children with 21st Century Mathematics Skills

CLASSROOM OBSERVATIONS

- I would like to take part in this project.
- I am happy for you to ask me questions and take notes while you watch me in the classroom during math’s lessons.
- I understand that I can change my mind about taking part in the project at any time.

Name of child			
😊	☹️	Date	

- I confirm that the student has had the information read to them and that they have independently provided/denied consent.

Signature of Teacher/Parent		Date	

- I confirm that I have provided the Information Sheet concerning this research project to the above child and parent/guardian, explained what participating involves and have answered all questions asked of me.

Signature of Researcher		Date	

Appendix D

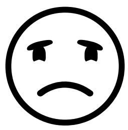
Child Focus Groups Consent Forms

Early Childhood Teachers' Preparation of Young Children with 21st Century Mathematics Skills

CHILD FOCUS GROUP

- I would like to take part in this project.
- I am happy for you to ask me questions and take notes while I am talking in a group.
- I am happy for you to audio record me.
- I understand that I can change my mind about taking part in the project at any time.

Name of child	

		Date	

- I confirm that the student has had the information read to them and that they have independently provided/denied consent.

Signature of Teacher/Parent		Date	

- I confirm that I have provided the Information Sheet concerning this research project to the above child and parent/guardian, explained what participating involves and have answered all questions asked of me.

Signature of Researcher		Date	

Appendix E

Participating Information Sheet
CHILD/PARENT PARTICIPANT INFORMATION SHEET

Early Childhood Teachers’ Preparation of Young Children with 21st Century Mathematics Skills

We would like to invite your child to participate in the research project: Early Childhood Teachers’ Preparation of Young Children with 21st Century Mathematics Skills.

What is the project about?

The research project will investigate how your child’s school supports the development of 21st Century skills within their teaching of mathematics in the early years. The study aims to guide the teachers to reflect on their current teaching with regard to 21st Century skills, and to assist the teachers to evaluate their intentional planning and assessment of the skills of mathematical creativity and innovation, critical thinking and problem solving, communication and collaboration. This project is one of four related school-university partnerships focusing on 21st Century learning occurring in this school. The broad aim of this school-university partnership is to improve teaching and learning in the school through an in-depth exploration of practices by employing multiple sources of information and data collection.

Who is undertaking the project?

This project is being conducted by Mistelle Moore and will form the basis for the degree of a Master of Philosophy, at The University of Notre Dame Australia, under the supervision of Dr. Derek Hurrell and Lorraine Day.

What will my child will be asked to do?

If you consent for your child to take part in this research study, it is important that you understand and discuss with your child the purpose of the study and what they will be asked to do. Please make sure that you ask any questions you may have and that all your questions have been answered to your satisfaction before you agree for your child to participate.

During the study the researcher will;

- Ask your child to participate in a student focus group which will meet three times, at the beginning, midpoint and at the end of the data collection period. Each focus group will consist of five to seven students from your child’s classrooms. Your child will be asked questions about their perceptions of their classroom mathematics learning and experiences in an informal group setting on school grounds. Focus groups will all be audio-recorded.

- Observe three mathematics lesson conducted by your child's classroom teacher. Each of the classroom observations will be the duration of the dedicated mathematics lesson.

All data collection will occur on school grounds.

What if I change my mind?

Participation in this study is completely voluntary. Even if you agree for your child to participate, you are free to withdraw them from further participation at any time without giving a reason and with no negative consequences. You are also free to ask for any information which may identify them to be withdrawn from the study.

Will anyone else know the results of the project?

Information gathered about your child will be held in strict confidence. This confidence will only be broken if required by law.

All audio recordings will be transcribed by the researcher. Your child will be attributed a pseudonym at this time. The original audio data file will be stored on a secure server under password protection. The de-identified transcription will be used for data analysis by the researcher. No audio file will ever be published or in the public domain.

Once the study is completed, the data collected from your child will be de-identified and stored securely in the School of Education at The University of Notre Dame Australia for at least a period of five years.

Who do I contact if I have questions about the project?

If you have any questions about this project please feel free to contact Mistelle Moore at Mistelle.moore@nd.edu.au. Alternatively, you can contact Dr. Derek Hurrell at derek.hurrell@nd.edu.au. We are happy to discuss with you any concerns you may have about this study.

Are there any possible risks from participation in this study?

There are no specific risks anticipated with participation in this study. The researcher collecting the data will attribute a pseudonym when transcribing the data, such that individuals will not be identifiable in the outputs, such as reports or research publications. While all efforts will be made to deidentify the individuals in the school community, it is important that participants in this study understand that this cannot completely ensure anonymity due to the specific nature of the case study and the small number of people involved in the study.

What if I have a concern or complaint?

The study has been approved by the Human Research Ethics Committee at The University of Notre Dame Australia (approval number 019169F). If you have a concern or complaint regarding the ethical conduct of this research project and would like to speak to an independent person, please contact Notre Dame's Research Ethics Officer at (+61 8) 9433 0943 or research@nd.edu.au. Any complaint or concern will be treated in confidence and fully investigated. You will be informed of the outcome.

How do I sign up to participate?

If you are happy for your child to participate, please sign on behalf of your child the attached form and return it to your child's classroom teacher.

Thank you for your time.

Yours sincerely,

Mistelle Moore

Appendix F

Parental Survey

Early Childhood Teachers' Preparation of Young Children with 21st Century Mathematics Skills

My name is Mistelle Moore, and I have been completely research in your child's pre-primary classroom this year. You received information on my studies at the start of the year. Your input is very important, and I would greatly appreciate you taking the time to complete a short survey. This survey is designed to gather information on how your school is preparing young children with 21st century mathematics skills.

Do you play any mathematics activities/games at home with your child? If so, can you give examples?

What 21st Century skills in general do you believe your child will need for living and working 2030 and beyond?

What mathematics do you believe your child will need for living and working 2030 and beyond?

What in the program at this school helps your child develop the mathematics for their future beyond school?

What would you like to see included in the program at this school to help your child develop the mathematics for their future beyond school?

<u>Thank you for taking the time to complete the survey, it is greatly appreciated.</u>

Appendix G

Lesson Observation One Matrix

21st Century Mathematical Skills Matrix
Pre-primary 12 students, 11 students present Teacher, Practicum Student, Educational Assistant
Mat Session-discuss terms, Task-draw a picture place on the chart
Key: Mat Session: M
Task: T
Summary: S
Summary Task: ST

DATE: 21/06/2021	TIME: 2:00- 2:45pm	Lesson Objective: Probability
Practice	**1. Communication**	
Related belief	Communication of mathematical ideas help students clarify and solidify their understanding of mathematics	Lots of questions by teacher during the task.
Evidence	Students participate in rich mathematical conversations **(4)**	MS√ T√ ST√ S√
	The teacher facilitates a balance of oral and written mathematical communication **(3)**	T√ ST√ S√
	The teacher facilitates dialogue of mathematical argumentation **(4)**	MS√ T√ ST√ S√
Practice	**2. Critical Thinking & Problem Solving**	
Related belief	Is a mode of thinking about a problem? It is the ability to think clearly and rationally The process of finding solutions to difficult or complex issues. Identify and ask significant questions that clarify various points of view and lead to better solutions.	
Evidence	The teacher promotes and challenges students to think and reason **(4)**	MS√ T√ ST√ S√
	The teacher encourages student to reflect on their thinking and learning **(2)**	T√ ST√
	Tasks engage students and maintain their involvement **(4)**	MS√ T√ ST√ S√
	The teacher promotes a positive disposition **(4)**	MS√ T√ ST√ S√
	Open ended tasks The teacher enables different possibilities, strategies and products to emerge **(2)**	T√ S√
	Closed tasks **(0)**	
	Students are encouraged to preserve **(4)**	MS√ T√ ST√ S√
	Students discuss mathematical reasoning **(4)**	MS√ T√ ST√ S√
	Students use trial and error **(2)**	T√ S√
Practice	**3. Mathematical Creativity and Innovation**	
Related belief	Mathematical creativity ensures the growth of the field of mathematics as a whole. It is a divergent process. Innovation is to act on creative ideas to make a tangible and useful contribution to the field in which the innovation will occur	Lots of opportunity to be creative with solutions.
Evidence	The teacher facilitates opportunities to image and mental imaginary encouraged **(3)**	T√ ST√ S√
	The teacher provides a place and space to work creatively i.e. not at a traditional desk, use of IT **(4)**	MS√ T√ ST√ S√
	Students try a variety of heuristics **(0)**	
	The teacher builds a connection to other domains and subject areas **(0)**	
	Students create a large range of ideas for solutions **(4)**	MS√ T√ ST√ S√
Practice	**4. Collaboration**	
Related belief	Mathematics is a social activity Collaborate with others. Demonstrate ability to work effectively and respectfully with diverse teams. Exercise flexibility and willingness to be helpful in making necessary compromises to accomplish a common goal. Assume shared responsibility for collaborative work, and	

	value the individual contributions made by each team member.	
Evidence	Students complete joint tasks **(0)**	
	Students work independently **(2)**	T√ S√
	The teacher questions the students **(4)**	MS√ T√ ST√ S√
	Student question the teacher **(4)**	MS√ T√ ST√ S√
	Students question their peers **(2)**	T√ S√

Evidence recorded in a tally with a final percentage.

(Adapted from Battelle for Kids, 2019).

Appendix H

Pre-Semi-Structured Teacher Interview Questions

Early Childhood Teachers' Preparation of Young Children with 21st Century Mathematics Skills

21st Century mathematics skills the study will be focusing on are mathematical creativity and innovation, critical thinking and problem solving, communication, and collaboration. When referring to the term '21st Century skills' in this study the term encompasses knowledge, skills, behaviours and dispositions.

Start the recorder.

1. How long have you been teaching?

2. How long have you been teaching at this school and this year level?

Pedagogy

3. Can you tell me how you structure your mathematics lesson?
    a. (How many times a week, duration of lessons, what does a usual mathematics lesson feature)

4. What is your understanding of 21st century learning?

5. Can you comment on what you believe to be important in terms of 21st century learning?

6. How confident are you in combining your use of pedagogy, content knowledge, and mathematical skills to effectively integrate 21st century learning?

Planning

7. How does the EYLF/ WA Curriculum guide your planning
8. Do you use the general capabilities in your planning? If so, in what ways?

9. Do you consider 21st century skills in your mathematics planning? If so, in what ways?

10. In what ways do you assess your students' mathematical learning?

11. Do you assess 21st century skills? If so, how?

Influences and barriers questions

12. What are the potential barriers you face when teaching mathematics? What strategies do you use to overcome these barriers?

Classroom Environment

13. Are there aspects of the school's classroom which enhance or limit the success of your mathematics teaching?

14. Are the majority of your mathematics lessons individual or groups tasks? (If group tasks, how are the groups selected?)

15. What kinds of materials do you use in your mathematics lessons?

Teacher Comfort

16. Do you enjoy doing mathematics activities with your students?

17. How comfortable to you feel about doing mathematics activities in your early childhood classroom?

School Environment

18. Do you discuss ideas and issues of mathematics teaching with other teachers?

19. What policies in regard to mathematics teaching/learning does your school?

20. Is there anything else that you think would be helpful to talk about?

Appendix I

Post-Semi-Structured Teacher Interview Questions

Early Childhood Teachers' Preparation of Young Children with 21st Century Mathematics Skills

21st Century mathematics skills the study will be focusing on are mathematical creativity and innovation, critical thinking and problem solving, communication, and collaboration. When referring to the term '21st Century skills' in this study the term encompasses knowledge, skills, behaviours and dispositions.

Pedagogy

1. Has your philosophy for teaching mathematics changed over the year? If so, why?

2. Has your structure of your mathematics lesson changed over the year? If so why? And how?

3. Have your perceptions/beliefs around 21st century learning changed over the year? If so, why? And how?

4. Can you comment on what you believe to be important in terms of 21st century learning?

5. If I said the 4C's (Creativity and Innovation, Critical Thinking and Problem Solving, Communication, Collaboration), in regards, to 21st century learning what would you think of?

6. How do you feel that you are able to combine your use of pedagogy, content knowledge, and mathematical skills to effectively integrate 21st century learning?

Planning

7. Over the last year have you changed how you use the EYLF/WA Curriculum to plan your mathematics lessons?

8. Do you use the general capabilities in your planning?

9. In what ways do you use 21st century skills in your mathematics instruction?

10. How do you assess your students mathematical learning?

11. Do you assess 21st century skills? If so, how?

Influences and barriers questions

12. What are the potential barriers you face when teaching mathematics? What strategies do you use to overcome these barriers?

Classroom Environment

13. Are there aspects of the school's classroom which enhance or limit the success of mathematics teaching?

14. Are the majority of your mathematics lessons (individual)/(groups tasks)? If group tasks how are the groups selected?

15. What kinds of materials do you use in your mathematics lessons?

Teacher Comfort

16. Do you enjoy doing mathematics activities with your students?

17. How comfortable to you feel about doing mathematics activities in the early childhood classroom

18. Is there anything else that you think would be helpful to talk about?

Thank you!

Appendix J

Principal Interview Questions

Early Childhood Teachers' Preparation of Young Children with 21st Century Mathematics Skills

21st Century mathematics skills the study will be focusing on are mathematical creativity and innovation, critical thinking and problem solving, communication, and collaboration. When referring to the term '21st Century skills' in this study the term encompasses knowledge, skills, behaviours and dispositions.

1. Can you comment on what you believe to be important in terms of 21st Century mathematics skills at your school?

2. Can you describe some of the key issues facing your school in regard to 21st century mathematics skills?

3. What reforms have you implemented to address these issues?

4. What would you like to be the key focus of your work in this school in the next few years, in 21st century mathematics skills?

5. What is your understanding of 21st century learning?

6. Can you comment on what makes 21st century skills worth teaching and learning?

7. How confident are your early childhood teachers of effectively integrating 21st century learning into their current teaching and learning processes are you? Why do you give this response?

8. Do they consider 21st century skills in your mathematics planning? If so, in what ways?

9. In what ways does your school assess your students' mathematical learning?

10. Do your teachers assess 21st century skills? If so, how?

11. What are the potential affordances and barriers your schools face when teaching mathematics? What strategies do you use to overcome these barriers?

12. Are there aspects of the school's classroom/environment which enhance or limit the success of your teacher's mathematics teaching?

13. Do your teachers discuss ideas and issues of mathematics teaching with other teachers?

14. What policies regarding mathematics teaching/learning does your school have?

Is there anything else that you think would be helpful to talk about?

Thank you for taking the time to answer the above questions it is greatly appreciated.