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Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics: The Proficiency Strands.

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Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the Proficiency Strands.

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Declaration

I hereby declare that this thesis is no more than 30,000 words in length including quotes and exclusive of tables, figures, appendices, bibliography and references. This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma. This thesis is entirely my own work and does not to the best of my knowledge breach any law of copyright and has not been taken from the work of others, save and to the extent that such work has been cited and acknowledged within the text of my work.

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Date: 1 September 2018

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Jim O'Neill

Abstract

This research examined how 14 Western Australian (WA) secondary mathematics teachers perceived effective mathematics teaching through the actions of teaching mathematics, described as Proficiency Strands. The research examined a variety of insights into what constituted effective teaching. Comparisons were made using an interpretive theoretical perspective of an instrumental case study and data were reviewed using a structured inductive framework with thematic analysis. Key findings of the research found that participants' beliefs and practices did help determine their perceptions of effective teaching but that understanding, and interpretation of mathematical proficiencies were less influential and inconsistently understood. The study found evidence that mathematical proficiencies are incorrectly regarded in a hierarchical sense. There was no evidence that teaching experience affected participants' understanding of mathematical proficiencies, but evidence was found that participants' lesson planning focused on classroom management and lesson content and less on the mathematical goals of the lesson. Participants felt that basic lesson structures employed by mathematics teachers could be improved, but had concerns over the volume of curriculum content, which makes that difficult to achieve.

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Chapter 1 Introduction

1.1 Research Question

This research examined how secondary teachers of mathematics perceive effective mathematics teaching through the actions of teaching mathematics. This was done to gather more information to help teachers, educators and managers gain insight into what features are considered important by classroom teachers in the effective practices they employ. This offers perspective into issues affecting mathematics such as falling uptake in students undertaking higher levels of mathematics and perceptions of falling standards in student performance (Hine, 2017). Adding to the understanding of effective teaching practices may offer further insights into the factors influencing student uptake in relation to the quality of mathematical instruction.

The populist press often simplistically presents Australian education, particularly mathematics education, as being below that of other nations when citing information from comparative international studies released on a regular basis (De Bortoli & Thomson, 2010; Thomson, De Bortoli, & Underwood, 2017). Generally, Australian students have failed to match standards in mathematics when compared to other neighbouring nations, particularly those around the Pacific Rim (Australian Government Department of Education, 2014). It is prudent to consider the curriculum and teaching philosophies in use in those countries to better understand the Australian perspective.

This research is important because there is little existing research evidence in Western Australia comparing the perceptions of effective teaching practices of teachers of mathematics to the actions of effective mathematics teaching. The Western Australian Curriculum Mathematics (WAC:M) (School Curriculum and Standards Authority, 2018a), indicates such actions as Proficiency Strands. The WAC:M declares the Proficiency Strands as Understanding, Fluency, Problem Solving and Reasoning. Those mathematical proficiencies are elaborated in the WAC:M (School Curriculum and Standards Authority, 2018b) where "The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored or developed" (p. 1). Sullivan (2011) posited that to address the aims of the curriculum teachers should ideally embed those Proficiency Strands in their regular practices. It is appropriate to collect information on the use and impact of Proficiency Strands in classroom practices. The research question, therefore is: What are Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the Proficiency Strands?

Gathering information on a range of aspects of the understanding and use of mathematical proficiencies, as described in the WAC:M, the research considered factors which may affect or influence those stated understandings. The research sub-questions are therefore:

a. Are teachers sufficiently familiar enough with the Proficiency Strands to identify them from a stated action?
b. What do teachers perceive as being effective mathematics teaching in a secondary classroom?
c. To what extent do teachers' beliefs of the purpose of mathematics in the secondary curriculum influence their teaching practices?
d. To what extent are teachers' perceptions of effective teaching in secondary mathematics reflected in their own practices?
e. To what extent does teacher experience influence the impact of teacher professional learning in their interpretation of the WAC:M?"

The investigation of factors focused on: (i) whether the Proficiency Strands were used and understood by teachers when planning lessons; (ii) what teachers of mathematics in Western Australia perceived as effective teaching through the examination of some staged teaching scenarios; (iii) the extent to which that perception of effective teaching had been influenced by the mathematical proficiencies of the WAC:M Proficiency Strands; (iv) whether Proficiency Strands were perceived by teachers as important when considering effective mathematics teaching, interrogated by comparing beliefs and perceptions of teachers with their common practices as described by Ball and Bass (2000); and (v) any influence on those teachers' perceptions of the importance of the Proficiency Strands in relation to the level of teacher mathematical academic background, length of teaching experience, and the amount and perceived quality of professional learning undertaken by teachers.

1.2 Research Plan

The study was conducted using quantitative and qualitative data gained from questionnaire and interview. It used a pragmatist epistemology using both an interpretive and positivist theoretical perspective (Crotty, 1998). This was thought an appropriate epistemology by the researcher as it used case study as its main theoretical qualitative perspective, where teachers of mathematics form a bounded system within the context of education (Gay, Mills, & Airasian, 2012), because they are constrained by the requirements laid down by the Department of Education in Western Australia. Instrumental case study was considered an appropriate perspective when attempting to understand something other than the general case (Johnson & Christensen, 2008). The positivist perspective was affirmed by the correlation and regression of beliefs and perceptions of teachers, triangulated against descriptions of effective teaching.

The research data were collected during the period November to December 2017 from a sample of 14 practising teachers. Data were collected in two rounds by (i) questionnaire, (ii) semi-structured interview and (iii) comprehension activity. In Round One the research, using questionnaires, examined: teacher perceptions of the purpose of secondary mathematics; what constituted effective mathematics teaching; and what were teacher beliefs about the importance of mathematics in education. In addition, teachers' regular enacted classroom teaching practices, teaching experience and background, including attended professional learning opportunities, were examined. In Round Two, (i) semi-structured interviews were used to elicit responses about effective teaching through examining written classroom scenarios depicting a Year 8 lesson, as well as (ii) gathering participant response to questions using the language of the Proficiency Strands through a comprehension activity.

A structured inductive framework, developed by Miles and Huberman (1994), was employed, using data reduction methods, to generate categories of perceptions of effective teaching. Those categories centred on modes of teaching instruction, lesson attributes such as modelling and differentiation, classroom management, the use of materials and presentation of content. Participant responses were also triangulated against beliefs and classroom practices responses gathered by questionnaire. The Proficiency Strand data were triangulated against participant beliefs and classroom practices gathered earlier and against participant teaching background and experience.

1.3 Background to Research

Teachers of mathematics in Western Australia have been using the Western Australian Curriculum: Mathematics (WAC:M) since early 2013 in government schools. In that time there has been opportunity for professional discussion and professional learning so that teachers could consider changes to teaching practices advocated by the WAC:M. Mathematical proficiency, termed Proficiency Strands, were expected to become a cornerstone of WAC:M implementation (Sullivan, 2011). Proficiency Strands describe the actions teachers should use in the planning and delivery of appropriate lessons as well as describing the range of required responses students might be expected to use in communicating answers (Sullivan, 2011).

To achieve an understanding of how the Proficiency Strands are used in everyday classrooms, researchers must also recognise other influences which impact on teachers' use of mathematical proficiencies. Those influences will include personal beliefs about: mathematics; learning; and teaching. Researchers must also understand the impact made by personal mathematical knowledge and the amount of teaching experience of any teacher. It is also appropriate to consider that individuals in this study, even though they may have volunteered for interview, may well be reluctant to express deeply held opinions to a stranger. Research instruments must be varied and be structured to allow insight into responses, also encouraging accurate responses. The instruments must also triangulate those responses for comparison to allow confidence in findings.

1.4 Thesis Structure

This thesis is structured into six chapters with sub-headings as appropriate. Chapter 1 deals with the introduction to the research and offers an insight into why the research is necessary, as well as detailing the research plan and the methodology underpinning its constructs. It then highlights limitations of the data and related conclusions.

Chapter 2 summarises the relevant literature associated with the study. The review aims to lead the reader through the historical background leading to the introduction of the current Western Australian Curriculum in Mathematics. It then considers the research rationale behind the initial Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2012) and examines the political and educational factors influencing the curriculum's development. Extant literature is also examined when discussing comparative curricula for other higher performing nations in comparative studies of student attainment. A prominent feature in this research is what constitutes effective teaching and learning. Different elements of effective teaching are then considered and discussed using research literature. The review then introduces other research with similar aspirations to this study and describes the information gained from that research. It highlights work carried out in England and South Africa into influences on effective teaching when considering teacher beliefs and practices and what teachers of mathematics themselves consider to be effective teaching.

Chapter 3 defines the methodology considered and used in this research. It discusses why a pragmatist epistemology is regarded as appropriate for the research and then details the instruments used in the study and their origins. The chapter also offers some understanding of the processes used when conducting the thematic analysis of teacher responses to the scenario prompts.

Chapter 4 describes the results gathered from the research instruments. It takes the information collected and presents to the reader a concise and summarised version of the data. Tables and figures are used to help display information in a succinct and understandable manner. Each research question is considered, and data is displayed appropriately, with a final data section linking triangulation elements to corroborate data across questions.

Chapter 5 details the analysis and resulting discussion about the data. It highlights important aspects of the research data to highlight the significance of features of that analysis. The results of each question are discussed and some perspective on what the information describes, linked to the relevant literature as appropriate, is offered.

Chapter 6 offers conclusions from the gathered data and discussion. It aims to point out the major findings of the research as well as offering insight into how this research could have been further strengthened, and the potential for future research. It concludes by suggesting potential actions relevant to teachers of mathematics and those delivering professional learning to teachers.

There are appendices detailing each research instrument used in the study. This is included for reference and consideration to allow the reader to better understand the interview process undergone by participants and how data were gathered and interpreted.

1.5 Limitations

This study interviewed 14 participant teachers currently teaching mathematics in Western Australian Department of Education schools. The size of the sample must statistically limit the scalability of the research findings. The researcher aimed to interview a broad sample of teacher experience and background mathematical knowledge. It was not possible to find subjects who had undergone the Department of Education 'Switch' program (Department of Education, 2016). The 'Switch' program for mathematics was designed to take qualified teachers with limited tertiary education through an accelerated and abbreviated understanding of mathematics so they would be qualified to teach mathematics at a secondary level. Having 'Switch' teachers participate in the study may have improved the data centred around the mathematical knowledge required by teachers and how that knowledge influences beliefs and practices.

Other conclusions are limited statistically by the sample size and conclusions are not generalisable as a result however, there is no reason to think that the sample used is not representative of a wider population and no bias is expected. A range of data findings in this study concur with larger sample data from other research, and this suggests the data in this study may be like that derived from a larger representative group of teachers.

Chapter 2 Literature Review

This research was designed to focus on Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the Proficiency Strands. To explore this proposition, it was necessary to consider different aspects of effective secondary teaching of mathematics as well as the attributes of the Western Australian Curriculum: Mathematics as currently applied to mathematics education in schools. This chapter will therefore consider an historical progression for the current curriculum, and it will reflect arguments for and against such curriculum development and how that discussion has shaped the curriculum. Deliberation of effective teaching and what factors influence perceptions of effective teaching will create a focus on the suitability of the curriculum design. Those factors relating to teacher ability to embrace and enact the Proficiency Strands will be considered. Considering research with similar aspirations to this study, and a description of information gained from other research, forms the final section of the chapter.

2.1 Historical Background to The Western Australian Curriculum: Mathematics

2.1.1 Research influencing the development of the curriculum.

In 2000 Jeremy Kilpatrick led a committee, commissioned by the National Research Council in the United States (Kilpatrick, Swafford, & Findell, 2001). Its remit was to conduct an examination of research to recommend best practices in the teaching of mathematics. The published report, *Adding it Up* (Kilpatrick et al., 2001), became a seminal work which influenced the teaching of mathematics in the United States, and later in the United Kingdom and Australia (Sullivan, 2011) among others. Prior to the work of the Kilpatrick Committee, mathematics education in the United States had suffered in the national press from the so called 'math wars' (Kilpatrick, 2001a). As an individual, Kilpatrick saw education researchers and politicians taking differing views on mathematics education. One side of the argument was negatively depicted by Cheney (1997) who claimed the use of 'fuzzy maths' marginalised teaching of numerical skills, overly advocated the use of cooperative learning strategies, and inappropriately amplified the relation between what is taught and why it should be taught. He felt that this promoted the view that a strong rationale for a wrong solution was as important as a correct solution. Cheney (1997) and others (Gardner, 1998; Stein, 1996) saw 'fuzzy maths' as choosing to employ nothing more than 'labels' of learning when describing the educational reform proposed by the California Department of Education (1992) in its curriculum framework. The Californian curriculum framework, proposing a different view to Cheney's and others advocated teachers develop a more inclusive approach to teaching mathematics with its emphasis on problem solving and investigation (Boaler, Wiliam, & Brown, 2000; California Department of Education, 1992), which epitomised the modern alternate approach Cheney (1997) found so objectionable.

The Kilpatrick committee attempted to avoid the extreme positions of those 'math wars' when proposing the arguments of *Adding It Up* (Kilpatrick, 2001b). The Kilpatrick committee's work was, however, not without criticism. Previously, Apple (2000) had warned that the political influence of neo-liberalism allowed the introduction of a national curriculum and national testing which had not always had the rationale of research behind it. Apple (2000) further emphasised that the link between national tests and other indicators of performance had been organised around a concern for regulation through the use of standardised external assessment, which Apple (2000) described as being "… connected with a strong mistrust of 'producers' (e.g., teachers) and to the need for ensuring that people continually make enterprises out of themselves" (p. 252).

Such criticism was echoed by other researchers (Bernstein, 2000; Gardner, 1998; McCulloch, 1997; Whitty, Power, & Halpin, 1998).

Attempting to find the centre ground of the 'math wars' arguments, Kilpatrick (2001a) developed the term "mathematical proficiency" (p. 106) to focus on the common disparities between research and practice in teaching. For the authors, the notion of mathematical proficiency could help teachers set achievement goals for students and act as a benchmark for teachers when considering expertise in teaching mathematics. Kilpatrick's 'mathematical proficiency' concept took the view that problem solving offered a context which would allow and encourage the mathematical development of every student, irrespective of their current mathematical skillset. The inclusion of problem solving in mathematics education has gained traction since the early 2000s. Boaler et al. (2000) suggested that using problems and structured tasks help to engage the widest range of student abilities. Others have agreed and developed arguments and material to support a task-based style of teaching mathematics (Back., Foster, Tomalin, Mason, Swan, & Watson, 2012; Boston & Smith, 2011; Clarke, Clarke, & Sullivan, 2012b; Sullivan, 2011; Sullivan et al., 2014). Earlier, Ernest (1991) had advocated that a major part of doing mathematics involves "… human problem solving and posing" (p. 281). By this he meant that when emerging mathematics problems are tackled, those problems often serve to develop major advances in mathematical knowledge and act as growth points in students' understanding mathematics. Ernest went on to propose a model that could be used to advance teaching and learning and this also involved teachers improving their own mathematical skillset so as to benefit students. In those discussions, the idea of a hierarchically structured curriculum had been an assumed pedagogical teaching model in the teaching of mathematics. Ernest (1991) concluded that if the mathematics curriculum was to reflect the discipline of mathematics it must not imply only a fixed hierarchical structure. He posited that there were multiple conceptual structures possible which would explain why students gain different concepts and skills in non-hierarchical ways.

2.1.2 Commission of the new curriculum.

In 2010, Sullivan was commissioned by the Australian Council for Educational Research to consider mathematics teaching in Australia. Sullivan (2011) recommended that the Australian Curriculum: Mathematics have a focus on mathematical proficiency as suggested by Kilpatrick et al. (2001). Sullivan (2011) stipulated that such a focus must employ tasks and other rich activities as a core of learning activity for all students. Yet Walshe (2015) attested that there is often a disconnect in the use of curriculum statements between the authors' intentions for outcomes, and how those outcomes are interpreted by teachers and others. Walshe also observed that there had been a move in Western cultures for curricula to be more student-centred and to lean towards further skills-based approaches, offering greater autonomy for teachers in the classroom. In an earlier argument, the Australian Government Department of Education review (2014) accepted that there could be a disparity over what the intended curriculum might detail as opposed to how that curriculum could be implemented in classrooms, and what students might actually attain as a result of the curriculum. This meant there could be a situation, even an expectation, where the task-based approach using the Proficiency Strands as advocated, might perhaps be adapted in some unforeseen way by teachers, and not necessarily as expected by Sullivan (ACARA, 2012). This could therefore influence the effectiveness of the curriculum.

Sullivan, Clarke, Clarke, Farrell, and Gerrard (2013) accepted that using a task-based teaching approach could lead to a diversity in approaches to planning lesson content, and to some confusion on the part of teachers about the important ideas contained in the curricular documentation. Liljedahl, Chernoff, and Zazkis (2007), when working with a group of teachers developing challenging teaching tasks, had noted that often teachers were faced with the challenge of constructing a task which was both mathematically and pedagogically sound, but that they could do so if provided with support. This was reinforced by Zhang and Stephens (2013) who examine the planning of a group of Chinese and Australian teachers. They found that Chinese teachers often spent more time planning lessons, and in greater detail with clearer teacher knowledge of content, than their Australian counterparts. Zhang and Stephens commented that Chinese teachers understood more deeply how to plan to achieve the intentions of the curriculum outcomes than Australian teachers.

2.1.3 General proficiencies and mathematical Proficiency Strands.

The pedagogy of how to structure and deliver chosen content also has an impact on effective mathematics teaching. The Australian Curriculum: Mathematics, as designed by Sullivan (2011) and ACARA (2012) aligned itself with the concept of mathematical proficiency as outlined by Kilpatrick (2001):

> The five strands of mathematical proficiency are (a) conceptual understanding, which refers to the student's comprehension of mathematical concepts, operations, and relations; (b) procedural fluency, or the student's skill in carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately; (c) strategic competence, the student's ability to formulate, represent, and solve mathematical problems; (d) adaptive reasoning, the capacity for logical thought and for reflection on, explanation of, and justification of mathematical arguments; and (e)

> productive disposition, which includes the student's habitual inclination to see mathematics as a sensible, useful, and worthwhile subject to be learned, coupled with a belief in the value of diligent work and in one's own efficacy as a doer of mathematics (p. 107).

As lead authors of the Australian Curriculum: Mathematics, Sullivan and other writers for ACARA (2012) reduced the number of proficiencies to four – Understanding, Fluency, Problem Solving, and Reasoning (ACARA, 2012). The curriculum (ACARA, 2012) used a graded model of conceptual development when detailing scope and sequence from Foundation to Year 10 as a content structure, thereby in part satisfying the non-hierarchical demands of conceptual understanding discussed by Ernest (1991) when he maintained that mathematics is not a linear subject. It maintained the need for teachers to include other teaching strategies by emphasising 'general capabilities'; that is the knowledge, skills, behaviours and dispositions that would support students to live and work fruitfully in the 21st Century. Those General Capabilities include Literacy, Numeracy, Information and Communication Technology, Critical and Creative thinking, Personal and Social Capability, Ethical Understanding and Intercultural Understanding (School Curriculum and Standards Authority, 2018c). This is in keeping with the goal of developing 21st Century learners (American Association of School Librarians, 2009) where learners are expected to access, process and use information from many sources to become useful citizens.

2.1.4 International comparisons.

Mathematical curricular reform in the early 21st Century has been driven by many factors both researched theory and actual statistical comparison. As stated, Kilpatrick et al. (2001) had developed the idea of mathematical proficiency. At the same time, comparative statistics were being generated using international studies by the Organization for Economic Cooperation and Development (OECD) in its Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). Such studies developed a by-product of comparison tables of performance (De Bortoli & Thomson, 2010; Thomson et al., 2017). As Apple (2000) asserted, political developments in many countries positioned education as a market driven structure which could be 'measured' by national results. In the international comparative analysis of PISA countries such as Finland and those of the Pacific Rim were seen as strong performers (Australian Government Department of Education, 2014). Researchers have looked to those countries to gain some insight into how they might learn from high performing nations. In Finland (Vitikka, Krokfors, & Hurmerinta, 2012) and in Canada (Gouvernement du Québec, 2018) there have been very similar changes in the curriculum structure to that designed in Australia, yet those nations have consistently performed better than Australia in comparative tests like PISA (Thomson et al., 2017).

When considering higher performing countries in comparative tests, the Australian Government Department of Education (2014) stated that Pacific Rim nations seemed to more effectively structure their curricular development to align with expected models of 21st Century employment and skills. Mikk, Krips, Säälik, and Kalk (2016) observed, when comparing practices in Finnish and Swedish mathematics education, that Finnish teachers spent more time discussing and preparing lessons suited to students as opposed to Swedish teachers who concentrated on pedagogy. In comparative studies Finland regularly scored higher than Sweden (De Bortoli & Thomson, 2010; Thomson et al., 2017). Kilpatrick (2001a) also commented on them when he observed that studies such as TIMSS were used by commentators and observers as evidence that teachers were 'better' in certain high performing countries, or that the information as presented ignored cultural and social factors which were not easily replicated. That such comparisons are often generally unhelpful is accepted (Boaler, 2016; Boston & Wilhelm, 2017; Krupa & Confrey, 2017; Mikk et al., 2016; Vashchyshyn & Chernoff, 2016; Zhang & Stephens, 2013). However, as Apple (2000) explained, information in this format has allowed authorities and stakeholders to frame an education debate which also always includes national standards, national curricula and national testing.

2.1.5 The Western Australian Curriculum: Mathematics.

In June 2013 the Schools Curriculum and Standards Authority in Western Australia (SCSA) issued a directive to establish phase 1 of the Australian Curriculum for English, mathematics, science and humanities (SCSA, 2014) in Western Australia. The directive established a timeline for complete implementation in 2015. The material provided assurance that the General Capabilities and the Proficiency Strands in mathematics would incorporate the ACARA curriculum of 2012 (ACARA, 2012; SCSA, 2018c). In 2017 SCSA amended the curriculum in mathematics, clarifying some content strands, and identified the mathematics curriculum as version 8.1 (SCSA, 2018c). The number of amendments to the ACARA (2012) K-10 curriculum were minimal. Version 8.1 is currently used in schools in Western Australia (SCSA, 2018b).

2.2 Factors Influencing Effective Teaching in Mathematics

2.2.1 Pedagogy and lesson goals.

Sullivan et al. (2013) offered support and advice on teaching mathematics which was expected to help teachers to embrace the reformed curriculum and to inform changes to their practices. When researching the nature of the content Australian teachers choose to teach Clarke , Clarke, and Sullivan (2012a) found that teachers usually talked to colleagues and referred to curriculum documents when planning lessons. They also found that teachers tend to use assessment information in deciding which mathematical conceptual understanding to incorporate into lessons. Clarke et al. (2012a) highlighted the importance of curriculum documents communicating the performances that are valued by the curriculum as well as how those concepts (with their related values) can be developed. To help direct the curriculum implementation Sullivan et al. (2013) introduced six key principles for effective teaching in mathematics. The principles included:

- articulating goals by identifying key ideas that underpin content (p. 25)
- making connections by building on current student knowledge (p. 26)
- fostering engagement by using rich and challenging tasks (p. 26)
- differentiating challenges by encouraging students to interact and question each other (p. 27)
- structuring lessons by allowing students to communicate ideas (p. 28)
- teachers summarising key ideas and promoting fluency and transfer through mental practice or regular written skills (p. 29).

However, Sullivan et al. accepted that using a task-based teaching approach would lead to a diversity in teaching approaches (Sullivan et al., 2013). To address this variety, Clarke et al. (2012b) developed reasoning tasks for the primary education sector, whilst Sullivan and colleagues worked with groups of primary and secondary teachers when developing further challenging tasks (Sullivan et al., 2014). The link between challenging tasks and key ideas as learning goals remains an area of interest for research.

The focus on activities rather than goals was reinforced by Hemmi and Ryve (2015) who, when comparing Swedish and Finnish teachers' planning, found that Swedish teachers tended to consider effective teaching as involving more exchanges with students, and using individual perceptions and everyday situations to relate mathematics to real life. In comparison, Finnish teachers tended to emphasise the importance of precise demonstrations of mathematical procedures, the completion of homework, and accentuating specific goals for every lesson. It appears that the Swedish model would align better to the description of the aims of teaching mathematics for understanding, as advocated by modern theorists such as Boaler (2016) and Sullivan et al. (2013). This appears to be less the case with the Finnish model. In a project using 60 Australian and 60 Chinese teachers, Zhang and Stephens (2013) found that the design of teaching was a critical dimension in enabling curriculum reform and that Chinese teachers often used more formal expressions of language, particularly in algebraic language, in their teaching of mathematics than their Australian counterparts. Notably, both Finland and China are higher performing nations than Australia in comparative international studies (Thomson et al., 2017). This raised questions of potential differences in the mathematical abilities of Chinese and Australian teachers (Zhang & Stephens, 2013) which Zhang and Stevens rejected.

2.2.2 Lesson planning.

Reigeluth (2013) looked at elements of effective mathematics teaching with an emphasis on lesson planning. When looking at instructional design theory, Reigeluth identified three levels of analysis to evaluate how well a method of teaching worked in achieving instructional outcomes: these are "effectiveness, efficiency, and appeal" (pp. 9-10). Effectiveness considered how well a method worked irrespective of the learning goals. Efficiency required a cost/time benefit analysis of the lesson; did the lesson reach the goals in an acceptable time-frame? Level of appeal related how much the students enjoyed and engaged with the lesson (Reigeluth, 2013). With a similar intent, Akyuz, Dixon, and Stephan (2013) used five headings to suggest a framework to improve the planning practices of teachers: preparation, reflection, anticipation, assessment, and revision. This again highlighted the importance of planning and preparation. For the development of mathematical proficiencies, Kilpatrick et al. (2001) were concerned that American teachers placed their emphasis on the procedures of the activities students go through, rather than the goals of those activities, and Reigeluth would frame this as privileging efficiency over effectiveness. Similar concerns were expressed in other research (Gardner, 1998; Stein, 1996).

2.2.3 Collaboration and classroom layout.

The Western Australian Curriculum: Mathematics highlighted student collaboration as one of its personal and social capabilities (School Curriculum and Standards Authority, 2018c). In mathematics education seating plans are common (Watt & Goos, 2017). The structure of those seating arrangements has been thought to influence student engagement as well as the advance of student collaboration. Gremmen, van den Berg, Segers, and Cillessen (2016) found, when interviewing 50 teachers, that seating was influenced by school culture as much as student learning. They also noted that arrangements can vary from single rows, pairs, small groups of four to larger U-shaped arrangements. In the Gremmen et al. study, teachers talked about classroom management as a major influence in deciding upon seating arrangements. Gremmen et al. (2016) also highlighted that student interactions played an important part in the success of seating arrangements. Fernandes, Huang, and Rinaldo (2011) noted that where a student sits in the classroom had an impact on student attainment and student engagement. Students who are far from the teacher, according to Fernandes et al., were less engaged in lessons and student collaboration was often off-task.

The importance of seating locations is supported by earlier work by Marx, Fuhrer, and Hartig (1999) who noted that students seated in rows asked fewer questions than students seated in semi-circular seating arrangements. Marx et al. suggested that seating influenced the promotion of an engaging classroom and allowed student collaboration as a learning goal. This implied that restricted seating designs, in single rows or other similar layouts, would affect the ability of students to interact and question each other, proposed by Sullivan et al. (2013) as one of the six principles of effective teaching.

Geiger, Anderson, and Hurrell (2017) also noted that new teachers were heavily influenced by the culture of the school and department in which they work. Employing Valsiner’s zone theory, Geiger et al. found that successful practices are closely tied to school context and cultural practices within that school, and that beginning teachers usually complied with colleagues and school values. Lee, Walkowiak, and Nietfeld (2017) noted that new teachers had the greatest concerns about their own student control influencing their classroom management and were therefore more likely to adopt restricted seating arrangements. This said, it is acknowledged that more experienced teachers may well adopt the same practices if experiencing classroom management issues (Roffey, 2004).

2.2.4 Mathematical knowledge for teaching (MKT).

There has been a long-standing debate, first highlighted by Shulman (1986) and continued by Ball, Thames, and Phelps (2008) and Rowland, Turner, and Thwaites (2014), concerning the knowledge teachers of mathematics require to complete the variety of tasks required in an effective classroom (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997a; Ball & Bass, 2000; Davis & Simmt, 2006; Hill & Ball, 2009; Park & Oliver, 2008). In the mid to late 1980s, research and standards in education tended to be constructed around how teachers managed their classrooms rather than the mathematical lesson content and effective questioning (Shulman, 1986). Shulman emphasised the importance of mathematical knowledge for teaching (MKT), that is, the very specific knowledge teachers of mathematics require to be effective teachers. Ball, Hill, and Bass (2005) later asserted that curricular reform cannot expect to improve educational performance without concentrating on what teachers of mathematics know and can teach. Ball et al. (2008) went on to investigate MKT and to develop the assertion "… that there is a need to carefully map [MKT] and measure it. This includes the need to better explicate how this knowledge is used in teaching effectively." (p. 404).

Other researchers have variously described the many forms of knowledge required for teaching mathematics (Adler & Davis, 2006; Ball et al., 2005; Davis & Simmt, 2006; Hill & Ball, 2009; Park & Oliver, 2008; Rowland et al., 2014). However, all agree that a teacher needs to not only know mathematical content, but how to engage that content in a manner students can understand. Teachers also, in the best cases, can predict mistakes that students will make. Park and Oliver (2008) asserted that the ability to apply those different forms of knowledge are indicators of the development from new to expert teachers. Rowland et al. (2014) developed a quartet of factors, "foundation, transformation, connection and contingency" (pp. 319-320), which link a teachers' understanding of content with their ability to present and adapt that content for others to understand. Rowland et al. asserted that their knowledge quartet complemented work done by Shulman (1986), Askew et al. (1997a) and Ball et al. (2005) on MKT.

In researching the different attributes displayed by experienced and expert teachers, Hattie (2012) characterised differences in their application of teaching knowledge. Drawing on information from his 2009 previous work on a meta-analysis of education research, Hattie (2012) calculated effect sizes of effective teaching from over 800 research studies across the education spectrum. He used this information to consider the importance of the teacher as a variable in the classroom equation. Hattie found a range of effect sizes suggesting that teachers employ a range of skills, some of which are more effective than others. Focussing on the most effective skills, Hattie developed a list of observations which might help distinguish expert from experienced teachers. The author estimated that a student in a high-effect teacher's class might have as much as a full year's academic advantage over those students in low-effect teacher classes. The contributing attributes, Hattie claimed, included being able to identify the optimal ways to represent different teaching topics, while creating an optimal learning environment and monitoring learning through providing effective feedback. Hattie also emphasised that high-effect teachers have high expectations that all students can meet success criteria and cited the work of Dweck (2010) to support the assertion.

Implicit in Hattie's (2012) observations is the ability of an expert teacher to choose from a wide range of teaching styles suited to the concept and learning outcomes required by the content strand. Those modes of learning could include direct instruction, using manipulative materials or other appropriate teaching methods (Hattie, 2012).

2.2.5 Ineffective teaching.

In determining effective teaching it is often useful to discuss how less effective teaching is considered and defined. Boaler et al. (2000) asserted that ineffective teaching often led to disaffection in students, and that this was linked to student ability groupings and less effective teaching methodologies. Boaler et al. asserted that it was the techniques of grouping, rather than the results achieved from grouping, that most affected student efficacy. Boaler et al. continued that often ability groupings lead to difficulties for both high and low ability students, estimating that more than 30% of students suffer mathematics anxiety and related performance issues in the classroom.

2.2.6 Task-based instruction.

Adler and Davis (2006) had noted that teacher knowledge of both content knowledge and pedagogical content knowledge, influenced their choice of challenging tasks when planning lessons, particularly lessons with task-based exploration at its core. Smith and Stein (2011) developed a group of five practices designed to help teachers use challenging tasks in the classroom. Those practices include "… anticipating, monitoring, selecting, sequencing and connecting" (p. 21). Smith and Stein advocated the use of the five practices as a method of improving the discussion for the goals of the lesson to be achieved. They also commented on the need for teachers to complete tasks fully as part of the knowledge required for open discussions, which they saw as a critical component of the learning involved in challenging tasks. Back et al. (2012) commented on the importance of teachers completing tasks before attempting to use them in a classroom and to consider the implied knowledge required to complete the tasks.

Ball et al. (2005) commented on the suitability of having non-specialist teachers of mathematics teaching mathematics using a task-centred reasoning approach. They highlighted potential issues about a lack of teacher understanding affecting mathematical development of students. Sullivan (2011) further engaged with concerns of teacher subject knowledge when addressing the question of whether it is appropriate to ask all teachers to engage with reasoning tasks, as an enactment of the Proficiency Strand of Reasoning (ACARA, 2012; School Curriculum and Standards Authority, 2018b) but decided that with appropriate training, preparation and support every teacher could undertake reasoning and rich tasks in regular classroom teaching.

2.2.7 Using manipulative materials and the CRA instructional approach.

Bouck and Park (2018) conducted a review of 36 articles involving manipulative materials in teaching mathematics. In more than 80% of the articles the use of manipulative materials was linked to the concrete - representational - abstract (CRA) instructional strategy. Whilst also highlighting the need for further research into CRA and using manipulative materials, Bouck and Park (2018) found that using manipulative materials had a positive effect on student performance. Strickland and Maccini (2013) cited some examples of successful CRA, and found that using a CRA approach using concrete manipulatives was an effective strategy to improve students' conceptual understanding and procedural fluency when multiplying two linear expressions in algebra, for example $[(a+2)(b+3)]$.

To develop a better understanding of the difficulties confronted when students moved from concrete to abstract representation (CRA) Fyfe, McNeil, Son, and Goldstone (2014) conducted a study which recommended starting with concrete manipulatives and then overtly and progressively phasing to a more abstract treatment.

Fyfe et al. argued that this methodology would go beyond the narrow benefits of using either concrete or abstract materials alone. They asserted that this model was favoured in research and was supported in newer mathematics curricula. The Fyfe et al. (2014) argument is supportive of the five practices of Smith and Stein (2011). However, Brown, McNeil, and Glenberg (2009) observed that teachers, when using manipulative materials, face substantial challenges in developing links between concrete non-symbolic concepts and abstract representations in lessons. Kaminski, Sloutsky, and Heckler (2009) pointed out that learners often find it difficult to transfer learning when completing tasks and actions when using concrete materials to more abstract symbols or depictions without adequate support, and that they often require specific direction from teachers. Brown et al. (2009) remarked that simply using concrete materials does not in itself guarantee the successful achievement of mathematical concepts.

The use of manipulative materials was described by Carbonneau, Marley, and Selig (2013) as an efficacious teaching strategy. In a meta-study of more than 7000 students from Kindergarten to Year 12 they identified minimally significant effect sizes in favour of using manipulative materials. (This was, however, moderated by the influence of both instructional and methodological strategies in the studies themselves.) In extension of the meta-studies into further areas, Carbonneau et al. (2013) concluded that such analysis revealed moderate to large effect sizes for retention, and small effect sizes for problem solving ability to relate to other topics and using appropriate language to justify working over the use of abstract mathematical symbols. Although Carbonneau et al. (2013) found small effect sizes for the same factors when using concrete manipulatives over more abstract methods, they did suggest there needed to be acknowledgement of the developmental experience of the user, the suitability of the objects used, and the level of guidance employed by the teacher when employing manipulative materials in classrooms. In a different meta-analysis into the use of virtual manipulatives, there was an acknowledgement of the ability for a student to explore different aspects of the concept with the learner in charge, leading to a moderate effect size on student attainment Moyer-Packenham et al. (2016).

2.2.8 Explicit instruction.

Hughes, Morris, Therrien, and Benson (2017) conducted a broad overview into the nature of explicit instruction. Hughes et al.’s (2017) review found five attributes in explicit instruction commonly identified in research. The attributes were to “segment complex skills, draw student attention to features of the content through modelling, promote student engagement by using systematically faded supports, provide opportunities for students to respond and receive feedback and create purposeful practice opportunities” (pp. 141-142).

Teachers of mathematics have used explicit instruction as a model of effective teaching for a number of years (Hattie, 2012). Archer and Hughes (2011) described explicit instruction as offering “instructional delivery which is characterised by clear descriptions and demonstrations of a skill, followed by supported practice and timely feedback” (p. 3). This style of teaching employed a direct approach which included design and delivery instructions. Archer and Hughes (2011) maintained that at the core of explicit teaching was a scaffolded structure which lead the student towards completion of the learning goal, using clear learning intentions and demonstrations. Askew, Rhodes, Brown, Wiliam, and Johnson (1997b) described explicit instruction as conforming to a transmission orientation where the teacher was at the centre of the instruction. However, Fisher and Frey (2008) adapted the explicit teaching model by adding the concept of gradual release. They advocated what became known as the ‘I do, We do, You do' aide-memoire to assist teachers in the gradual release of learning repsonsibility during an explicitly taught task.

In a report from New Zealand, Tait-McCutcheon, Drake, and Sherley (2011) suggested that, through the reflection process, the single teacher in their case study came to understand that a better model of teaching would also incorporate more task-based activity learning rather than explicit instruction alone. Tait-McCutcheon et al. (2011) also suggested that knowing basic facts allowed students to free short term memory because it allowed more time for reasoning in challenging problems, although they did confirm that there was no agreement in the research literature as to what constituted 'basic facts'. The importance of students knowing basic facts was a feature of that teacher's understanding and her employment of explicit instruction. Perry (2007) noted that teachers, who were described as experts, revealed their belief that there was a place for practice once understanding had been gained, but that they were not inclined towards drill or repeated skill-based questioning.

2.2.9 Direct instruction.

Hughes et al. (2017), when exploring explicit instruction, found it necessary to include both Direct Instruction (upper case) and direct instruction (lower case) as aspects of cited research. Direct Instruction is based upon the work of Englemann and Becker (Engelmann & Carnine, 1982). In exploring differences in terminology, Hughes et al. (2017) indicated that direct instruction had its origins in the early 1960s, although most of the research was conducted in the 70s and 80s. Hughes et al. (2017) identified Direct Instruction as a mode of instruction which includes scripted lessons and highly structured content learning sequences. Direct Instruction included both curriculum and instructional direction, what to teach and how to teach it (Hughes et al., 2017). Explicit instruction, by comparison, only had a focus on how to teach. Hughes et al. (2017) also described 'direct instruction', as based on effective teaching behaviours as well as reflective ones. Hughes et al. (2017) concluded there seemed little to separate lower case direct instruction and explicit instruction in much of the research literature.

Huitt, Monetti, and Hummel (2009), when discussing direct instruction, asserted that whatever direct instruction a teacher deems to be essential or important should be demonstrated using an active presentation. This presentation should describe the process in a clearly presented sequence, based on some form of task analysis, leading to completion. Hattie (2009) found that using similar structures could give an effect size of learning improvement up to 0.7, or almost two years progress. A popular version of the direct instruction approach was the use of scripted lessons. Huitt et al. (2009) commented that commercially produced direct instruction materials had often been subjected to normalisation and trialling that was unlikely in teacher-made materials. According to Englemann (2004) such standardisation of curriculum, when used as intended and, when measured by standardised tests of basic skills, had an established positive impact on student learnin. Further, in Englemann's opinion, this positive impact would be more than would be expected from any of the cognitive approaches using less structured methods, such as those proposed by Boaler (2016) and Sullivan (2011). Both explicit and direct instruction require sufficient time on task and structured questioning as essential features (Huitt et al., 2009).

Direct instruction, however, has also been critisised as a teaching approach. Ewing (2011), in a review of the literature, found direct instruction had been used and developed for more than 30 years. She proposed that direct instruction had limitations when used with Indigenous Australians, using evidence from PISA studies analysed by De Bortoli and Thomson (2010). She argued that the low attendance and a poor sense of self-efficacy in mathematics of many Indigenous Australian children in schools, meant that other pedagogical strategies could support conceptual development in better ways. Roussouw, Rhodes, and Christiansen (1998) saw direct instruction as lacking in enquiry procedures. They questioned whether, without engagment in problem solving and creating, if the higher order aspects of Bloom's taxonomy[1] would be missing in direct instruction, thereby limiting the cognitive development of the student. This lack of higher order thinking would also fail to match ACARA's (2012) requirement of creative and critical thinking, a general capability in the Australian curriculum.

The more important consideration, however, in this study, was whether reasoning and rich task-based materials, rather than explicit or direct instruction, led to improved mathematical learning outcomes (Roussouw et al., 1998). Research findings (Hattie, 2009; Roussouw et al., 1998) have deemed it better for a teacher to use varied approaches to suit the concept and to offer a wider range of learning activities to give a broad range of teaching and learning experiences. McQualter (2016) observed that teachers of mathematics must make many decisions, including which teaching models to use, and society expects that all those decisions should be wise. Hughes et al. (2017), in discussing whether explicit instruction, Direct Instruction or direct instruction is effective, concludes that more work needs to be done to establish links to more cognitive forms of instruction, as well as to identify which strategies are most effective in improving learning.

2.2.10 Teacher professional learning.

Western Australian teachers, as part of the requirements of continued registration, must maintain 100 hours of professional learning over a five-year period (Teacher

[1] (remembering, understanding, applying, analysing, evaluating, and creating (Krathwohl, 2002))

Registration Board of Western Australia (TRBWA), 2018). However, most of this learning can be self-directed or developed in consultation with the school. The TRBWA (2018) defines professional learning as “… activities that will improve the teacher's knowledge, practice and competencies” (p. 1), yet this needs to be professionally based. Askew et al. (1997a) found partaking in protracted off-campus courses of teacher professional learning improved student performance whereas in-house development was much less effective. Barrett, Butler, and Toma (2012) found that teachers who were considered as less effective were more likely to participate in professional learning given the opportunity, (which they nevertheless admitted may have been mandatory in some instances). They nonetheless argued that the evidence on the effectiveness of professional learning is less definitive than it should be, partly because previous evaluations did not account for the pre-existing levels of effectiveness in participants. Barrett et al. (2012) argued that teachers already considered effective may not have added any value from professional learning activities to their existing effectiveness. Barrett et al. also asserted that the reverse would hold for less effective teachers if they were only encouraged, rather than mandated, to attend activities. Barrett et al. concluded that a suggested framework or roadmap of targeted professional learning for less effective teachers may offer greater benefits over the current model of self-requested development.

Farmer, Gerretson, and Lassak (2003) found that professional learning for teachers of mathematics should involve teachers completing a rich and challenging student task with time to reflect on personal and professional implications for teaching that task. Other researchers suggested using similar processes in improving teachers’ pedagogical effectiveness (Back et al., 2012; Liljedahl et al., 2007; Sullivan et al., 2014). Farmer et al. (2003) and Liljedahl et al. (2007) both found that allowing time for reflection when completing professional learning activities encouraged teachers to adopt more inquiry-based learning tasks in their everyday teaching, and that this had an impact on their regular teaching practices. Bates, Phalen, and Moran (2016) warned, however, that the current increase in online or virtual professional learning risks minimising effectiveness gained from working with colleagues. They cited evidence that when left to choose, teachers often chose video materials selecting the most practical video rather than ones most likely to lead to learning. Relying on a self-directed professional learning model may not lead to the required gains necessary to upskill teachers in the essential skills to make development as effective as designed (Barrett et al., 2012).

Day and Hurrell (2013) proposed that offering dedicated professional learning structured to upskill teachers, both primary and secondary, helped improve teacher efficacy. In a model designed to clarify pedagogical approaches appropriate to different curriculum outcomes and different stages of development, Day and Hurrell incorporated mathematical proficiencies and worked from concrete through to the representational stages of learning. They found that their described approach was seen by participants as successful and generated a positive disposition towards the topics covered. Their model supported the work of Brown et al. (2009) and Fyfe et al. (2014) who posited that to be an effective strategy, using concrete materials required teachers to use the correct materials for the situation and to be aware of the importance of connecting concrete representations to abstract representations.

2.2.11 Adoption of teaching paradigms.

When inviting teachers to undertake curricular reform it is necessary to understand factors which influence those changes. The method undertaken to initiate such changes is important to the successful implementation of change (Nisbet, 1978).

Froyd et al. (2017) indicated two competing paradigms, dissemination and propagation when initiating change. The authors proposed that dissemination was the traditional paradigm for initiating change, but also asserted that paradigm was less effective than the propagation paradigm. Froyd et al. (2017) described the propagation paradigm as one “… which solves a local instructional problem or improves some aspect of student learning” (p. 39). The emphasis for the propagation paradigm was predicated on working locally to improve outcomes for stakeholders (Froyd et al., 2017). The ACARA (2012) implementation model for the introduction of the new mathematics curriculum would be closer to the dissemination model described by those authors and others (Seymour, 2002; Walshe, 2015).

When examining key concepts on teachers’ use of mathematics curricula, Remillard (2005) found that adopting curricular reform for classroom teachers centred on the distinction between the intended and the enacted curriculum. Remillard noted that curricular use centred on whether teachers were textbook dependant. The author cited further research by Manoucherhi and Goodman (1998), who attributed a teachers’ mathematics knowledge, and their understanding of pedagogy, as factors when teachers adopted curricular reform. Those arguments reflect the MKT discussions examined earlier. Drake and Sherin (2006) asserted that, for many teachers, detailed explanations of the meaning behind the content needed elaboration in curricular documentation. Remillard (2005) concluded that “teacher knowledge, beliefs and dispositions will not alone result in uniform mathematical instruction” (p. 239). The author also indicated a need for teachers to examine a new curriculum with other colleagues, making reflective decisions about both content and delivery.

Merely publishing a curriculum rationale or detailing content and pedagogical suggestions will not satisfy teachers in adopting the new paradigm (Remillard, 2005;

Drake and Sherin, 2006; Walshe, 2017; Ball, 2003). When discussing the adoption of a new mathematics curriculum in Cyprus, Christou, Eliophotou-Menon and Philippou (2004) found that many teachers often expressed concerns whether they were qualified to teach according to the expressed rationale. Those concerns focussed on the time afforded to teachers to develop effective adoption strategies. The authors used a model designed to measure the adoption of change which elaborated seven stages of necessary development when accepting change. Those stages moved from awareness (stage 0) through to refocussing (stage 6) where teachers accepted changes and concentrated on innovations to make such change more effective. Christou, Eliophotou-Menon and Philippou (2004) used this measure on a sample of 655 teachers and estimated that, in the Cyprus context, effective adoption (stages 5/6) had not yet reached a high level of adoption after three years. Implementing effective curricular change, therefore, indicates that teachers need to be offered time, training, and materials which are targeted to encourage awareness and lead to reflection and collaboration.

2.3 Insights into School Mathematics in Western Australia

2.3.1 Participation rates.

Underlining the importance of mathematics as a discipline, Watt and Goos (2017) quoted the Australian Academy of Science (2016) when stating that "mathematics is regarded as the enabling discipline not only for STEM-related fields but many other areas of intellectual inquiry" (p. 134). Watt and Goos (2017) also highlighted that Australian students recorded a decline in absolute performance in comparative studies such as PISA and TIMSS. Watt and Goos reported a decline in positive student engagement within mathematics in classrooms over the past two decades. Hine (2017) noted that students cited "an expressed lack of interest or enjoyment" (p. 313) in the mathematics subject as reasons for this dissatisfaction. Hine also commented that mathematics was considered a difficult subject, which acted as a deterrent for many. Falling numbers in mathematics, and in particular by girls, was highlighted in research by Kennedy et al. (2014) and Wilson and Mack (2014).

2.3.2 Gender differences.

Watt and Goos (2017) highlighted that student's loss of interest, own perceived self-efficacy and abilities when facing repeated failure in school, are all indicators as to why Australian students move away from mathematics and other science based courses. This is particulalry true of girls. Forgasz and Leder (2017) found that 15 year old Australian students thought that learning mathematics was male-oriented in accordance with common expectations and, for girls in particular, ability in mathematics was often met with derision. Thomson et al. (2017) noted similar findings of reduced self-efficacy and confidence in mathematics in Australian girls in the 2015 PISA study. Boaler (2016) asserted that ineffective, traditional teaching coupled with ability grouping leads to further gender discrepancies without other interventions. Boaler further asserted that without change towards her concept of a 'growth mindset' in mathematics, as well as changes to the teaching models in use, inevitably means a continuation of low expectations of girls. Holmes, Gore, Smith, and Lloyd (2018) reported that careers using mathematics "remain a highly gendered concern but the reasons remain unclear" (p. 671). Holmes et al. (2018) further observed that careers in technology were often considered unfeminine and acted as a disincentive for girls.

2.3.4 Efficacy and mindset.

Fredricks, Filsecker, and Lawson (2016) and Watt, Carmichael, and Callingham (2017) commented that without positive engagement, student attainment in general and participation in mathematics are diminished. When analysing the results from the PISA 2009 study, Mikk et al. (2016) found that relationships between students and their teacher played a significant role in academic performance, discipline and student motivation. Student self-efficacy was also emphasised by Lee et al. (2017) as a major factor in student engagement. Watt et al. (2017) give emphasis that the classroom environment is linked to student self-efficacy. For Watt et al. (2017), a mastery classroom environment was linked to students valuing mathematics. Watt et al. commented that a performance learning environment had no measurablly positive impact. Student-oriented classroom strategies, according to Schukajlow et al. (2012), helped promote students' interest when compared with traditional directed teaching.

Boaler (2016) and Dweck (2010) both asserted that the ability of teachers and students to adopt a 'growth mindset' would improve student efficacy and engagement. Dweck (2010) described understanding of brain plasticity as critical in helping teachers better understand student learning potential. Boaler (2016) affirmed that without equity, students would fail to participate fully in mathematics. For Boaler et al. (2000), equity meant not placing students into ability groupings as that "suggests that students are constructed as successes or failures by the set in which they are placed" (p. 643). Boaler et al. (2000) concluded ability groupings affect student efficacy in a negative manner.

2.4 Research into Teachers' Beliefs and Perceptions of Effective Teaching

2.4.1 Beliefs and perceptions of teachers of mathematics.

Teacher beliefs are considered important in the teaching and learning of mathematics. According to Swan (2006), beliefs underpin personal thought and behaviour. Beliefs, as described by Clark and Peterson (1986), often affect teachers' pedagogical choices more than the curriculum guidelines they follow or the skills and knowledge they require to complete the content strand. Teachers' beliefs also guide their teaching rationale, classroom practices and teaching processes (Clark & Peterson, 1986; McQualter, 2016). To categorise the beliefs of teachers of mathematics, Ernest (1989) devised a series of three components: "conception of mathematics as a subject for study, the nature of mathematics teaching and the process of learning mathematics" (p. 251). Askew et al. (1997b) discussed how the belief orientation of teachers influenced their classroom practices. They further developed the terminology of Ernest (1989) when elaborating the concepts of transmission, connectionist and discovery teachers.

Askew et al. (1997b) described the transmission teacher as one for whom mathematics was a collection of rules and routines which needed to be understood and applied at the correct time. The transmission orientation did not include efficient methods of solution over other methods of finding solutions; the method becomes more important than its perceived efficiency. They further posited that the transmission teacher may not take enough account of student prior knowledge, asserting that any failure in understanding was due to a lack of ability in the student. This belief orientation may related to discussion about explicit instruction and direct instruction modes of lesson structure and teaching style described earlier (Hughes et al., 2017; Tait-McCutcheon et al., 2011).

Askew et al. (1997b) reported that connectionist teachers were those for whom mathematics was taught by emphasising the links between mathematical concepts. Connectionist teachers believed that all students could learn mathematics given the correct methods of teaching for these students. The connectionist style of teaching mathematics also required some mathematical understanding of the common errors made by students as they acquired understanding. Being able to predict and understand student misconceptions was a fundamental aspect of the mathematical knowledge for teachers' premise of previous research (Ball et al., 2008; Hill & Ball, 2009). Askew et al. (1997b) found that students who had teachers with a connectionist orientation made greater gains than students with teachers of other orientations.

The third orientation described by Askew et al. (1997b) was the discovery teacher who viewed mathematics as a human creation to be discovered through exploration and reflection. Discovery orientation teachers accepted all forms of calculation as acceptable, with no emphasis on the most efficient or appropriate expression possible. Students determined their own pace of learning when developing their own strategies for reasoning and problem solving learning through practical problems. However, it is interesting that Šapkova (2013), in a study of 390 Latvian teachers, found that although teachers espoused a constructivist approach, likened to a discovery approach, their classroom practices more closely related to the transmission orientation. This movement between orientations was thought to be typical for many teachers. Dayal (2013) also asserted that the use of a discovery orientation depended upon how a teacher views mathematics. The confidence of a teacher to build upon and use student responses was influenced by the teacher's background knowledge of mathematics (MKT). In turn, this had a direct impact on teachers' use of challenging tasks.

2.4.2 Measuring teacher beliefs and perceptions.

Swan (2006) used the work of Ernest (1989) and Askew et al. (1997b) when devising scales to measure teacher beliefs and teacher classroom practices. To allow validation of the scale on teacher practices, Swan conducted questionnaires with students of the participant teachers. The use of such a questionnaire allowed comparison between stated practices and student perceptions of those practices. Those scales proved to be a reliable predictor for measuring beliefs and practices of classroom teachers (Cronbach alpha = 0.85). Swan's belief scales measured for transmission, connectionist and discovery beliefs, and validated those beliefs against classroom practices measured as student-centred or teacher-centred. Matching teacher-centred practices against the transmission belief orientation and student-centred practices against the connectionist orientation, Swan found that the principal classroom practices used by teachers in his study were almost entirely teacher-centred. Swan presented that "most of the teachers in our sample reported that they were constrained to work in ways they did not believe in" (p. 69) and noted those same teachers reported finding it difficult to reduce their governing position in a classroom. In discussing the connectionist orientation, Swan found that "the connectionist position was distinguished from the others mainly by the emphasis that teachers gave to students discussing mistakes and ideas, and the use made of prior learning" (p. 65), but noted that there was no agreement in any orientation on what learning is.

2.4.3 Perceptions of effective teaching.

The question of how teachers decide what constitutes an effective lesson also requires consideration. Stols, Ono, and Rogan (2015) argued that to encourage an improvement in learning in mathematics classrooms, teachers need to understand their own teaching style first and consider how that style affects students in the classroom.

Stols et al. (2015) investigated whether teacher perceptions could be measured reliably. They undertook a study, using eight video vignettes of mathematics lessons, to elicit responses as to what was deemed effective teaching of mathematics. Results of the Stols et al. research showed that using such a research instrument was a reliable predictor for the perception of effective teaching in the group, and that this could be used to understand teaching styles. Teacher comments on the content of the video vignettes focused on the use of materials and modes of instruction and less on the concept of fractions which was the core of the video lesson. Stols et al. found this to be significant. This reinforced findings by Hemmi and Ryve (2015), who, as mentioned earlier, have discussed the focus of teachers in higher performing countries in international measures, such as Finland, where they centred on lesson goals rather than pedagogy. Stols et al. (2015) also commented that it is important that teachers find time to reflect on their practice which reinforced arguments made by Farmer et al. (2003) and Liljedahl et al. (2007). Interestingly, the teachers in the Stols et al. study agreed on what constituted the most ineffective lesson, a teacher-centred example, but were undecided as to the most effective lesson that they observed. Further, Stols et al. (2015) offered potential reasons for teachers selecting as the most ineffective lesson which highlighted classroom over-crowding and a lack of resources. Whilst those reasons may be particular to South Africa, the teacher-centred lesson style supports the finding of Swan's study (2006), that this is the most common lesson orientation employed.

2.5 Conclusion

This chapter has outlined the historical background to the Western Australian Curriculum: Mathematics and the research which had influenced the development of the curriculum. The discussion also provided context for Australian developments in mathematics education whilst explaining the probable significance in comparative international statistical data. The chapter also offered detailed discussion of effective teaching practices in mathematics and links between research, pedagogy and the core actions of the mathematics curriculum, the Proficiency Strands. Considering research with similar aspirations to this study allowed discussion of factors and influences on teachers' perceptions of effective teaching.

Chapter 3 Research Plan

3.1 Introduction

The literature review suggested the benefits of engaging in further research into the impact teachers have on effective learning in mathematics classrooms. At present there is little evidence of such research specific to Western Australia (WA). Further impact of the K-10 revised mathematics curriculum (School Curriculum and Standards Authority (SCSA), 2018b), which is almost wholly based on the matheamtics curriculum as laid out by ACAR(ACARA, 2012), has, in WA, had limited research attention in terms of influence on classroom teaching strategies. The alignment of the Proficiency Strands to regular enacted teaching practices is a useful research topic. This research was focused on five aspects of teaching mathematics in WA: (i) familiarity with the Proficiency Strands; (ii) consensus on effective teaching, (iii) impact on beliefs of teachers; (iv) teacher practices in classrooms; and (v) teacher experience and professional learning.

This chapter will describe the research epistemology pertinent to this study as well as indicate reasons for its selection. Research instruments will be considered, including a detailing of their background and/or development. The design process will discuss population selection, data collection methods, analysis techniques and ethical considerations. The thematic analysis undertaken to develop the themes and categories as described in participant responses to instruments will describe how the major themes were explicated. Ethical considerations pertinent to the research will be examined and considered. The chapter will conclude with the design plan including the timeline and features of the research process. Table 3.1 shows sub-headings and descriptions employed in this chapter.

Table 3.1

Overview of Chapter 3: Research Plan

Sub-Heading	Sub-Division	Timeline
3.1 Introduction		August - September 2017
3.2 Theoretical Framework	3.2.1 Epistemology. 3.2.2 Theoretical perspective.	
3.3 Methodology		
3.4 Method	3.4.1 Case study. 3.4.2 Instrumental case study. 3.4.2.1 Semi-structured interviews. 3.4.3 Questionnaires. 3.4.3.1 teacher beliefs. 3.4.3.2 classroom practices. 3.4.3.3 background data. 3.4.4 Comprehension activity. 3.4.5 Research participants.	November – December 2017
3.5 Data Analysis	3.5.1 Data reduction. 3.5.2 Data display. 3.5.3 Drawing verifications and conclusions.	January – April 2018
3.6 Ethical Considerations		
3.7 Conclusion		May – August 2018

Table 3.1 indicates the structure of the chapter and indicates the relative timing of each section or heading. Section 3.4 indicates the data collection phase when interviews were conducted with participants. Section 3.5 indicates the data analysis phase when thematic analysis and statistical reviews were conducted.

3.2 Theoretical Framework

A theoretical framework according to Neuman (2011) is "a general theoretical system with assumptions, concepts and specific social theories" (p. 85). Neuman elaborated that those frameworks allow "sweeping ways to see the world and that every person will be subject to several frameworks in their lifetime" (p. 86). A theoretical framework allows researchers to understand what reality is like for an individual, and how that individual relates to the world as they see it whilst allowing links between multiple factors in a natural setting (Denzin & Lincoln, 2011; Johnson & Christensen, 2008; Punch, 2014).

Crotty (1998) proposed four elements to consider when undertaking research. He named them: epistemology, theoretical perspective, methodology, and methods. Each term will be discussed in the following sections. Crotty used those terms to help refine the research process for researchers, and advocated that the researcher needed to consider and justify what methodologies and methods would be used in the research. Johnson and Christensen (2008) suggested researchers need to be unobtrusive and have minimal impact on the behaviour being studied and Bryman (2012) argued that existing knowledge about the particular area of research, gained from literature, should influence the background to that research. Figure 3.1 shows the theoretical framework applied to this research.

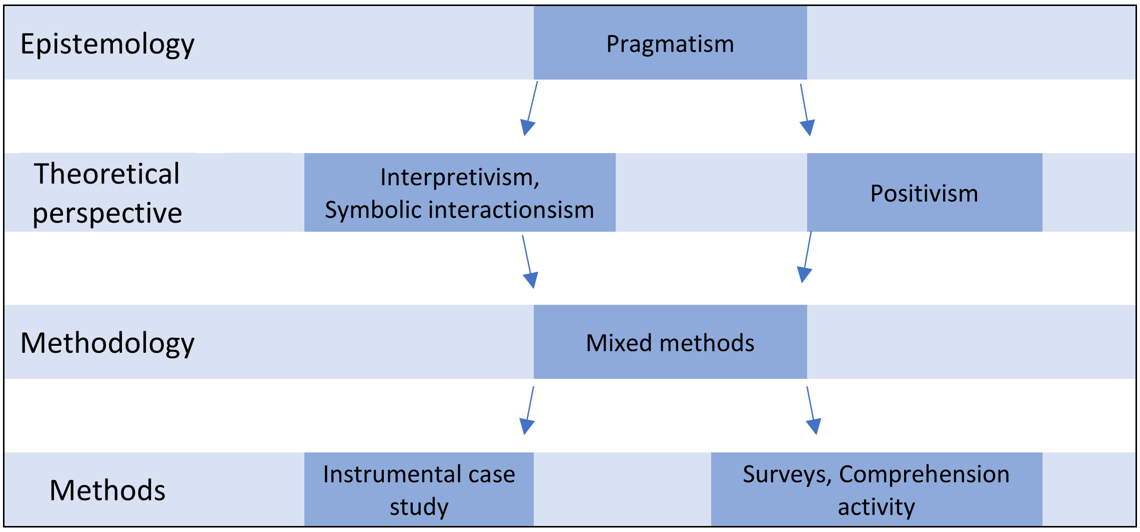


Figure 3-1 Theoretical Framework for This Research Study

Figure 3.1 indicates that a pragmatism approach is applicable to the research overall. This is justified as there are different theoretical perspectives applicable to the different research instruments which will be used in the study. Such instruments will be discussed in Table 3.2.

3.2.1 Epistemology.

Crotty (1998) described epistemology as "how we know what we know" (p. 8). Crotty described the importance of understanding the nature and theory of knowledge which relates to the research topic as being fundamental to the research. This research will define its epistemology as pragmatism. Johnson and Christensen (2008) described pragmatism as a "philosophical position that what works is what is important or valid" (p. 33). In this epistemology the basis of what is important will not be based in abstract philosophy but rather on what is deemed to work in practice. The research will be planned and conducted based on what evidence is required to answer the research questions (Crotty, 1998; Gay et al., 2012; Johnson & Christensen, 2008).

In this research there are five related questions. Each question needs to be considered for the best allocation of a suitable research method. Punch (2014) declared that data be subdivided into two main types: quantitative data which is numerically measured and qualitative data which is not in the form of numbers. Punch considered qualitative data to be mostly word or language based. Those definitions are shared by other researchers (Bryman, 2012; Gay et al., 2012; Neuman, 2011). Table 3.2 shows the related questions and data type indicated by that question. Data will be required in both the qualitative and quantitative format as Punch (2014) describes "mixed methods research" (p. 88).

Table 3.2

Research Questions Matched to Instruments and Data Type

Question	Instrument	Data Type
Are teachers sufficiently familiar with the Proficiency Strands to identify them from a stated action?	Comprehension Activity	Quantitative
What do teachers perceive as being effective mathematics teaching in a secondary classroom?	Interview	Qualitative
To what extent do teachers' beliefs of the purpose of mathematics in the secondary classroom influence their teaching practices?	Questionnaire	Quantitative
To what extent are teachers' perceptions of effective teaching in secondary mathematics reflected in their own practices?	Questionnaire	Quantitative
To what extent does teacher experience influence the impact of professional learning in their interpretation of the curriculum?	Questionnaire	Quantitative

Table 3.2 indicates the research questions and the related instrument used to collect evidence for that question. The table also indicates the data type for such evidence. It shows the necessity for both qualitative and quantitative data to be collected.

In mixed methods research the researcher uses a mixture of quantitative and qualitative methods in a single study (Gay et al., 2012; Punch, 2009, 2014). The advantage of such an approach links to the fundamental principle of mixed research, as described by Johnson and Christensen (2008), which says that "it is wise to collect multiple sets of data using different research methods and approaches in such a way that the resulting mixture or combination has complementary strengths and non-overlapping weaknesses" (p. 51). Whilst commenting on mixed research methods, Bryman (2012) explained that unless data resulting from each method is not "mutually illuminating" (p. 628) then the researcher is just using the methods in tandem. Gay et al. (2012) indicated that a true mixed methods research plan needs to lead to a better understanding of the phenomenon under investigation. In this research, triangulation between instruments that discuss effective teaching and those that examine espoused classroom practices will satisfy the requirements for both Bryman (2012) and Gay et al. (2012).

3.2.2 Theoretical perspective.

As this is a pragmatist mixed methods approach, the theoretical perspective most relevant to this study was a combination of interpretivism and positivism, reflecting the mixed methods perspective. Crotty (1998) described the theoretical perspective as "the philosophical stance that lies behind our chosen methodology" (p. 7). He added that the theoretical perspective adopted in the research is intrinsic to the reason and context of the study.

Quantitative data is normally formed using a desire to describe, predict and explain (Johnson & Christensen, 2008; Punch, 2009, 2014), and this is a positivist perspective. Crotty describes positivism as relating to what has been posited. He further describes the need for a positivist perspective to have a strong association to a scientific approach. In two of the qualitative questionnaires, data gathered are linked to a recognised scale to which associated scientific methodology are ascribed (Swan, 2006). In any research the determination of the research design is critical to the analysis and description of data, and as the current research does not conduct any form of experiment, or gather and randomly group any variable, it is best classified as non-experimental research (Johnson & Christensen, 2008).

In the concurrent mixed methods design, the gathering of qualitative data emphasises the need for an inductive approach to data collection (Bryman, 2012). Collecting data and linking it to an emphasis on understanding behaviour, as in this study, is best described as interpretivism, a contrasting epistemology to positivism. Interpretivism views relationships as having aspects which cannot be measured accurately by science (Bryman, 2012; Crotty, 1998; Neuman, 2011) as it concentrates on the meaning people bring to situations and behaviour (O'Donoghue, 2007). To better understand the behaviour of the subject when conducting research, the German sociological philosopher Weber (1947) termed the phrase *verstehen* when describing empathic understanding. Weber offered the explanation of a “science which attempts the interpretive understanding of social action to arrive at a causal explanation of its course and effects” (p. 88). Although Weber’s treatment of *verstehen* was much more detailed than the simplistic summary quoted, others adopted the term to discuss a method to investigate social phenomenon (Bryman, 2012; Neuman, 2011) where the researcher interprets actions with an understating of the context. As teachers, there are linguistic and symbolic stimuli which elicit responses known to other teachers, but alien to an outside observer, termed symbolic interactionism. There is reason to include symbolic interactionism as a major factor in the interpretivist framework for this research study (Crotty, 1998).

3.3 Methodology

This research will be described by a mixed methods methodology. Such a methodology is generally expected when gathering data which is both quantitative and qualitative in nature. Bryman (2012) indicated that mixed methods research must be more than simply using both methods in tandem, and that it should offer illumination of data otherwise obscured. In this study, triangulation of data will highlight participant perceptions of effective teaching through semi-structured interview when compared to espoused classroom practices gathered through a qualitative questionnaire. Other qualitative factors, such as teaching experience and quality of professional learning undertaken by participants will also enlighten effective teaching perceptions. A mixed methods methodology is therefore appropriate for this research.

3.4 Method

The methods used in this research were a semi-structured interview, three questionnaires and a comprehension activity. The case study design was selected as the appropriate methodology for the qualitative, semi-structured interview as there was the need to draw out participant observations related to the bounded system of effective classroom teaching strategies (Neuman, 2011; Punch, 2009). Use of the quantitative instrument by questionnaires was based on the position that data gathered were indicating beliefs, perceptions, behaviours and personal background of participant teachers and that such data is value free and objective (Bryman, 2012), although one could question whether that is ever truly so. The comprehension activity was designed around a matching activity designed to ascertain participants' understanding of the language of the Proficiency Stands of the WAC:M (School Curriculum and Standards Authority, 2018b). The research design is shown in Figure 3.2.

Figure 3-2 The Research Design Sequence

Figure 3.2 indicates that the research sequence required contact and recruitment of participants who were then interviewed. The interview process invited responses by questionnaire, then by semi-structured interview, and finally through the comprehension activity. Each interview took between 40 and 60 minutes to complete. Data analysis was conducted during and after the interview cycle.

3.4.1 Case study.

Bryman (2012) described case study design to be concerned with the "complexity and particular nature of the case in question" (p. 52). Neuman (2011) described a case study as a "suitable method when looking to examine many aspects of a few cases" (p. 177). Neuman elaborated that by following case study methodology, the researcher will end up with in-depth knowledge of the facts rather than a surface knowledge across many cases. Neuman asserted that case-study has many strengths, such as the fact that it clarifies thinking and allows researchers to link abstract ideas in specific ways. Stake (1995) observed that case-study research is concerned with the complexity of the case in question. Johnson and Christensen (2008) suggested that to use a case-study means defining the boundaries of the case in question. In this research the emphasis on participants' judgements of effective teaching elicited responses covering aspects of teaching. Those aspects form many bounded systems such as student engagement, use of materials or pedagogy, and will invoke several cases being examined, requiring an extension from case study to instrumental case study.

3.4.2 Instrumental case study.

Instrumental case study is the method used when the researcher requires an understanding of something other than the primary case (Johnson & Christensen, 2008). Johnson and Christensen stated that "the researcher studies the case to learn about something more general" (p. 408). In this research the general goal is evidence of effective teaching, although each response will contribute to the overall picture. Both

Stake (1995) and Johnson and Christensen (2008) advanced the conviction that instrumental case study is appropriate when the study needs to gain a broader perspective of the scope of an individual case. By using instrumental case study, responses collected from participants were grouped into relevant themes and clusters, allowing exploration of ideas that linked opinions about effective teaching, both positive and negative.

There are potential issues over the use of the case study approach. Bryman (2008) expressed concerns that findings from case studies cannot be generalised. Neuman (2011) countered that concern by proposing that case studies are highly heuristic. Neuman specified that case studies "help with constructing new theories, developing or extending concepts, and exploring the boundaries among related concepts" (p. 42). Case studies, according to Newman (2011) "allow researchers the ability to capture complexity and trace processes" (p. 43). In this research, data collected from participant responses was triangulated against quantitative data gathered from other instruments so as to offer better insight into thoughts, knowledge and opinions expressed by participants.

3.4.2.1 Semi-structured interviews. Semi-structured interviews were conducted using classroom teaching scenarios as a prompt to gather perceptions of effective teaching. A semi-structured interview is a style of conversational interview. Neuman (2011) described it as a flexible technique in which "the interviewer adjusts interviewing questions to the understanding of specific responses" (p. 341). Interview materials were tested on volunteer colleagues to gauge potential responses, identify potential issues and predict a likely timeframe for conducting the interview. Bryman (2008) thought it important to test and develop an interview guide to allow consistency for the researcher and clarification of any potential failings in the questions. The guide is shown in Appendix E.

All interviews were conducted after arrangement through school principals. Each interview was conducted in the participant's school or at a mutually convenient venue selected by the participant. Participants were contacted by email, after gaining approval from the school principal, which included the following information (Appendix H):

- Letter of information for participants
- Outline of the research
- Potential significance of the research
- Approval from the Human Research Ethics Committee at the University of Notre Dame Australia, Fremantle
- Approval from the Department of Education, Western Australia
- The right to withdraw at any stage
- A contact email and phone number to indicate acceptance

Interviews were arranged and took between 40 and 60 minutes. Being interviewed in a familiar or neutral setting encouraged a relaxed and genial atmosphere between researcher and subject. Each subject knew the researcher was a practising teacher, and that the information would be confidential and form part of a thesis for a higher degree. To ensure confidentiality, each subject was asked to select a random number from a list of numbers. After the interview, the researcher then allocated a random letter to each set of responses ensuring no participant could be identified in any way. All participants worked in government schools in the Perth Metropolitan area and had a range of teaching experience within a range of schools. Only two participants worked in the same school, and those interviews were conducted on separate occasions.

With the permission of participants, interviews were recorded and transcribed to ensure accuracy. Participants were informed of their right to withdraw at various stages of the interview. No participants withdrew. It was not possible to offer transcripts of interview for member checking without sharing the lesson scenarios, potentially affecting future participant responses. Each participant was invited to check transcripts at the end of the data collection process, but no participant elected to read their transcript.

Each interview was recorded to allow accurate representation of participant response and participants were assured of anonymity in their responses. By allowing for individual responses, some points were open for elaboration and further insight by participants, extending the knowledge base gained by the research. The participants commented on four brief scenarios of classroom teaching practices, inspired by the vignettes used by Stols et al. (2015). The scenarios depicted a Year 8 lesson using content strand ACMNA194 from Number and Algebra of the WAC:M (School Curriculum and Standards Authority, 2018b) [Appendix G]. The content strand covered the teaching of a pattern and algebra lesson on solving linear equations using algebraic and graphical techniques. The scenarios described four different teachers displaying a variety of classroom interactions and teaching approaches. The participants were asked to answer a question in two parts regarding each scenario: that is, what were the most effective and least effective aspects of the lesson? At the end of all four scenarios, the most and least effective lessons were identified by the participants, and they were asked to provide a justification for their choices. These selections of the least and most effective lessons were then used as evidence of the participants' perceptions of effective teaching and used to triangulate against the beliefs and classroom practices data gathered by questionnaires. The rationale used to develop the scenarios is shown in Appendix F.

3.4.3 Questionnaires.

3.4.3.1 Teacher background data. Teacher background data were collected to help understand the range of experience, formal mathematical background and professional learning of each participant. In addition, some general information about range of curriculum experience was gathered to triangulate background information against data from other instruments.

3.4.3.2 Teacher beliefs about mathematics, teaching and learning. Swan (2006) designed a scale to gather data on teacher beliefs about mathematics, teaching and learning. The Swan scale was used without adaptation. The scale was based on Swan's interpretation of the work of Askew et al. (1997b) in determining the factors influencing teachers of mathematics and what they believe about mathematics. The scale used three questions, which gathered data on what mathematics is, what mathematics teaching is and what mathematics learning is.

Each question invited participants to rate three statements, giving each statement a percentage score, to a total of 100% across the three statements. The strategy expected participants to offer fine-grained responses in preference to a Likert-type scale. Participants therefore graded each statement in both ranking and importance. Statements were indicative of the belief categorisations defined by Askew et al. (1997b) as transmission, discovery and connectionist orientation.

3.4.3.3 Teacher classroom practices. Swan (2006) designed a different scale to gather data on teacher classroom practices. The Swan scale was used without adaptation. The questions (n = 25) asked participants to rate a series of statements about their regularly enacted classroom practices. The scale gathered responses on a 5-point Likert scale with one indicating ‘almost never’ and five indicating ‘almost always’.

Swan (2006) used the scale to compare teacher beliefs to enacted classroom practices. Each statement was allocated to a category of teacher-centred practice or student-centred practice. By gathering data on enacted classroom practices, it was expected to compare indicated beliefs with those practices. In addition, the classroom practices data were compared to responses from semi-structured interviews on effective teaching, which allowed triangulation of response from multiple sources (Bryman, 2012).

3.4.4 Comprehension activity.

A comprehension activity (Appendix D), in the form of a card shuffle, was used to gather participant knowledge of the use and language of the Proficiency Strands of the WAC:M (School Curriculum and Standards Authority, 2018b). The card shuffle asked participants to identify the relevant Proficiency Strands when allocated a series (n = 14) of teaching ‘action’ descriptions used as learning objectives. The ‘actions’ were elaborations written by the researcher, and which had been taken from various strands within the WAC:M content descriptors (School Curriculum and Standards Authority, 2018b). The actions were selected by the researcher using verbs and clauses associated with the Proficiency Strands as listed in content documents covering curriculum Years 7 to 10 of the WAC: M. The researcher then placed those verbs and clauses into statements designed to reflect common teaching activities or incorporate learning goals commonly designed by teachers.

3.4.5 Research participants.

The participants in this research were secondary teachers of mathematics from Department of Education schools in Perth, WA. Sampling was taken from colleagues known to the researcher, a purposive sample, and from that group other subjects willing to participate, the snowball sample, were invited to participate. Bryman (2012) defined "purposive sampling as a non-probability form of sampling. The researcher does not seek to sample research participants on a random basis" (p. 418). Snowball sampling is a technique where subjects refer or recommend other suitable subjects who match the research criteria (Cohen & Crabtree, 2006; Walter, 2013). The strategy was to interview teachers from a range of teaching experience and background e.g. Head of Learning Area, graduate teachers, classroom teachers, senior teachers (those teaching for more than 10 years), 'Switch' teachers and teachers teaching mathematics as a minority subject in their teaching qualification.

In all, 26 subjects were invited with 14 accepting the invitation to participate in the research. Of the 14 sampled, 8 were purposive sample participants and 6 snowball sample participants. No attempt was made to ascertain why the other 12 teachers did not choose to participate. Table 3.3 shows the demographic of participants involved in the research. Only one participant had primary teaching experience. Definitions used to describe length of service are described in the table.

Table 3.3

Participants' Teaching Experience and Sample Source Used in This Research

Job Description	Number of participants (n=14)	Sample type
Head of Learning Area	4	All purposive
Graduate Teacher	3	1 purposive, 2 snowball
Classroom teacher	3	All snowball
Senior Teacher	4	3 purposive, 1 snowball

Graduate teacher < 5 years' experience, Senior Teacher > 10 years' experience, Classroom teacher between 5 and 10 years' experience, Head of Learning Area – administrative position with no length of service described.

Table 3.3 indicates the range of teaching experience of the participants. The range of teaching experience was welcomed, as was the inclusion of Heads of Learning Area who offered managerial perspectives to responses. Sample types indicated that eight participants were purposively sampled and that six participants were interviewed by referral from other participants.

3.5 Data analysis and display

This research used the quantitative scales developed by Swan (2006) to investigate relationships of regression and correlation between teacher beliefs and perceptions of classroom practices. Regression and correlation are suitable statistical measures when investigating non-experimental data (Punch, 2014), as in this study. Further quantitative data, the background data, was integrated into those relationships to allow filtering by other factors such as teaching experience and professional learning.

Punch (2014) described qualitative data as offering the greatest diversity in analysis techniques. Punch (2014) claimed "the term data-analysis itself has different meanings among qualitative researchers" (p. 168). This research used a systematic thematic analysis (Braun & Clark, 2006), and the structured inductive framework of Miles and Huberman (1994), to analyse qualitative data. The Miles and Huberman framework filters data through data reduction, data display and drawing and verifying conclusions. Miles and Huberman saw those activities as running concurrently and iteratively. Figure 3.3 shows the iterative process applied in this research. The processes occurred during data collection and post data collection as well as during the planning phase when designing the interpretivist approach.

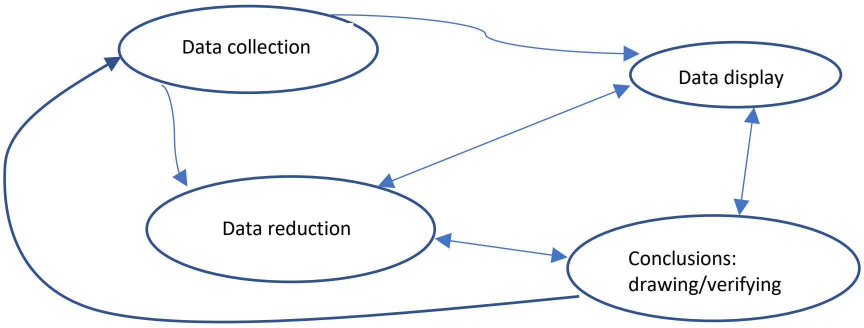


Figure 3-3 Components of Analysis of the Qualitative Data
(Miles and Huberman, 1994 p.12)

Figure 3.3 shows the cyclical nature of the data analysis where data were collected and reduced into broad responses which were then further reduced into related themes and categories. Emergent themes then indicated variations in the data reduction which is discussed in the following section.

3.5.1 Data reduction.

Miles and Huberman (1994) described data reduction as "the process of selecting, focusing, simplifying, abstracting and transforming the data" (p. 10). To reduce and organise data meant structuring transcripts and coding content into a manageable system. To enact this process the researcher constructed a bespoke database in Microsoft Access. Structuring the database allowed the selection and categorisation of comments and subsequent development of themes. According to Miles and Huberman (1994), early coding requires the researcher to be particularly attuned to the research epistemology and theoretical perspective, in this research the interpretivist orientation. Being attuned is necessary because, as described by Miles and Huberman (1994), data reduction "… organises data in such a way that 'final' conclusions can be drawn and verified" (p. 11). In this research the analysis included counting the number of comments on emerging themes, but the comments themselves were regarded as important elements of data. During the coding process transcripts were reviewed as further codes were created and comments were re-allocated in whole or in part. As an example, one code (SBEFSI) allocated to Scenario B (SB), an effective comment (EF), with student involvement (SI) as its topic, was extended from Scenario B to other scenarios. A similar coding system was then allocated across all four scenarios and extended to include positive and negative comments. The database had eight codes, four positive - four negative, under the category (SI) student involvement across four scenarios. Once initial coding had taken place then pattern coding of groups generated categories and themes, displayed in Figure 3.4, was generated. The figure shows the systematic process of analysing data gathered in transcripts.

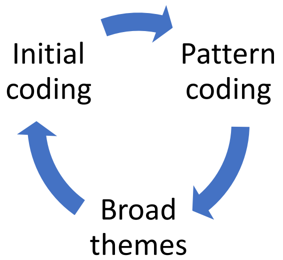


Figure 3-4 Schematic Process of Systematic Analysis of Qualitative Data

Figure 3.4 shows the cyclical development of initial coding leading to emergent patterns and the development of broad themes. Further thematic coding was then applied and adjusted, prompting further thematic development.

An organisational conceptual diagram outlining the thematic analysis process for collected data is shown in Figure 3.5.

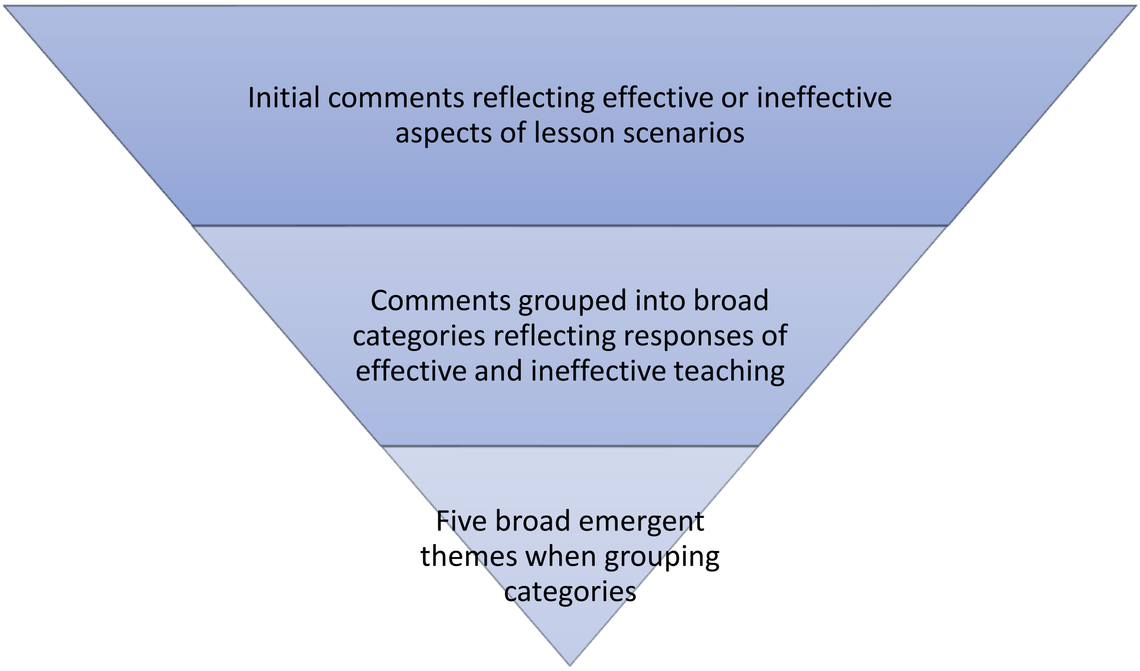


Figure 3-5 Organisational Conceptual Diagram Outlining the Thematic Analysis Process

Figure 3.5 shows the conceptual outline for the thematic analysis used when analysing the qualitative data. Initial responses were headed by effective teaching and ineffective teaching. Comments were later added about the attributes of the most common lesson taught by teachers of mathematics as described by participants. Responses gathered were then grouped into general categories which linked responses across headings. Those emergent categories were then reduced into five themes as detailed in Table 4.8.

3.5.2 Data display.

Data display is the means where data are organised to allow inspection and conclusions considered. Miles and Huberman (1994) described the process as "an organised, compressed assembly of information that permits conclusion drawing and action" (p. 11). In this research, displays were categorised into two major components:

effective teaching and ineffective teaching. Within those components the data allowed different categories to emerge, in both components, which included: using manipulatives, explicit instruction, graded exercises and more. Gathering categories together allowed five broad themes to emerge: use of materials; modes of instruction; presentation of content; classroom management and lesson attributes. Table 3.4 shows examples of coding of emergent themes for Scenario A as effective comments by participants.

Table 3.4

Examples of Coding when Identifying Emergent Categories and Themes for Scenario A, Effective Responses

Initial Code	Topic		Pattern Coding	Broad theme
SAEFCM	Concrete manipulatives	1a	Manipulative materials	Use of materials
SAEFDL	Discovery learning	3a	Discovery learning	Presentation of content
SAEFGW	Group work	4b	Working in groups	Classroom management
SAEFUM	Using mistakes	3a	Mistakes as teaching points	Presentation of content
SAEFCL	Collaborative learning	4b	Student collaboration	Classroom management
SAEFDT	Demonstrate thinking	3c	Proficiency Strands	Presentation of content
SAEFVR	Visual representation	2e	Mathematical method	Presentation of content
SAEFHWRK	Homework	5c	Homework	Lesson Attributes

SAEFCM – (SA) Scenario A, (EF) Effective comment, (CM) Concrete manipulatives

Table 3.4 indicates the type of coding applied to a series of comments discussing effective teaching responses. Codes were then applied to indicative topics. As example SAEFCM indicates SA as Scenario A, EF indicates an effective teaching discussion by the participant and CM indicates commentary relating to the use of concrete materials. Patterns were then further grouped into broad themes where comments related to concrete materials combined into the theme 'use of materials'. Such themes were used across all responses. Table 3.5 shows the developed themes for modes of instruction linked to associated categories to earlier topics.

Table 3.5

Example of Topics Linking to Categories and into Theme of Mode of Instruction

Topic	Categories	Theme
Direct instruction Traditional teaching approach Explicit teaching Preventing student errors	2a Explicit instruction	**Modes of instruction**
Graded examples	2b Graded exercises	
Meeting student needs Student engagement	2c Student engagement	
Using mistakes as teaching points Lack of information Teacher direction Teacher questioning Mistakes and motivation	2d Teacher questioning	
Taking / making notes Visualise the problem Lesson structure Change of sign algorithm Modelling substitution	2e Mathematical method	

Table 3.5 offers an example of the thematic analysis leading to the broad theme of modes of instruction. It indicates the range of topics highlighted by participants which were reduced into related categories, finally described as the theme modes of instruction. This process was used in each theme.

3.5.3 Drawing verifications and conclusions.

The final stage of analysis is drawing and verifying conclusions from emergent themes. Miles and Huberman (1994) suggested "this is the point where the competent researcher holds conclusions lightly, maintaining openness and scepticism" (p. 11). In this research the purpose of the qualitative interview process was to allow triangulation between participants' expressed classroom practices, their beliefs as gathered by a quantitative questionnaire, and what they understood as effective teaching strategies. Verification of the data required the researcher to revisit the data on multiple occasions, refining allocations of categories or verifying constant application of coding across the 14 transcripts.

The organisation of the data allowed a large volume of information to be classified and categorised into coherent themes. Those themes were verified against participant comments on effective or ineffective teaching and then compared to expressed beliefs or admitted classroom practices. Miles and Huberman (1994) asserted that "the meanings which emerge from the data have to be tested for their plausibility, their sturdiness, their confirmability – that is, their validity" (p. 11). Whilst this chapter has focused on explaining the process of analysing the qualitative data, the following chapters will detail the triangulation between themes and expressed perceptions from other data instruments.

3.6 Ethical Considerations

Habibis (2006) posited that ethical research is concerned with ensuring that ethical principles and values always govern research involving humans. It was therefore incumbent on the researcher to ensure those principles were met and that they were maintained throughout the research. The research proposal was deemed as being ethically acceptable when submitted to the Human Research Ethics Committee (HREC) of the University of Notre Dame Australia (UNDA), Fremantle (reference 017019F) and the Department of Education, Western Australia (DoEWA) (reference D17/030672). The researcher also ensured that the National Health and Medical Research Council (NHMRC) guidelines were strictly adhered to (NHMRC, 2017). To meet the ethical standards expected of educational research, and in agreement with ethical conditions set by the DoEWA evaluation and accountability section, this study was conducted with prior consent from all participants. The ethical process is detailed:

1. Approval gained from UNDA HREC and DoEWA accountability section.
2. School principals completed consent and approval for participant contact.
3. Participants completed consent forms prior to involvement.
4. Participants and schools de-identified.
5. All records and interviews stored electronically on password protected computers, password protected files of the researcher.
6. All recorded data stored as in guidelines of UNDA Fremantle for a period of five years upon final submission of research and then destroyed in accordance with UNDA guidelines.
7. All participants and DoEWA will be offered copies of the research upon final submission.

3.7 Conclusion

The research was designed to explore the beliefs and perceptions of secondary teachers of mathematics in Western Australia into what was considered effective teaching with an emphasis on the 'actions' of the curriculum, the Proficiency Strands. The chapter explained the research rationale detailing the methodology and theoretical perspective underpinning the study. The selection of the relevant epistemology, theoretical perspective and methodology of the study were justified. The chapter then detailed the research instruments and how those instruments were analysed. The distribution of participant teachers were considered; the manner in which those participants were contacted, and interviewed were outlined; and the ethical propriety of the research process was assured. The following chapter will detail the data collected across the five instruments and triangulation of the information across the data.

Chapter 4 Results

Teacher experience is used throughout this chapter to filter participant data. To aid an understanding of the grouping of teacher experience, participants were categorised by length of teaching service as: a) Graduate (G), a teacher with less than five years' teaching experience; b) Classroom Teacher (CT), a teacher with between five and ten years classroom experience; c) Senior Teacher (ST), a teacher with more than 10 years classroom experience; and d) Head of Learning Area (HOLA), a teacher with management level experience leading the mathematics department. The HOLA group did not have a defined length of teaching experience but typically had at least ten years' classroom experience.

4.1 Are Teachers Familiar Enough with the Proficiency Strands to be Able to Identify them from a Stated Action?

The instrument used to establish participant familiarity with the language of the Proficiency Strands was a card sorting exercise. The purpose of the card sorting exercise was to establish the participants' familiarity with the language used and detailed in the WAC:M (School Curriculum and Standards Authority, 2018a) in relation to typical mathematical actions detailed in curriculum content strands, relating those actions to the applicable Proficiency Strands. Participants' responses are indicated in Table 4.1.

Table 4.1

Proficiency Strand Selections by Respondents

QNO	Question	Fluency	Problem Solving	Reasoning	Understanding	Total
1.	Can you represent or calculate … in different ways?	6 (26.1%)	7 (30.4%)	2 (8.7%)	8 (34.8%)	23
2.	In what ways can you prove?	2 (9.5%)	9 (42.9%)	5 (23.8%)	5 (23.8%)	21
3.	Can you work flexibly with a concept?	4 (28.6%)	3 (21.4%)	7 (50%)	-	14
4.	How can you test your idea?	-	7 (46.7%)	8 (53.3%)	-	15
5.	Can you choose a suitable algorithm?	4 (28.6%)	2 (14.3%)	2 (14.3%)	6 (42.9%)	14
6.	How reasonable is your answer?	3 (18.8%)	-	9 (56.3%)	4 (25%)	16
7.	Can you rearrange this formula?	8 (47.1%)	-	2 (11.8%)	7 (41.2%)	17
8.	Use mathematical language to describe …	7 (46.7%)	-	3 (20%)	5 (33.3%)	15
9.	What is the same about?	2 (12.5%)	2 (12.5%)	6 (37.5%)	6 (37.5%)	16
10.	Is there a rule we can use to describe …?	4 (21.1%)	2 (10.5%)	5 (26.3%)	8 (42.1%)	19
11.	In what ways can you model and plan?	2 (10.5%)	11 (57.9%)	4 (21.1%)	2 (10.5%)	19
12.	What patterns, connections, relationships can you see?	4 (25%)	3 (18.8%)	3 (18.8%)	6 (37.5%)	16
13.	What can you recall?	4 (28.6%)	-	-	10 (71.5%)	14
14.	What can you infer?	-	-	14 (100%)	-	14
Total		50	46	70	67	233

Parentheses indicate percentage of individual question responses.
Multiple selection of responses possible

The total number of responses could vary from 14 x 14 (196) if each participant matched each statement to only one proficiency, to a potential 784 if each participant selected all four Proficiency Strands for every statement. Only one question (Q14) had complete consensus between participants and seven statements had selections under every heading. Two statements had two selected headings and three statements had three selected headings. The total of each heading from Table 4.1 indicates that Reasoning and Understanding were the most selected classification (70 and 67 responses) with Problem Solving the least selected (46) Proficiency Strand. Participants found Q1 the most difficult question to classify as it drew the most diverse number of responses (23) whereas Q13 and Q14 had only 14 responses, with Q13 having responses in two categories. Overall, participants found it difficult to match other participant classifications of the statements, disagreeing on 13 of 14 descriptions. Each statement was then classified to a common heading of the Proficiency Strands by the researcher. Responses are quantified in Table 4.2.

Table 4.2

Consistency of Participant Selection (n = 14)

QNO	Statement / action	Researcher categorisation	No. of participants matching researchers' categorisation	
1	Can you represent or calculate … in different ways?	FLUENCY	6	42.8%
2	In what ways can you prove?	REASONING	5	35.7%
3	Can you work flexibly with a concept?	FLUENCY	4	28.5%
4	How can you test your idea?	REASONING	8	57.1%
5	Can you choose a suitable algorithm?	FLUENCY	4	28.5%
6	How reasonable is your answer?	PROBLEM SOLVING* (reasoning)	0 (9)	0 (64.3%)
7	Can you rearrange this formula?	FLUENCY	8	57.1%
8	Use mathematical language to describe …	UNDERSTANDING	5	35.7%
9	What is the same about?	UNDERSTANDING	6	42.9%
10	Is there a rule we can use to describe …?	UNDERSTANDING	8	57.1%
11	In what ways can you model and plan?	PROBLEM SOLVING	11	78.6%
12	What patterns, connections, relationships can you see?	REASONING	3	21.4%
13	What can you recall?	FLUENCY* (understanding)	4 (10)	28.5% (71.4%)
14	What can you infer?	REASONING	14	100%

* More than one response attributed to the statement. Parentheses indicate number and percentage of responses for the alternate classification.

In only one question (Q14) of the statements against the Proficiency Strands was there a unanimous response by participants in matching the researchers' categorisation. Q6 and Q13 are judged by the researcher as being reasonable to have more than one response therefore the numbers in parentheses indicate the selections for the alternate heading deemed appropriate by the researcher. The median measure of central tendency was chosen to avoid any potential outlier influencing the mean because of the 0 in selection agreement by participants in Q6. The median value for the first variation across all questions is 5.5. For the alternate responses the median value is 7. The range in median values indicates that participants might be located between 42 and 50% in agreement with their interpretation of the Proficiency Strand descriptors matching those of the researcher in the WAC:M content as currently written.

There was 100% agreement between participants and the researcher in Q14 and 78.6% agreement in Q11. There was only minor agreement in Q12 at 21.4% and 0% agreement in Q6 for Problem Solving but 64% agreement when Q6 was re-classified as Reasoning.

4.2 What Do Teachers Perceive as Being Effective Mathematics Teaching in a Secondary Classroom?

Teacher perceptions of effective mathematics teaching were gathered by semi-structured interview in participant responses to four selected scenarios depicting a Year Eight lesson in number and algebra; ACMNA 194, "Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution" (School Curriculum and Standards Authority, 2018b). It was expected across four scenarios that all four Proficiency Strands would be encapsulated. Each lesson was characterised by the researcher as depicting an orientation of teacher practices as described by Askew et al. (1997b), seen again in Table 4.3. Details of the lessons are shown in Table 4.3.

Table 4.3

Scenarios and Linked Orientations and Proficiency Strands as Matched by the Researcher

Scenario	Orientation	Major resources	Dominant proficiency strand
A	Discovery	Manipulatives	Reasoning, problem solving
B	Transmission	-	Understanding, fluency
C	Transmission	Student response boards	Understanding, fluency, reasoning
D	Connectionist	Video, Manipulatives	Reasoning, problem solving

Table 4.3 indicates the lesson style expected for each scenario. It offers an expected orientation thought appropriate by the researcher as well as the likely Proficiency Strands applicable to that lesson.

Participants were asked to select the most and least effective lesson from the range of scenarios, results are listed in Table 4.4.

Table 4.4

Participant Selections of Effectiveness in Lesson Scenarios (n =14)

Scenario	Most effective	Least effective	Most common
A	5	-	-
B	1	12	13
C	4	-	1
D	4	2	-

Participants perceived the least effective lesson as Scenario B, a transmission lesson. Participants also selected Scenario B to typify the most common lesson scenario employed by other teachers of mathematics in schools. There was no agreement on the most effective lesson. Scenario A had the largest response but there were multiple comments indicating that participants could have selected Scenarios C and D instead of A. It is of interest that Scenario B was selected on only one occasion as being effective

Table 4.5 indicates the number of comments collected for each scenario.

Table 4.5

Frequency of Participant Feedback by Scenario

Scenario	Effective n	Ineffective n	Total N
A	51	32	83
B	39	59	98
C	67	32	99
D	54	41	95
Total	**211**	**164**	**375**

Each scenario had participant responses under headings of effective and ineffective teaching. Scenario C gathered the most comments regarding being effective whilst B had the most comments about being ineffective which supports the results displayed in Table 4.4. There were more positive than negative comments.

Table 4.6 displays the categorisations and number of responses of effective and ineffective pedagogical and content descriptions for each scenario.

Table 4.6

Frequency of Categorisations of Pedagogical and Content Descriptions Made by Participants

Effective			Ineffective		
Theme	**Category**	**No of responses**	**Theme**	**Category**	**No of responses**
Scenario A					
1a	Manipulative materials	10	5d	Quality of content	11
4b	Working in groups	9	4d	Classroom control	4
3a	Discovery learning	8	5d	Lesson time	4
3a	Mistakes as teaching points	8	2e	Visual representation	3
4b	Collaboration	7	5d	Curriculum pressure	2
2e	Visual representation	5	3a	Teacher direction	2
	Other responses*	4		Other responses*	6
	Total	**51**		**Total**	**32**
Scenario B					
2d	Teacher questioning	11	2a	Explicit instruction	14
2a	Explicit instruction	6	5a	Modelling substitution	9
2b	Graded exercise	5	4c	Seating plan	7
2e	Change of sign algorithm	4	2d	Teacher questioning	6
4e	Ability of group	4	2b	Graded exercise	6
5e	Lesson introduction	5	2e	Change of sign algorithm	5
2e	Taking notes	3	5d	Speed of lesson	5
	Other responses*	3	2e	Taking notes	4
			5c	Homework	3
	Total	**39**		**Total**	**59**
Scenario C					
4c	Seating plan	19	2c	Student participation	11
2b	Graded work / differentiation	16	2b	Graded work/differentiation	7
1b	Student response boards	14	4a	Student accountability	4
2c	Student participation	9	4b	Student collaboration	3
5a	Modelling and explanation	5	1b	Student response boards	2
4a	Student accountability	4		Other responses*	5
	Total	**67**		**Total**	**32**
Scenario D					
1a	Manipulative materials	12	1a	Manipulative materials	9
2e	Justification of answers	12	1c	Student engagement	7
5d	Real life context	9	5d	Lesson timing	5
5a	Teacher modelling	6	5a	Teacher modelling	5
2e	Using the inverse operation	5	5c	Homework	4
2c	Student engagement	4	4c	Classroom management	3
2b	Graded examples	3	2b	Graded examples	3
	Other responses*	3		Other responses*	5
	Total	**54**		**Total**	**41**

***. Other responses** relate to categories with at most two responses and are accumulated in each scenario.

The results indicate that, for effective lesson characteristics, classroom organisation was the most frequent response as an indicator of effective teaching. The next most frequent categorisation was discussion on graded exercises in lesson scenarios. The most frequently mentioned ineffective description was explicit instruction and was followed by elicitation of student participation and quality of lesson content.

Categorisation groups reflected common responses in some key areas. Category 2a grouped responses on explicit instruction, often incorrectly termed direct instruction, by participants. Participant responses indicated that in the view of respondents, direct instruction equated to rote learning or teacher directed drill lessons. For instance, one participant stated: "This is possibly about the worst lesson you could ever do … it's chalk and talk … it's purely direct instruction" (Participant K). This misunderstanding of direct instruction was a common aspect of the participants' responses.

Several responses focused on the use of graded exercises as a form of differentiation. The text in Scenarios B and C described the use of graded questions structured in a hierarchical sense from easy to difficult. Participants accepted that identification as a standard textbook layout when commenting on a lack of effectiveness "and getting all the students to start at question 1, well that's kind of poor" (Participant E) and "if the teacher knows their students then they know which student should start at question one and which students should start at question ten or somewhere else" (Participant D).

Participant responses on teacher direction and using mistakes as teaching points were categorised as both effective and ineffective, depending upon how well the teacher managed the questioning. A typical response as to effectiveness was "I like the fact that the groups with the incorrect decisions are encouraged to demonstrate their thinking to other students so that they can get their feedback; so, it becomes more a peer involvement. I do like that aspect of it" (Participant J). An ineffective response was typified by "but the weaker kids get so demoralised by making mistakes that they give up … and they will give up" (Participant K).

Table 4.7 offers examples of responses for the more frequent effective and ineffective categories detailed by category, participant and teaching experience.

Table 4.7

Participant Response by Scenario for Effectiveness by Category, Participant and Teaching Experience

Scenario	Effective	Ineffective
A	• I believe that students learn best when aiding or teaching other students. (Collaboration, Participant F, CT) • Because it helps those kinaesthetic learners, those students that actually need physical … the physical helps them develop patterns, physically touching and manipulating objects rather than all just intellectual. (Manipulative materials, Participant C, ST) • So, in terms of effectiveness, the students are being led through to discover something for themselves and if you do that then obviously they are more likely to think back to that activity and recall it later. (Discovery learning, Participant N, ST)	• It feels like it has a beginning and a middle but I am not sure if it has an end. (Quality of content, Participant M, ST) • A lot of teachers, in one hour, will never really get to the point of gathering all that material together … and then discussing the strengths and the weaknesses of it. (Lesson time, Participant M, ST)
B	• And if the questioning process is thorough … going around the room, and the teacher is responding to that feedback, then I think it sounds like it's an effective lesson. (Teacher questioning, Participant J, ST) • Modelling of the changing the sign … It's an important part of the process to make sure that they understand. (Algorithm, Participant A, G)	• I wouldn't do a lesson this way anymore. (Explicit instruction, Participant M, ST) • … and substitution is very very difficult for a lot of them …so not modelling that and asking them to do it is just downright appalling … and that's setting students up for failure. (Modelling, Participant K, ST) • Oh my God, let's just teach some tricks … there's no understanding there … that's teaching a trick! (Algorithm, Participant K, ST) • I am thinking of a teacher who has to get through a curriculum and who has been disrupted with all sorts of things on a practical basis and I think that typifies that sort of a lesson. (Speed of lesson, Participant L, HOLA)
C	• I also like this … that there was differentiation in the lesson ... different students could do … understanding, fluency or reasoning … so differentiated in the tasks. (Differentiation, Participant I, G) • Arranging your students into small groups with a mixture of low ability, high ability and middle ability as a group … they are working together or they have got some ability. (Seating plan, Participant F, CT) • I like that the students use the show me white boards to answer some quick-fire questions because it gives the teacher immediate feedback about which students get the concept and which don't without actually pointing to particular students or requiring calling out and maybe embarrass themselves. (Student response boards, Participant N, ST)	• I would have another option besides the text book because a lot of kids don't like the reading … there's too many words … so I would have a worksheet option as well for those kids who need or want that. (Graded work, Participant E, HOLA) • I think it's not going to stick in the students' head … it's not going to be something that will make them think about it. (Student participation, Participant N, ST)
D	• It de-mystifies it … inverse operations … that's great … there are no tricks there … they are actually using mathematics. (Inverse operation, Participant K, ST) • It is important that kids should do their own thinking because in doing that they are creating those pathways in their brains about how to solve this type of problem. (Justification of answer, Participant C, ST) • Love seeing manipulatives … and relating it to the world … especially when it's talking about algebra … too many people do it as just solve the algebra without relating it to what it is. (manipulatives, Participant L, HOLA)	• They would need to understand what they are doing, which means they get it already. (Manipulatives, Participant H, HOLA) • It would require a lot of planning from teachers I imagine. It would require a lot of the preparation aspect. (Lesson preparation, Participant F, CT)

HOLA – Head of Learning Area, ST – Senior Teacher, CT – Classroom Teacher, G - Graduate

For example, F, CT indicates participant F, a classroom teacher. The table details responses which were concentrated into grouped themes. The emergent themes are detailed further in Table 4.8 which presents the breadth of themes raised during the interview.

Table 4.8

Frequency and Percentage of Coded Comments Per Theme and Category

	Theme	Percentage	Theme value	Code and category	Effective n	Ineffective n	Total N
1.	Use of materials	14.36	1a	Manipulative materials	22	9	31
			1b	Student response boards	14	2	16
			1c	Audio-visual materials	5	5	10
2.	Modes of instruction	35.01	2a	Explicit instruction	6	4	10
			2b	Graded exercises	24	16	40
			2c	Student engagement	13	18	31
			2d	Teacher questioning	11	6	17
			2e	Mathematical method	29	12	41
3.	Presentation of content	12.34	3a	Conceptual development	16	2	18
			3b	Method / layout	14	10	24
			3c	Proficiency Strands	7	0	7
4.	Classroom management	16.12	4a	Student accountability	4	4	8
			4b	Student collaboration	16	3	19
			4c	Classroom organisation	19	10	29
			4d	Behaviour management	0	4	4
			4e	Setting / Ability	4	0	4
5.	Lesson attributes	22.17	5a	Modelling	11	14	25
			5b	Differentiation	10	12	22
			5c	Homework	0	4	4
			5d	Lesson preparation	9	23	32
			5e	Prior learning / understanding	5	0	5
	Total	100			239	158	397

Table 4.8 summarises the outcomes of the coding process. Five over-arching grouped categories were developed leading to themes of use of materials, modes of instruction, presentation of content, classroom management and lesson attributes. Two themes 'Modes of instruction' (Theme 2) and 'Lesson attributes' (Theme 5) together account for 57% of responses.

Table 4.9 shows selected, indicative, verbal responses made about the themes with higher frequency. These themes do not include the responses listed earlier.

Table 4.9

Examples of Comments Made by Participants About Effectiveness of Lesson Scenarios

Theme	Effective	Ineffective
Lesson attributes include: Lesson preparation, modelling of explanation, prior learning and checking for understanding.	• The students have to demonstrate their thinking … so that they actually have to explain it rather than the teacher just saying 'no, that is incorrect'. They have to extend on the pattern which encourages thinking … and they need to show that they can apply their thinking … and students are encouraged to look at it in a different perspective. (Participant D, HOLA, Scenario A) • There is accountability … making sure that most students are on board and then getting them to where you want to go. (Participant G, CT, Scenario C) • Need to know the expected layout … because eventually that's what we have to assess them on … (Participant L, HOLA, Scenario D)	• This lesson required a certain level of classroom management skills and if you have those skills then this is a great way to do it. (Participant G, CT, Scenario A) • I think it's not going to stick in the students' head … it's not going to be something that will make them think about it. (Participant N, ST, Scenario C) • The teacher needs to guide them through that [modelling] and encourage them into formal language. (Participant D, HOLA, Scenario D)
Modes of instruction include: explicit instruction, student engagement and teacher questioning.	• I like this lesson … it is a nice range of style and activity and I would teach this lesson. (Participant G, CT, Scenario C) • I think this is a good lesson … and it is certainly going to be more interesting for the students than scenario B would have been. (Participant N, ST, Scenario C) • The students are encouraged to do their thinking rather than the teachers thinking. (Participant C, ST, Scenario D)	• I think this has potentially a risk of less student engagement than the others. (Participant J, ST, Scenario C) • Starting them at different places and go around and see how that's going … so it might fall down a little bit there. (Participant H, HOLA, Scenario C) • Students just sit there and chill out and don't have a go. (Participant C, ST, Scenario D)
Use of materials includes: the use of concrete manipulatives and student response boards.	• I like that it is interactive ... that the kids are kinaesthetically involved in the problem. (Participant L, HOLA, Scenario A) • And we've got some manipulatives that we are going to use … so manipulatives are always good … that will be something they can think back on and remember. (Participant N, HOLA, Scenario D) • I think that the show me personal boards give the teacher a rapid-fire feedback around the room so he can see what the understanding is like. (Participant J, ST, Scenario C)	• Those whiteboards … they can turn into instruments for graffiti quickly … so we don't use them here. (Participant B, G, Scenario C) • I find the materials distracting … if I was a year 8 student in that class I would be annoyed that I had to use these things to do something that seems a bit cumbersome. (Participant G, CT, Scenario D) • … and do the manipulatives really get the message across? Using colours for positive and negative might not work for all kids … the addition of two negatives might be problematic … (Participant B, G, Scenario D)

HOLA – Head of Learning Area, ST – Senior Teacher, CT – Classroom Teacher, G - Graduate

Some themes are commented across all scenarios and could be thought of as important. Themes linked to elements of effective teaching across all scenarios (also see the fifth column in Table 4.8), include developing teacher-student collaboration, meeting student needs, teacher modelling and explanation, as well as classroom control and management. Themes linked to less effective lessons included behaviour management issues and student engagement, as well as mistakes leading to a lack of motivation, as described in Table 4.6. Participants strongly supported the use of graded exercises to ensure that students did not mindlessly complete every question on an exercise, as detailed in Table 4.6. There was a lack of agreement over the use of algorithms used without understanding, it being thought of as both effective and ineffective. There was no consistent response of the elements of effectiveness when grouped by teaching experience, which is of interest.

An important overall result was that there was strong agreement about both the least effective lesson style, and the most commonly viewed lesson style being a traditional transmission orientation lesson, Scenario B. Moreover, it is notable that the least effective and most commonly viewed lessons are the same, a transmission lesson often described by participants and drill and practice.

Comments included:

> Solving a linear equation by taking notes, but that doesn't necessarily mean they are learning. *(Participant B, G)*
>
> Because it is all teacher-centred. It is teacher modelling, teacher notes, students just copying material, the teacher is modelling the situation – not the kids. *(Participant D, HOLA)*
>
> But I think it comes down to being time efficient in structuring the lesson … you can get in there … open your book … go through examples on the board ... get guys to take notes … do some examples … check understanding and minimal planning. (Participant B, G)
>
> We have people who are not maths trained trying to teach maths, so they just go for the text book. *(Participant E, HOLA)*
>
> It doesn't sit with fluency and reasoning … that much. *(Participant F, CT)*

Participants were prepared to point out potential failings or offer warnings about what may go wrong in certain situations. In general, effective teaching included the use of manipulative materials and building from concrete concepts to extending lessons, employing strong student engagement. Positive comments also included extending thinking and reasoning through well differentiated work. Ineffective teaching was highlighted by a strong teacher-centred lesson, with little student interaction, and a stronger emphasis on repetitive activities, usually characterised as fluency.

Table 4.10 shows the selections of teaching groups by experience.

Table 4.10

Participant Scenario Responses (A, B, C and D) by Teaching Experience

	Most Effective				Least Effective				Most Common			
Groups	A	B	C	D	A	B	C	D	A	B	C	D
HOLA	2	1	1			3		1		4		
ST	1		1	2		4				4		
CT	1		1	1		2		1		3		
G	1		1	1		3				2	1	

HOLA – Head of Learning Area, ST – Senior Teacher, CT – Classroom Teacher, G - Graduate

There was no consensus across participants, when grouped by experience, about the most effective lesson. No member of the HOLA group selected Scenario D as most effective. There was consensus in each range of experience about the least effective lesson and the most common lesson, both identified as Scenario B. No member of any group selected Scenario A or Scenario C as least effective nor Scenario A or Scenario D as the most common teaching style seen in classrooms.

4.3 To What Extent do Teachers' Beliefs of the Purpose of Secondary Mathematics Influence their Teaching Practices?

The purpose of the Swan (2006) instrument was to establish participants' beliefs about what mathematics is, what teaching is and what learning is. Data gathered here, indicated in Appendix B, were triangulated against other data collected by other research instruments, detailed in Section 4.6. Response data are shown in Table 4.11.

Table 4.11

Participant Beliefs About Mathematics, Learning and Teaching.

Category	Question	Orientation	These data (n=14) Mean %	SD	Swan Data (n=64) Mean %	SD
Mathematics is	• *a given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to students.*	MT	45.2	15.4	45.2	21.3
	• *a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.*	MD	29.5	9.8	29.3	14.6
	• *an interconnected body of ideas which the teacher and the student create together through discussion.*	MC	25.2	10.1	25.5	12.8
Learning is	• *an individual activity based on watching, listening and imitating until fluency is attained.*	LT	35.6	15.4	34.8	18.1
	• *an individual activity based upon practical exploration and reflection.*	LD	33.9	8.5	33.4	12.8
	• *an interpersonal activity in which students are challenged and arrive at understanding through discussion.*	LC	30.3	14.8	31.9	15.8
Teaching is	• *structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught; assessing when a student is ready to learn.*	TT	34.6	20.8	41.3	18.0
	• *providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.*	TD	40.4	18.0	29.9	11.7
	• *a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.*	TC	25.0	10.7	28.8	16.5
	Transmission Mean	MT, LT, TT	38.5	17.6	40.4	17.3
	Discovery Mean	MD, LD, TD	34.6	13.3	30.8	10.0
	Connectionist Mean	MC, LC, TC	26.8	12.0	28.8	12.1

MT – mathematics is with a transmission orientation, LT – learning is with a transmission orientation, TT – teaching is with a transmission orientation
MD – mathematics is with a discovery orientation, LD – learning is with a discovery orientation, TD – teaching is with a discovery orientation
MC - mathematics is with a connectionist orientation, LC – learning is with a connectionist orientation, TC – teaching is with a connectionist orientation

The questions for each category and orientation are listed. Response data show the mean weighting of all participants along with the standard deviation for each question. The results of a larger study by Swan (2006) are indicated for comparison purposes

The ternary graph, Figure 4.1, indicates the mean percentage discovery, transmission and connectionist beliefs of each interviewed participant. Table 4.12 shows the mean weighted value for each participant plotted in Figure 4.1.

Table 4.12

Mean Percentage Weighting for Participants in Each Belief Orientation

Participant	Group	T	D	C
A	G	36.67	30.00	33.33
B	G	24.33	47.67	27.67
C	ST	50.00	23.33	26.67
D	HOLA	26.67	33.33	40.00
E	HOLA	33.33	53.33	13.33
F	CT	30.00	36.67	33.33
G	CT	39.33	36.00	24.33
H	HOLA	73.33	16.67	10.00
I	G	30.00	40.00	30.00
J	ST	63.33	25.00	11.67
K	CT	28.67	42.00	28.67
L	HOLA	23.33	36.67	40.00
M	ST	43.33	26.67	30.00
N	ST	36.67	36.67	26.67
Mean		38.5	34.6	26.8

T -Transmission, D – Discovery, C – Connectionist
G – Graduate, ST – Senior Teacher, CT – Classroom Teacher, HOLA – Head of Learning Area

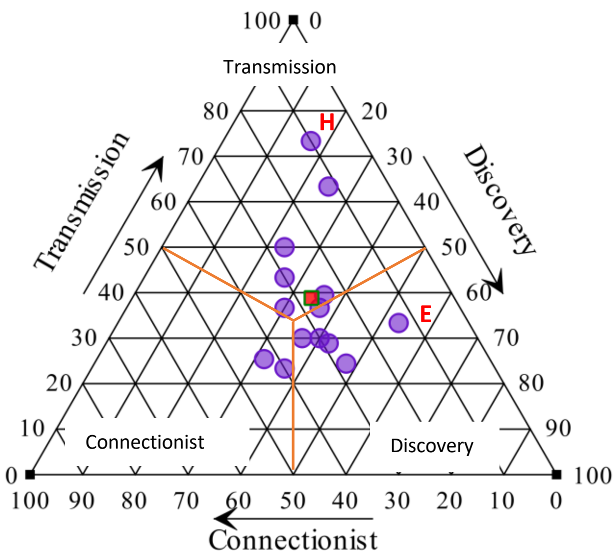


Figure 4-1 Ternary Graph Showing the Mean Weightings For Each Participant

The square mark in Figure 4.1 indicates the overall mean percentage score for each orientation. Responses are clustered around the centre. There are few extreme participants, but H and E have high attributes for transmission and discovery respectively. Both have lower scores in the connectionist orientation. There is a similar number of participants in transmission (seven) and discovery (six) with fewer values in connectionist (two) orientations. One participant sits on the transmission/discovery cusp. Plots for participants E and H, both HOLAs, have been indicated for reader clarity, as these can be argued to be more 'extreme' responses which is interesting.

Table 4.12 shows there are six particpants who had higher responses in transmission, five in discovery, two in connectionist and one who expressed no preference. In considering the strength of related participant beliefs, correlation analysis was conducted between the weighted mean in each category against the other categories. The expectation was that this analysis would establish links between belief orientations, highlighting similarities and differences in association between variables. Those results are shown in Table 4.13 where Pearson's product-moment correlation coefficient is used to measure association. Significance is two tailed to allow for a negative correlation factor.

Table 4.13

Correlation Between Belief Orientations (n =14)

		Transmission	Discovery	Connectionist
Transmission	Pearson correlation (r)	-	-0.777**	-0.752**
	Significance (2-tailed) (p)	-	0.001	0.002
Discovery	Pearson correlation (r)	-0.777**	-	0.169
	Significance (2-tailed) (p)	0.001	-	0.563
Connectionist	Pearson correlation (r)	-0.752**	0.169	-
	Significance (2-tailed) (p)	0.002	0.563	-

** Correlation is highly significant at the 0.01 level (2-tailed).

There were significant negative correlations observed between the overall transmission weightings and the corresponding discovery and connectionist orientations although the correlation between discovery and connectionist orientation is just thought to be significant at the 99% confidence level. Table 4.14 shows participant data when filtered by teaching experience through participant groups.

Table 4.14

Correlation Between Orientations by Teaching Experience

Group		Transmission	Discovery	Connectionist
G	Transmission	-	-1.000*	0.999*
(n=3)			(0.019)	(0.035)
	Discovery	-1.000*	-	-1.000*
		(0.019)		(0.016)
	Connectionist	0.999*	-1.000*	-
		(0.035)	(0.016)	
CT	Transmission	-	-0.674	-0.790
(n=3)			(0.529)	(0.420)
	Discovery	-0.674	-	0.080
		(0.529)		(0.949)
	Connectionist	-0.790	0.080	-
		(0.420)	(0.949)	
ST	Transmission	-	-1.000*	0.999*
(n=4)			(0.019)	(0.035)
	Discovery	-1.000*	--	-1.000*
		(0.019)		(0.016)
	Connectionist	0.999*	-1.000*	-0.862
		(0.035)	(0.016)	(0.138)
HOLA	Transmission	-	-0.720	
(n=4)			(0.280)	
	Discovery	-0.720	-	0.268
		(0.280)		(0.732)
	Connectionist	-0.862	0.268	-
		(0.138)	(0.732)	

* Correlation is significant at the 0.05 level (2-tailed)
Values are displayed as Pearson's *r* and significance *p* in parentheses
G – Graduate, ST – Senior Teacher, CT – Classroom Teacher, HOLA – Head of Learning Area

It shows that although the samples are small, there is a strong negative correlation for the G group between transmission and discovery. They are, however, positively aligned between transmission and connectionist. This is also reflected in the ST group. There is also a significant relationship between discovery and connectionist orientations for both groups which is not replicated in the whole group. The small sample may well be a factor in this result.

Analysis of the selections for the participants when grouped by experience gives some finer detail in common beliefs. The HOLA group was the most diverse as it contains both participants E and H in the group of four, both being potential outliers. The HOLA group weighted mean was almost equal in transmission (39.2) and discovery (35) with connectionist ranked lowest (25.8). The G group had an equal weighting between the transmission (30.3) and connectionist (30.3) orientations. Discovery (39.2) orientation had the highest weighting for this group. The CT group displayed a consistent weighting between the transmission (32.7) and discovery (32.2) orientations with the connectionist (28.8) orientation having a lower weighting. Both the CT and G groups had weightings clustered around the centre suggesting similar practices between participants. The ST group presented a significant weighting to the transmission orientation (48.3) in this set of four participants. The other orientations were similarly weighted at discovery (27.9) and connectionist (23.8). Weightings were not tightly clustered and were influenced by one strong transmission orientation (63.3) result.

4.4 To What Extent are Teachers' Perceptions of Effective Teaching in Secondary Mathematics Reflected in their Own Practice?

The purpose of this research instrument was to gather perceptions on classroom practices which could corroborate participant belief orientations previously expressed in question 3. Responses were gathered using a Likert-type five-point scale. One on the scale indicated 'almost never' where five indicated 'almost always' when responding to statements about classroom practices. The scatter graph, Figure 4.2, shows the student and teacher-centred practices of all 14 participants. Table 4.15 shows the mean categorical response for each participant.

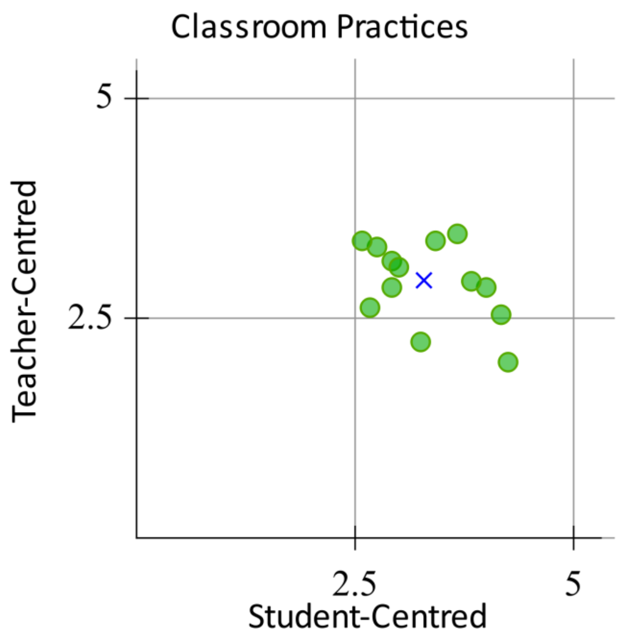


Figure 4-2 Student v Teacher-Centred Weighted Means for All Participants' Classroom Practices

Table 4.15

Participant Mean for Classroom Practices

Participant	Group	Student Centred	Teacher Centred
A	G	2.75	3.77
B	G	4.00	2.85
C	ST	2.75	3.31
D	HOLA	4.25	1.92
E	HOLA	2.67	2.62
F	CT	3.42	3.38
G	CT	2.92	3.15
H	HOLA	2.58	3.38
I	G	3.67	3.46
J	ST	3.00	3.08
K	CT	4.17	2.54
L	HOLA	3.25	2.23
M	ST	2.92	2.85
N	ST	3.83	2.92
Mean		3.30	2.91
SD		0.57	0.44

G – Graduate, ST – Senior Teacher, CT – Classroom Teacher, HOLA – Head of Learning Area

The overall mean score is indicated by the X around which the dots are clustered. Student-centred practices scored slightly higher than teacher-centred practices. The individual participant scores suggest that few respondents were distinctly one category or the other except for participant D who recorded the lowest on teacher-centred practices and the highest student-centred practices. This would be expected given the previous results in Table 4.12.

Analysis of the selections for each group gives finer detail of classroom practices. The HOLA group were influenced by one strongly student-centred score (4.25). One other participant was notably teacher-centred (respondent H). In general, there was an indication, albeit a minimal one, of a student-centred preference in this group. Mean scores were 3.2 for student-centred and 2.5 for teacher-centred. Although the sample size was too small to be statistically significant, there was an observed negative correlation, using Pearson's product-moment correlation coefficient, between student and teacher-centred practices in this group, $r = -0.8472, p < 0.2$. The G group showed little preference for teacher or student-centred practices, the mean of student-centred practices being 3.47 and teacher-centred practices 3.36. As with the HOLA group it was noted there was an observed negative correlation between practices $r = -0.8995, p < 0.3$ however the small sample restrains its statistical significance. The CT group recorded a student-centred mean of 3.50 with a teacher-centred mean of 3.02. As with previous groups it was noted there was an observed negative correlation between practices

$r = -0.7995, p < 0.5$, but again, the small sample restrains any claim of statistical significance. The ST group showed little preference for teacher or student-centred practices with a student-centred mean of 3.13 and a teacher-centred mean of 3.04. The ST group was the most tightly clustered and did not show the negative correlation apparent in other groups.

Table 4.16 shows the mean response per question as well as the overall mean response. The table employs the z-score, a standardised score which indicates how many standard deviations an element is from the mean. Typically, 68% of standardised responses are within one standard deviation, $z = \pm 1$, from the mean in a normal distribution. In this distribution, 76% of responses fell within one standard deviation of the mean of the results of a larger study by Swan (2006), which are indicated for comparison purposes.

The individual questions with the largest variation were Q11, 14, 19, 20, 21 and 23. Of the ten highest recorded practices, four were teacher-centred and six were student-centred.

Table 4.16

Mean Response Per Question for Participant Classroom Practices (n=14)

QNO	Type	Question	Mean	SD	Swan Mean	Z score
1	T	Students learn through doing exercises	3.07	0.83	3.83	-0.92
2	T	Students work on their own, consulting a neighbour from time to time	2.64	0.74	3.33	-0.92
3	T	Students only use the methods I teach them	2.86	0.95	3.22	-0.38
4	T	Students start with easy questions and work up to harder questions	3.57	1.02	4.19	-0.61
5	S	Students choose which questions they tackle	2.50	1.02	1.95	0.54
6	S	I encourage students to work more slowly	2.57	0.76	1.98	0.78
7	S	Students compare different methods for some questions	3.07	1.07	2.43	0.60
8	T	I teach each topic from the beginning, assuming they know nothing	2.43	1.34	3.33	-0.67
9	T	I teach the whole class at once	3.43	0.94	3.98	-0.59
10	T	I try to cover everything in a topic	3.43	0.85	3.63	-0.24
11	S	I draw links between topics and move back and forth between topics	4.14	0.86	3.03	1.29
12	S	I am surprised by the ideas that come up in a lesson	2.79	0.80	2.08	0.88
13	T	I avoid students making mistakes by explaining things carefully first	2.64	1.01	3.29	-0.64
14	T	I tend to follow the textbook or worksheets closely	2.36	0.63	3.11	-1.19
15	S	Students learn through discussing their ideas	3.43	1.28	2.68	0.58
16	S	Students work collaboratively in pairs or small groups	3.71	1.14	2.65	0.93
17	S	Students invent their own methods	2.86	1.10	1.83	0.93
18	S	I jump between topics as the need arises	3.43	0.94	2.57	0.92
19	T	I tell students which questions to tackle	3.29	0.73	4.10	-1.12
20	S	I encourage students to make and discuss mistakes	4.29	0.91	2.70	1.74
21	T	I only go through one method for doing each question	1.64	0.93	3.03	-1.49
22	S	I find out which parts students already understand and don't teach those parts	3.00	1.36	2.38	0.46
23	S	I teach each student differently according to individual needs	3.79	1.05	2.44	1.28
24	T	I know exactly what mathematics the lesson will contain	4.14	0.77	3.83	0.41
25	T	I tend to teach each topic separately	3.00	0.88	3.15	-0.17
		Mean	**3.12**		**2.99**	
		Mean student-centred values	**3.30**		**2.53**	
		Mean teacher-centred values	**2.91**		**3.61**	

S – Student-centred, T – Teacher-centred

The most highly scored questions were Q20, Q11 and Q24. The acknowledgement of the importance of student error as an element of effective practice reinforced similar responses made in question two, teachers' perceptions of effective mathematics teaching through depicted lesson scenarios. Q11 emphasised links between topics as an important element in effective lessons which linked to the connectionist belief orientation in question three, teachers' beliefs of the purpose of mathematics. The lowest scored question was Q21. Acknowledgement by participants of student error suggested that participants expected to use more than one method of explaining working to questions, also linked to the connectionist belief orientation highlighted in question three.

Participants were then allocated to a single belief category, transmission, discovery or connectionist, according to their predominant overall response from the beliefs questionnaire shown earlier in Table 4.12. One participant (N) expressed no overall preference and was excluded from this part of the analysis. Analysis in this form allowed triangulation of responses between those participants' beliefs expressed in Table 4.12 with the same participants' classroom practices from Table 4.15. Table 4.17 shows the mean reported frequency for each classroom practice for each predominant belief orientation.

Table 4.17

Participants Reported Practices by Belief Orientation

QNO	Question	type	C	D	T
1	Students learn through doing exercises	T	2.00	3.40	3.17
2	Students work on their own, consulting a neighbour from time to time	T	2.00	2.60	3.00
3	Students only use the methods I teach them	T	2.00	2.60	3.33
4	Students start with easy questions and work up to harder questions	T	2.00	3.60	4.00
5	Students choose which questions they tackle	S	3.50	2.80	2.00
6	I encourage students to work more slowly	S	3.00	2.40	2.50
7	Students compare different methods for some questions	S	4.00	3.40	2.33
8	I teach each topic from the beginning, assuming they know nothing	T	1.50	2.80	2.67
9	I teach the whole class at once	T	2.00	3.20	4.00
10	I try to cover everything in a topic	T	3.00	3.60	3.17
11	I draw links between topics and move back and forth between topics	S	5.00	4.40	3.67
12	I am surprised by the ideas that come up in a lesson	S	3.00	3.20	2.33
13	I avoid students making mistakes by explaining things carefully first	T	1.50	2.40	3.17
14	I tend to follow the textbook or worksheets closely	T	1.50	2.40	2.50
15	Students learn through discussing their ideas	S	4.00	4.20	2.33
16	Students work collaboratively in pairs or small groups	S	4.00	4.40	2.83
17	Students invent their own methods	S	4.00	3.20	2.17
18	I jump between topics as the need arises	S	3.50	4.00	2.83
19	I tell students which questions to tackle	T	2.50	3.20	3.67
20	I encourage students to make and discuss mistakes	S	4.50	4.40	4.00
21	I only go through one method for doing each question	T	1.00	1.40	2.17
22	I find out which parts students already understand and don't teach those parts	S	2.50	2.40	3.33
23	I teach each student differently according to individual needs	S	4.00	4.20	3.50
24	I know exactly what mathematics the lesson will contain	T	4.00	4.00	4.33
25	I tend to teach each topic separately	T	2.00	3.40	3.17
	Student-centred mean		3.75	3.58	2.82
	Teacher-centred mean		2.08	2.97	3.26

C – Connectionist, D – Discovery, T- Transmission

S – Student-centred, T- Teacher-centred

In Table 4.17, C represents the connectionist orientation, D the discovery orientation and T the transmission orientation. The small sample size for connectionist teachers makes it difficult to make meaningful associations other than the teachers, here labelled as connectionist, clearly employ more student-centred than teacher-centred practices in their classroom. Transmission orientation teachers were more teacher-centred than student-centred, but in a much less pronounced way than the other orientations. Connectionist orientation participants were the most student-centred of all, although both connectionist and discovery orientation participants recorded higher student-centred than teacher-centred practices. Those findings were in keeping with the data collected in research questions two and three. Participants viewed themselves as more student-centred teachers, but all had high teacher-centred practices in their classrooms. There was a degree of consistency between expressed belief orientations and reported classroom practices.

4.5 To What Extent Does Teacher Experience and Background Influence the Curricular Impact of Teacher Professional Learning?

Data collected in this area is detailed in Appendix A. The group of 14 participants had a range of experience in teaching. The least experienced participant had been teaching one year and the most experienced teaching for 39 years. The mean length of service in secondary schools was 16.8 years. One participant had two years of primary school service, two had taught in district high schools and one had tertiary teaching experience. Participant teacher qualifications are summarised in Figure 4.3.

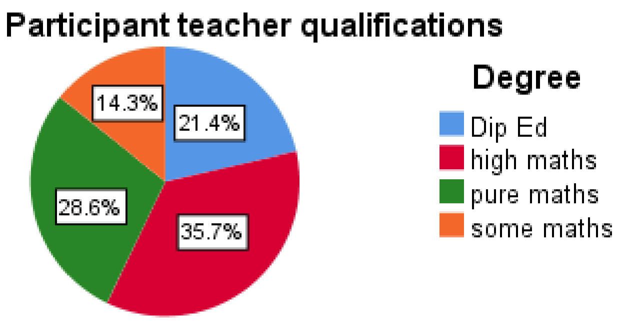


Figure 4-3 Background Tertiary Education of Participant Group

Four of the participants had completed a degree in pure mathematics, five had degrees with high mathematics content and five had some mathematics in their background, however 36% of participants reported not having studied mathematics beyond year two in university. Table 4.18 records participants' self-expressed confidence in teaching different levels of the WAC:M in Years 1 to 12.

Table 4.18

Participant Confidence Across the Western Australian Mathematics Curriculum Content Years 1 to 12

	Y11/12 Mathematics Specialist	Y11/12 Mathematical Methods	Y11/12 Mathematics Applications	Y11/12 Mathematics General	Year 10A	Year 7 to 10	Years 1 to 8
Not confident	3	1	0	0	0	0	0
Some confident	2	2	0	0	0	0	2
Mostly confident	4	2	0	0	0	0	2
Highly confident	5	7	13	14	14	14	9

The five teachers who selected 'not' or 'some confidence' in Y11/12 Mathematics Specialist (Table 4.18) were those participants with the least formal qualifications and four of those five teachers also had the shortest length of teaching experience, suggesting a lack of exposure to all courses offered in schools. All participants expressed comfort in teaching all courses from Year 7 up to Year 11 Mathematics General. Nine participants recorded being highly confident in teaching Years One to Eight mathematics although only one participant had declared primary teaching experience. Not every participant chose to answer each question or level and no inference about this was made.

All Western Australian teachers are required to complete a minimum of 100 hours of professional learning within a five-year period to maintain registration with the Teacher Registration Board in Western Australia (Teacher Registration Board of Western Australia, 2018). When asked about the type of professional learning (PL) undertaken over the last five years, participants were invited to classify that PL under three categories: formal mathematics, classroom practices and whole school developments. Participants were asked to allocate an approximate percentage to each category to a total of 100%. Results are shown in Table 4.19.

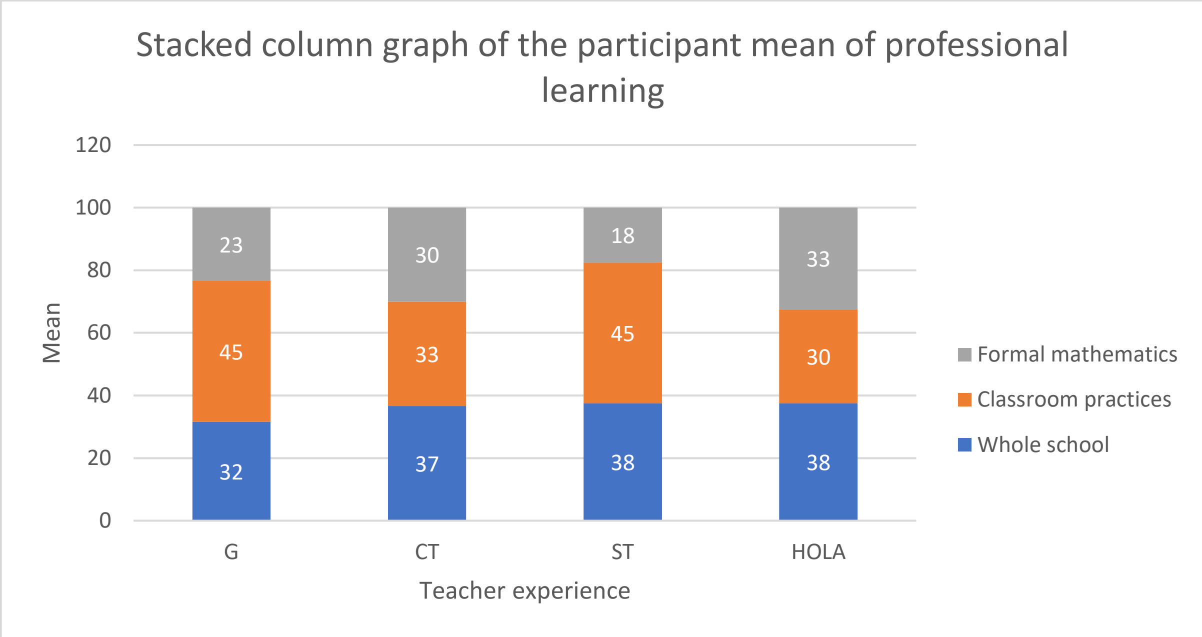
Table 4.19

Percentage Professional Learning for Participants Over the Past Five Years

Participant	Group	classroom practices	formal mathematics	other school developments
A	G	20	20	60
B	G	75	10	15
C	ST	60	20	20
D	HOLA	20	60	20
E	HOLA	30	60	10
F	CT	50	0	50
G	CT	20	40	40
H	HOLA	40	0	60
I	G	40	40	20
J	ST	20	10	70
K	CT	30	50	20
L	HOLA	30	10	60
M	ST	40	30	30
N	ST	60	10	30
	mean	38.21	25.71	36.07

G – Graduate, ST – Senior Teacher, CT – Classroom Teacher, HOLA – Head of Learning Area

PL was heavily weighted in favour of classroom practices and whole school developments. Three participants, (D, E & K), recorded highest scores in formal mathematics. At most, 26% of PL involved formal mathematics, some of which could be expected to be targeted at teachers of Years 11 and 12 students. Summarised data are displayed in Figure 4.4.



G – Graduate, CT – Classroom Teacher, ST – Senior Teacher, HOLA – Head of Learning Area

Figure 4-4 Professional Learning by Teacher Experience Over The Last 5 Years

Two HOLA participants indicated some 60% of PL was designated as formal mathematics (Figure 4-4). Without that involvement, the mean figure for formal mathematics PL reduces to 20% over the remaining participants. HOLA participants had the highest mean formal mathematics PL at 32.5% with the ST group lowest at 17.5% mean time. The CT group conducted 30% of their PL time in formal mathematics with the G group conducting 23% of PL in the same area. The G group spent more time with elements of classroom practices and school development PL than formal mathematics which was matched by the ST group. The ST group spent most time in whole school development PL (38%).

Participants were also asked to rate the PL undertaken out of 100 as to its capacity to improve effectiveness in teaching and learning. This is shown in Figure 4.5.

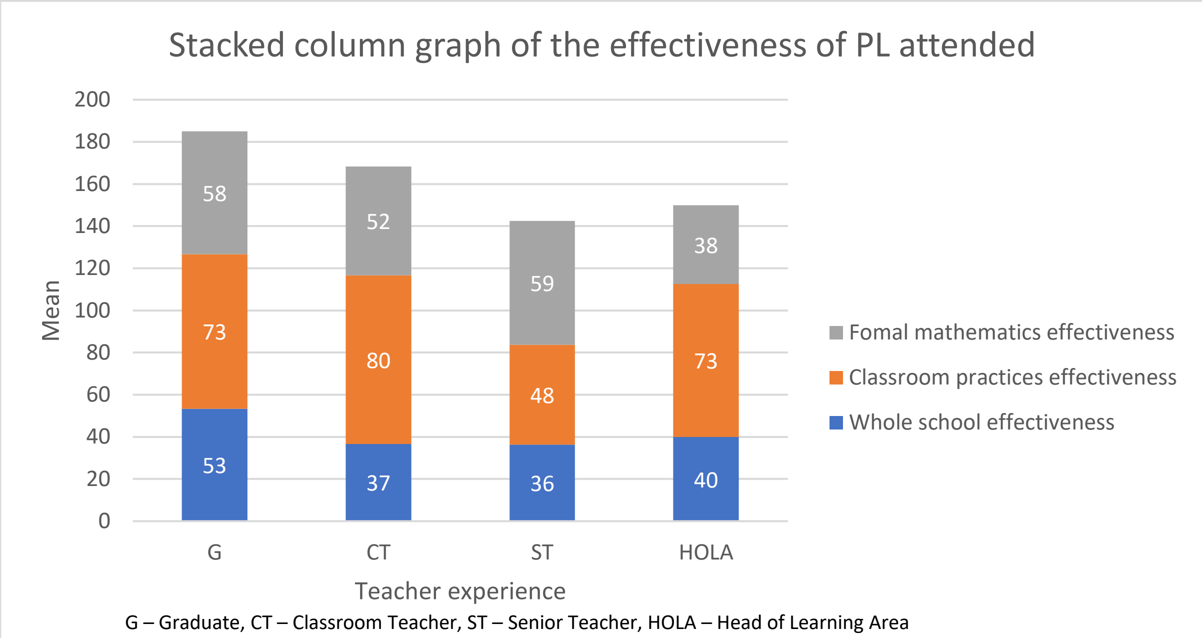


Figure 4-5 Stacked Column Graph of Effectiveness of Attended PL

The data shows that the G group were more positive in general about PL attended and that the ST group were least positive about PL attended. The CT group gave classroom practices the highest mean score for PL attended that improved effectiveness in teaching and learning. Generally, all groups regarded whole school developments as having the lowest impact on improving effectiveness in teaching and learning. The Participants rated formal mathematics PL as 55% effective in improving their classroom effectiveness. This compared to rating classroom practices at 67% and whole school development at 41% effective in improving classroom effectiveness. The data were also analysed by teacher experience. HOLA group did not rate formal mathematics as being effective in improving effectiveness in teaching and learning, which is of interest.

4.6 Triangulation of Data

Triangulation of data assisted in making comparisons of participant selections across the various questions posed as well as establishing consistency of beliefs and practices in the effective teaching of mathematics. Data is shown in Tables 4.20A and 4.20B.

Table 4.20A

Proficiency Strand Response by Participant Group (n =14)

Group	Group size	Total number of correct responses	Average Response Rate
G	3	17	5.67
CT	3	25	8.33
ST	4	35	8.75
HOLA	4	24	6.00

The Proficiency Strand responses (research question one) by participant group and belief orientation show that of the teaching groups, the ST group had the highest average response rate (8.75) and the G group had the lowest (5.67).

Table 4.21B

Proficiency Strand Response by Belief Orientation (n =14)

Group	Group size	Total number of correct responses	Average Response Rate
C	2	13	6.50
D	5	42	8.40
T	6	37	6.17
None	1	9	9.00

Participant responses were also compared with declared participants' beliefs as highlighted in Table 4.12. In these data, the highest average response was by the single participant (9) with equal weighting in transmission and discovery orientation. Discovery orientation participants had a value (8.4), albeit with a larger group size.

Transmission orientation participants had the lowest average group response. Table 4.21 shows participant data across questions.

Table 4.22

Triangulation of Data by Participant

	Belief Orientation Mean score			Classroom Practices Mean score		Scenario Lesson Choices			Teaching experience and background	
Participant	T	D	C	SC	TC	EL	LEL	CL	Group	B
A	36.67	30.00	33.33	2.75	3.77	D	B	B	G	Some
B	24.33	47.67	27.67	4.00	2.85	A	B	C	G	Dip
C	50.00	23.33	26.67	2.75	3.31	D	B	B	ST	Some
D	26.67	33.33	40.00	4.25	1.92	A	B	B	HOLA	Dip
E	33.33	53.33	13.33	2.67	2.62	C	B	B	HOLA	High
F	30.00	36.67	33.33	3.42	3.38	A	B	B	CT	Dip
G	39.33	36.00	24.33	2.92	3.15	C	D	B	CT	High
H	73.33	16.67	10.00	2.58	3.38	B	D	B	HOLA	High
I	30.00	40.00	30.00	3.67	3.46	C	B	B	G	High
J	63.33	25.00	11.67	3.00	3.08	D	B	B	ST	PM
K	28.67	42.00	28.67	4.17	2.54	A	B	B	CT	PM
L	23.33	36.67	40.00	3.25	2.23	A	B	B	HOLA	High
M	43.33	26.67	30.00	2.92	2.85	C	B	B	ST	PM
N	36.67	36.67	26.67	3.83	2.92	D	B	B	ST	PM

T – Transmission, D – Discovery, C – Connectionist

SC – Student-centred, TC – teacher-centred

EL – Most effective lesson, LEL – Least effective lesson, CL – Most commonly taught lesson

G – Graduate (<5 years), ST – Senior Teacher (≥10 years), CT – Classroom Teacher (≥5 and <10 years), HOLA – Head of Learning Area (not time specific)

B – Background, Some – Some mathematics in the degree (<2 years), High – high mathematics content in degree, PM – Pure mathematics degree, Dip – little formal mathematics within the qualification

The participants with the four lowest mean scores in the transmission belief orientation (L, B, D, K) all selected Scenario A, the proposed discovery lesson, as their most effective lesson. Three of the four highest scoring connectionist-oriented participants also selected Scenario A as their most effective lesson. Similarly, the three participants with highest student-centred classroom practices scores selected Scenario A as their most effective lesson. Comparing the five highest teacher-centred classroom scores showed no common selection of an effective lesson.

All three teachers with a Diploma of Education background selected scenario A as their most effective lesson, while both participants who asserted some mathematical content in their qualification selected Scenario D (connectionist) as their effective lesson. There was no agreement between participants based on their length of teaching experience as to the most effective lesson. There was strong agreement across all orientations and practices about the least effective lesson and the most commonly taught lesson, both Scenario B (transmission).

Chapter 5 Discussion

The research was structured to answer the question: What are Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the Proficiency Strands? Finding a response to the research was involved and complex. The basis for the research was to consider teachers' perceptions of effective teaching and to investigate links between those perceptions and the Proficiency Strands of the WAC:M. It examined whether teachers' beliefs about mathematics, teaching and learning influenced their practices and whether their practices were alternatively driven by the curriculum content and the curriculum Proficiency Strands. Sullivan et al. (2013) categorised the use of Proficiency Strands as the 'process' of teaching the curriculum. The responses from this group of participants (*n*=14) are varied and complex. The discussion will examine responses to each of the five sub-questions and will consider the range of participants' responses in those areas, concluding by identifying pertinent features of those participants' responses and then linking those responses to extant literature.

5.1 Are Teachers Familiar enough with the Proficiency Strands to be Able to Identify them from a Stated Action?

The Western Australian Curriculum: Mathematics (WAC:M) is firmly based on the curriculum developed by ACARA in 2013 (ACARA, 2012; School Curriculum and Standards Authority, 2018b). Mathematical Proficiency Strands are a feature in both curricula. As the current WAC:M has been embedded in all government schools for a period of approximately three years (School Curriculum and Standards Authority, 2014) a sensible null hypothesis for identification of Proficiency Strands, therefore, is that participants would be able to identify Proficiency Strands from stated actions in the process of teaching mathematics.

Participants were asked to match the four Proficiency Strands (Fluency, Understanding, Reasoning and Problem Solving) to 14 statements. The statements were assigned to strands by the researcher from a variety of curriculum content descriptors after careful consultation with the curriculum documents (School Curriculum and Standards Authority, 2018a). The Proficiency Strands were not well matched by the group, as the participants found it hard to agree on statements linked to actions in teaching with only one statement (What can you infer? $n = 14$) gaining unanimous agreement between participants and the researcher (Table 4.1, Q14). Seven of the statements had participants selecting all possible responses, for example 'In what ways can you prove?' gathered participant responses in each Proficiency Strand. Overall, participants matched at most a median of 50% of actions to the assigned proficiency, as allocated by the researcher. This is a surprising result given that the curriculum has been in place for at least three years. It would be expected that processes might well be embedded incorporating proficiencies as described by Sullivan (2011), and that the language of the curriculum might be more consistently applied.

By far the most common selections made by the participants of the Proficiency Strands were Reasoning and Problem Solving (Table 4.1). This could be seen to be in keeping with the current emphasis on mathematical classroom content and may well reflect the perceived emphasis on task-based modes of teaching and learning. Extant literature highlights the potential differences in student performance in countries where content strand emphasis contrasts with those nations with an emphasis on the mathematical tasks. An important element of categorising a nation would be the identification of countries where teachers make important links to the goals of the content strand rather than the task process (Mikk et al., 2016; Vashchyshyn & Chernoff, 2016; Zhang & Stephens, 2013), a problem which seems to reflect the attitudes and practices of WA teachers. With Australia's apparent lower performance in international comparative studies (Thomson et al., 2017) Australian teachers might consider reviewing Sullivan et al.'s (2013) six key principles for effective teaching of the curriculum. It would not be unreasonable to think that without a more detailed understanding of the purpose behind the actions of the Proficiency Strands and the related language of the content strand, then those goals may be obscured or even lost for participants. Research by Zhang and Stephens (2013) with Chinese teachers and Mikk et al. (2016) with Finnish teachers indicated goal setting and targeting improve student attainment when comparing international measures of success in mathematics. It could therefore be tentatively concluded that emphasis on content strands in a curriculum design framework offers different benefits in learning to a focus on task processes in mathematics education.

The desire to deliver the curriculum as it was intended, has implications for educators and curriculum designers. Existing teachers of mathematics in WA should benefit by undertaking targeted and detailed professional learning centred around the Proficiency Strands, with an emphasis on how those proficiencies underpin the curriculum. The teaching and learning goals such an emphasis might inspire could allow growth of the ideas underpinning mathematical development, or as Sullivan et al. (2013) describes "the important ideas" (p. 464). Designing professional learning to support such a structure will be a challenge for curriculum designers and others delivering professional learning opportunities, whilst meeting Australian Institute for Teaching and School Leadership Standards (Australian Institute for Teaching and School Leadership, 2018), especially standards 1,2,3 and 5. Can they offer a professional learning session which clearly articulates both the mathematical content and the philosophical basis underpinning the curriculum to be covered? As yet, this has proven to be problematic, although the model illustrated by Day and Hurrell (2013) may help illuminate possibilities when linking algebra and teacher content knowledge with the actions undertaken by students in a classroom. Further, pre-service teachers need to spend time discussing the philosophy behind the Proficiency Strands rather than merely focussing on the content. Whilst pre-service teacher courses may provide such learning, the graduate teachers in this study have not demonstrated that they have found that philosophy easy to understand or apply. Curriculum adoption for this group of teachers has offered parallels to the research literature described by Drake and Sherin (2006) and Remillard (2005), who both described adoption requiring time for teachers to explore, examine, and discuss changes in collaboration with colleagues. Linking findings in this question with results from Table 4.19 on amount and quality of PL would indicate that, for this group of teachers, such collaboration and time for reflection has not taken place.

For many participants, the 'default' proficiencies were Reasoning and Understanding. It seems that participants hope to engage students in reasoning activities to improve learning and make their teaching more effective. Participants may mistakenly believe that undertaking reasoning will inevitably lead to understanding in students. While it may often be the case that student understanding can follow from reasoning, as this group of participants observed, without expert teacher direction that is not a guaranteed outcome. Whilst using reasoning activities is commendable, effective mathematics teaching needs to use all four Proficiency Strand actions to make adaptable, rounded learners and provide effective teaching and learning in mathematics classrooms. It appears the need for a balanced emphasis on the use of all four

Proficiency Strands as actions for teaching mathematics has been obscured. There is a need for that emphasis to be restated and supported in further detailed research.

Data reported in Tables 4.1 and 4.2 indicate that participants in this group of teachers were unable to match the Proficiency Strands to curriculum content actions. At best, the median success rate of matching statements to allocated proficiencies was 50%. This means that this group of participants were largely not able to identify the Proficiency Strands from stated actions taken from content strands of the WAC:M.

5.2 What do Teachers Perceive as Being Effective Mathematics Teaching in a Secondary Classroom?

To understand what teachers themselves understand as effective mathematics teaching this study posed four teaching scenarios for discussion. The study described four lessons chosen to depict belief orientations described by Askew et al. (1997a) and featured by Swan (2006). There were two transmission lessons, one discovery and one connectionist lesson (Table 4.3). Amongst the participating teachers, choosing the most effective lesson of the four scenarios was problematic (Table 4.4). There was agreement around the participants' perception of ineffective teaching. It was clear that a traditional lesson with the teacher teaching from the front of the room and students being led through some truths towards prescribed learning and potential understanding, that is a transmission orientation (Swan, 2006), was not seen as highly effective. However, a similar lesson albeit with better student engagement in its design, was seen by many to be successful and in some cases optimal. Teacher-centred, transmission-oriented teaching was marked as traditional, but all participants articulated that this style of teaching happens regularly in Western Australian schools (Table 4.4). Participants did not state that traditional forms of teaching were inferior, but they were more positively focused on contemporary classroom strategies such as those typically supported by

Boaler et al. (2000). The reasons proffered as to why the transmission style of teaching is used centred, for the participants, around the time available for content delivery or the volume of outcomes needing to be covered (Table 4.7). For some participants their reasons related to classroom management, such as student collaboration and engagement, or behaviour management issues. A transmission style of teaching was seldom perceived to be an optimal form of teaching. Participants further perceived that transmissive teaching was only ineffective if students were not engaged in the process and the teacher had not checked for understanding during the lesson (Tables 4.6 & 4.7).

The participants' responses indicated that ineffective lessons and ineffective aspects of lessons had common attributes (Table 4.8). These included: highly teacher-centred modes of delivery (employed what was described by many as direct instruction, which is described by Hughes et al. (2017) as instruction based on effective teaching behaviours); a lack of student engagement; and modes which required little reasoning, collaboration or activity by students. This contrasts with Hattie's (2009) findings regarding Direct Instruction, when he described the outcome of Direct Instruction as offering a positive effect size of almost two years' instruction. It appears the participants agree with Hughes et al. (2017) when considering Direct Instruction. They saw this as being evidenced through teaching by example which students often copied from a board and followed by using many repetitive short response questions to check for understanding. The use of algorithms would be a feature of the Direct Instruction approach described by Scenario B, which was interpreted as the least effective lesson by almost all the participant group (Tables 4.4 & 4.6). Scenario B was characterised as typifying classes with poor classroom management and student misbehaviour and was a style which allowed a teacher to minimise student interaction whilst controlling behaviour. The findings concur with the work of Stols et al. (2015) who found that teachers are good at defining effective and ineffective lessons in terms of methodology used, and assigning student engagement as an indicator of effective teaching. How seats are arranged in the classroom were highlighted as one of the reasons for the least preferred lesson. Students were placed in rows in this scenario, making collaboration difficult and limited. A lack of collaboration in this lesson, contrasted with the emphasis on collaboration in other lessons (Table 4.9) indicated participants were convinced of the importance of collaboration in student learning. Collaboration is an integral part of the General Capabilities of the Western Australian Curriculum: Mathematics (SCSA, 2018c) and complies with the key principles for effective teaching articulated by Sullivan (2011). It is therefore significant that participants recognised the importance of collaboration as a contributor to effective teaching, in alignment with the General Capabilities as described in the curriculum (School Curriculum and Standards Authority, 2018c).

Participants were clear in their opinion of the ineffective use of direct instruction as a teaching strategy. It must be clarified that participants appeared to fairly consistently use the term 'direct instruction' to mean the use of mostly rote and repetitive skill-based work, and this not in the strategic sense as advocated by Englemann (2004) or Huitt et al. (2009). Where direct instruction is linked to single row seating patterns participants described "boring, thoughtless lessons produced by stressed teachers rushing to complete a full curriculum" (Participant L, Table 4.7). Many comments centred on the use of this strategy to cover curriculum content in a time-efficient manner, rather than an educationally efficacious way. Participants' comments highlighted underlying concerns about the time taken to complete what was perceived to be a busy curriculum in the time allocated by schools. Many participants claimed that using manipulative materials, and other engaging strategies, were rejected due to time constraints when planning lessons (Table 4.9). Strategies which focus on pedagogical stratagems may be related to the issues where lesson focus is centred on achieving a clear mathematical learning goal, as opposed to planning for engaging strategies to encourage participation and relevance commonly asserted by researchers (Mikk et al., 2016; Thomson et al., 2017; Zhang & Stephens, 2013). Lesson focus, therefore, remains an important aspect of effective teaching. The question of whether it is effective to simply import a pedagogy such as Finland's, in isolation, from countries higher ranked in comparative studies than Australia, requires detailed research and should not be decided solely based on international comparisons.

Participants made few links with the Proficiency Strands and the varied actions of mathematical thinking these actions encourage. When Proficiency Strands were mentioned, the context was reinforced by participants commenting on the hierarchical progression through the Proficiency Strands from Understanding and Fluency to Reasoning and Problem Solving. Participants often related the Proficiency Strands to lesson differentiation strategies structuring the weakest students into Fluency and the more able into Reasoning and Problem Solving. This would be akin to ability grouping through setting described by Boaler et al. (2000) and does not exemplify the key skills ideal of Sullivan (2011). Graded questioning, a phrase used in the classroom scenarios to describe book questions ranging from easier to harder, was regularly used by participants as a positive feature in a lesson (Table 4.8). Effectiveness was characterised through students choosing, or being directed towards, the correct level of difficulty in their questioning and work, but again there was no link to mathematical Proficiency Strands enunciated by participants. This meant that students potentially lost the opportunity to consolidate understanding of a dependant concept or skill before progressing to a more demanding concept or skill. It seems that participants have clear ideas of what effective differentiation involves but do not link differentiation strategies to research based methods or the mathematical Proficiency Strands as set out by Sullivan et al. (2013) and the WA curriculum rationale (SCSA, 2018b).

Although the Proficiency Strands describe the actions that Sullivan (2011) expected to be used by teachers to offer a wide range of challenge and instruction to students, proficiency strand use seems an elusive aspect of the actual practice of effective teaching. Many participants positioned the proficiencies into a hierarchical structure of difficulty, similar to Bloom's taxonomy (Krathwohl, 2002). It appears they expect students to progress through the proficiencies in some order towards completion. This may well be reinforced by textbook writers who often order questions under a particular Proficiency Strand heading, in many cases, with questions becoming seemingly more difficult or involved. Teachers would do well to familiarise themselves with the stated actions of mathematical proficiencies in curriculum documentation (School Curriculum and Standards Authority, 2018b) to assist in planning more effective lessons and goal setting for classes.

It is reasonable to argue that specific knowledge of the Proficiency Strands does not by itself make or break a lesson. However, are teachers who are conflicted in their understanding of the processes enacting effective practice as envisaged by the curriculum designers? What may be happening in schools is that mathematics is either being taught in the ways it has traditionally been taught, as evidenced by the selection of the most commonly taught lesson made by participants in this research (Table 4.4), or teachers have interpreted or misinterpreted the use of Proficiency Strands, as described by Walshe (2015). Whether the teaching is effective or not, will largely depend upon what is measured and the person measuring its effect. Many 'good' teachers teach effective lessons without applying reference to the Proficiency Strands.

However, if the common lesson style includes transmission lessons, seen by participants as traditional or restricted in their delivery, those lessons may fail to improve mathematical understanding for students who would benefit from more student involvement. Teacher-centred practices were featured heavily by many of the participants' responses during this study (Table 4.16), although, interestingly, as a teaching philosophy it was rejected by almost all participants. It seems that teachers in this study do not see themselves as traditional, but show varied understanding of current curriculum processes, and see traditional teaching traits in other professionals around them. A lack of familiarity and understanding of the Proficiency Strands may well be limiting the success of teaching mathematics by participants in this study. Those participants would do well to become more familiar with the actions and range of learning opportunities advocated in the type of activity suited to different Proficiency Strands as part of regular classroom practices.

Thematic analysis of the recurrent comments from the participants highlighted the importance of lesson attributes and modes of instruction, with the use of materials described as important by participants, who advocated the use of manipulative materials (Table 4.8). Typical lesson attributes, highlighted by participants, included modelling and differentiation. Modes of instruction included direct instruction and student engagement, when linked to instruction. This is interesting and reflects similar findings by the Stols et al. (2015) study. Stols et al. noted that their participants failed to highlight the educational objectives of the lesson topic. Few of the WA participants in this research commented on the objective of the lesson to depict the teaching of linear equations using algebraic and graphical techniques. Not linking effective teaching to lesson aims reflects similar findings in other research (Mikk et al., 2016; Šapkova, 2013; Zhang & Stephens, 2013) where Australian teachers concentrated on planning of lessons, pedagogy and classroom management without a designated focus on the objectives of the lesson.

Participants did not select any one lesson as the most effective, suggesting that no single lesson had all the characteristics of collaboration, modelling and questioning detailed as effective by the themes established in Table 4.8. This is reasonable and reflected similar findings of Stols et al. (2015). The choices of what constituted an effective lesson will be compared to an individual's beliefs of effective teaching and how those beliefs influence that choice will be explored later. Against expectations, teaching experience was proven not to be a major factor in determining if lessons were effective or ineffective (Table 4.10). Many experienced teachers did not highlight the range of effective aspects in lesson scenarios which again affirms the finding by Swan (2006) of a general lack of agreement of what constitutes effective learning. As an example, Heads of Learning Area were notably mixed in their agreement of what aspects of effective learning a lesson contained. That was an unexpected result as there would be the belief that the role of a Learning Area leader would include monitoring, modelling and managing the learning in a mathematics department, leading to a shared definition of effective teaching. Such a response may be because the described scenarios did not meet the criteria for those individuals.

For participants, effective teaching was characterised by student collaboration, meeting student needs, teacher modelling and explanation as well as classroom control and management. There were many who expected to see some concrete modelling in lessons, particularly in algebra. A recurring comment was on the use of questioning and how teachers deal with student mistakes. There were concerns over opening students to ridicule, yet many participants wanted teachers to explore the thinking behind student misconceptions as a method of gaining understanding. When discussing similar mathematical teaching pedagogies, using prior learning and exploring student misconceptions, indicated considerable disagreement between participants, similar to the disagreement between Cheney (1997), Boaler (2016) and Dweck (2010). It seems practical to conclude that further research is required into what constitutes effective learning in mathematics for teachers of mathematics as opposed to researchers and observers, as there appears to be some dichotomy between teacher expectations of learning and research evidence of student attainment.

Modelling and explanation were also highlighted as important for effective lessons. It was noted that the use of structured algorithms for changing the sign of variables and constants in algebraic manipulation garnered both positive and negative comments. It was interesting to note that the positive, supportive comments in using the change of sign algorithm were generated by the participants favouring a transmission orientation. This may well be in keeping with what many might see as a traditional teaching style, a teaching style generally regarded as a negative feature of lessons by participants. Pertinent comments acknowledging the reasoning component of modelling non-algorithmic thinking reinforced the fact that such practices help 'de-mystify' mathematical reasoning to students and encourage students to begin to think like mathematicians. Negative comments around modelling and explanation highlighted that some understanding must already be present to engage and develop relevant thinking, thereby reducing the potential impact of the practice. The differences in opinion here relate to the mathematical beliefs of teachers. Askew et al. (1997b) described whether mathematics relates best to the transmission set of rules to be learned and followed or the connectionist perspective where mathematics constitutes a network of inter-related concepts best understood by experiential learning. Ball et al. (2005) included the mathematical thinking pertinent for teachers which characterises research in this area.

More participants in this study related to transmission rather than the connectionist perspective (Table 4.12).

The ratio of comments in the current study related to effective and ineffective lessons differed to that found by Stols et al. (2015). Stols et al. found that teachers were reluctant to comment on ineffective aspects of video vignettes. This may well be because, as Stols et al. observed, teachers did not want to criticise another colleague. For the WA participants that was not evident and may well be because there was no identifiable colleague involved. Table 4.5 shows that more than one-third of comments described ineffective lessons. The lesson gathering the highest number of effective comments was a transmission lesson with strong student involvement through student whiteboards (scenario C). The least effective lesson as selected by the participants was also a transmission lesson, but with a more traditional approach and description (Scenario B). Indeed, this was the least effective lesson seen in mathematics classes in WA as selected by 13 of the 14 participants (Table 4.4). The discovery lesson also featured as ineffective with participants' reasons reflecting the concerns of the ability of students to translate work completed using concrete manipulative materials into written algebra. This is in concordance with the research of Brown et al. (2009) in their study on the use of concrete materials, who also found that although the use of concrete materials adds to the understanding of the student in that particular task, transfer of skills to other tasks can be difficult to achieve.

The lesson style thought to be used most frequently by the participants' colleagues, in typical mathematics classes in schools, was clearly a teacher-centred transmission orientation lesson (Table 4.4). Although there is no reason to doubt this sample as atypical, this would be even more concerning if it were true of a larger sample, as the research clearly shows that teacher-centred transmission is the style of teaching in Western Australia. It follows that many teachers, in the opinion of this group of participants, appear to be struggling to find effective ways to engage with the curriculum in its conceptual delivery aspirations, the Proficiency Strands. Many participants put this delivery method down to pressures of time, teacher knowledge and lesson content. Transmission lessons are simple to construct and when teaching a five-period day, according to at least one participant (e.g. Participants C & J), that is a critical consideration. Transmission lessons also benefit teachers teaching out of field, described by participants as teachers not majoring in mathematics at tertiary level (Participant K), in that it allows such teachers control over the content and delivery of a lesson (Ball et al., 2008). Transmission lessons also avoid the problem-solving discovery lesson, which has the possibility of 'discovering' some unexpected formula or pattern which the teacher does not comprehend or cannot explain. According to the participants, transmission lessons also make it easier to cover the curriculum content in the time accorded in school planning. Although as one participant wryly observed, "that is why we end up teaching the same thing year after year" (Participant N). There is a challenge for schools to address the unproductive nature of mathematics teaching, as evidenced in the current research. Concerns over planning time and content overload are real and need to be addressed if schools are committed to changing the pedagogical practices of teachers of mathematics. This would be a fruitful topic for further research and analysis.

5.3 To What Extent do Teachers' Beliefs of the Purpose of Secondary Mathematics Influence Their Teaching Practices?

A teacher may well be influenced by many factors in deciding what and how to teach mathematics. Research has indicated that one of the factors is how a teacher's belief orientation influences decisions, according to Ernest (1989) and Askew et al.

(1997a). The beliefs of teachers involved in this research were thought to influence their teaching practices, which in fact constitute the null hypothesis for this research question.

The participants were quizzed about their beliefs of what mathematics is, what learning is and what teaching is. The data collected from this participant group supported the findings of Swan (2006) (Table 4.11). Both studies asserted that the participating teachers saw mathematics as a system of truths and rules teachers need to impart to students. There was also agreement between both studies that teaching is effective when teachers provide a stimulating environment to facilitate exploration but there also needed to be a linear structure for students to follow. There was no agreement, however, in both studies about what constitutes effective learning of mathematics. This finding is consistent with data from question two in the current study where there was no agreement by participants on what constitutes effective learning of mathematics (Table 4.11).

How beliefs affect teachers' practice depends on the mathematics teaching orientation with which participants most align. The participant group of the current study was clustered around the transmission and discovery cusp (Figure 4.1), descriptions first described by Askew et al. (1997b), with a minor influence from the connectionist orientation (Figure 4.1). What this means, in practical terms, is that this group of teachers generally believe a teacher-directed orientation is effective, but at the same time the use of problem-solving and reasoning activities is appropriate. There is no clear indication that the amount of teaching experience influenced the participants, although new teachers had a more balanced view of effective teaching using both transmission and discovery orientations (Table 4.14). There was an observed negative correlation between strong teacher led lessons and student discovery lessons (Table

4.13). That correlation is not surprising as it supports descriptions of common practices of transmission and discovery-oriented teachers according to research (Adler & Davis, 2006; Ernest, 1989; Swan, 2006).

It is valuable to connect the findings of Swan (2006) and the group of Western Australian (WA) teachers in the current study (Table 4.11). Swan's research was conducted in England in 2005-06 (n=64). The fact that there is a shared set of beliefs about what mathematics is, suggests that there may well be similarities in the development of teachers in both the English and WA systems. Although most participants (11 out of 14) associated their overall beliefs with the transmission and discovery orientations, there was a strong association with mathematics seen as a given body of knowledge. This strong transmission orientation does not have the creative aspect or interconnectedness, central to connectionist and discovery orientations, of what mathematics is. It would be interesting to compare belief orientations of what mathematics is between other nations and, in particular, those countries performing higher in PISA (Thomson et al., 2017) than Australia.

The participants in Swan's (2006) study were more transmission oriented in their assertion of what effective teaching is and this may have been influenced by the time when the study was conducted. In 2006 the movement towards more discovery-based 'rich tasks' in teaching was still gaining traction in the UK, evidenced by Swan (2007). Rich task development contrasted with more traditional pedagogies in the UK, where teachers explicitly directed learning through a detailed sequence of learning interactions (Swan, 2007). Twelve years on, this group of Western Australian participants were agreed that the focus of effective teaching is more about exploring interconnectedness in mathematics (Table 4.9). They were only slightly more inclined to see teaching as facilitating exploration above a structured linear curriculum (Table 4.11). The participants acknowledged the use of mitigating misunderstandings through careful sequencing of experiences along with assessing to determine what and when a student is ready to learn. The mixed discovery and transmission orientation is reflected in the previous responses to effective teaching from question two (Table 4.8) with themes in comments made about teacher questioning, checking for understanding and prior learning and using mistakes as teaching points (Table 4.8). It would have been useful to gather more detailed information from the participants to illuminate and examine this point further and this could well be a topic for further research.

There is still work to be done in helping teachers to develop a better and more consistent understanding of what learning is. This Western Australian group were as conflicted as Swan's (2006) group about what learning mathematics entails. Knowing more about learning, in a mathematical context, requires better understanding. It may well be the case that effective learning includes aspects of watching and imitating, practical exploration and reflection and arriving at understanding through discussion. Without some clearer delineation of effective learning teachers can never be sure when lesson planning encourages learning or learning is a by-product of instruction. Greater prominence for research into brain plasticity (Boaler, 2016; Dweck, 2010) and how it impacts learning may promote a necessary discussion for teachers of mathematics. Sullivan et al. (2014) and others (Akyuz et al., 2013; Boaler, 2016; Clarke et al., 2012b; Star, 2016; Watt & Goos, 2017; Zhang & Stephens, 2013), have declared that the interpersonal, discussion-based activities in lessons should be prominent parts of learning in the classroom. However, a better scale, or a refinement of the existing Swan scale, as used in the current research (Table 4.11), may yield clearer indications of what mathematical learning is. A better understanding of what learning in mathematics is would benefit the mathematics teaching community and would be useful further study.

The results of the effect of teacher belief orientations on other orientations was of interest because research suggested a difference in beliefs-influenced teacher orientations (Askew et al., 1997a). The negative correlations between transmission and the other orientations was notable (Table 4.13). Given evidence from other studies (Adler & Davis, 2006; Ernest, 1989; Swan, 2006), this might be as expected, whereby transmission orientation is considered traditional teaching whereas discovery and connectionist orientations would be considered more contemporary pedagogy. For participants, that negative correlation may be linked to the arguments between explicit and direct instruction models and more student-centred constructivist models (Archer & Hughes, 2011; Englemann, 2004; Ewing, 2011; Reigeluth, 2013; Tait-McCutcheon et al., 2011). There was a lack of highly significant correlation between discovery and connectionist orientations which may weaken the previous argument, suggesting there may not be links between contemporary pedagogies (Table 4.13). Whilst it may have been reasonable to expect contemporary models of effective teaching to share common attributes, such as student collaboration, use of the Proficiency Strands in lesson exemplification or mathematical differentiation. It was, nevertheless, not evident in the data. Such a view needs to be moderated by the positive correlation displayed by the graduate early teacher group and the senior teacher group with more than ten years' teaching experience, who shared a strong transmission orientation (Table 4.14). It is a limitation of the research when attempting to explore common attributes of teacher beliefs more deeply given the small sample sizes being used. Notably, there was no observed movement away from transmission to other belief orientations based on length of teaching service, as might have been expected. However, one participant commented, with respect to research question two, "… I would no longer teach in that way" when referring to a transmission lesson suggesting a movement from transmission to a more student-centred orientation for that individual.

The results suggest that the beliefs of this group of Western Australian teachers are thought to influence their teaching practices. Those beliefs may well be influenced by the Proficiency Strands and the teaching processes they in turn encourage, leading to reduced transmission orientation styles of teaching.

5.4 To What Extent are Teachers' Perceptions of Effective Teaching in Secondary Mathematics Reflected in Their Own Practice?

Research by Swan (2006) indicated that the perceptions of teachers about effective teaching were evident in student perceptions of classroom practices. Dayal (2013) posited that the type of classroom practices employed by teachers influenced their instructional behaviour. The perceptions of effective teaching were thought to influence teacher and student-centred teaching practices, which will constitute the null hypothesis for this question.

Swan (2006) looked to connect teacher beliefs with their classroom practices. With regards to their classroom practices the participant group in this study were broadly aligned with Swan's results. However, there were a few differences in practices. Swan (2006) found that teachers aligned themselves strongly with teacher-centred classroom practices which would be typified by students starting with easy questions and working up to more difficult questions whilst being led by the teacher. The teacher would know what mathematics the lesson would contain. The Western Australian group were marginally more student-centred in their responses, having students working collaboratively, expecting them to make mistakes and discuss these mistakes guided by teacher facilitation (Table 4.16). The fact that only one participant in this study was categorised by indicating low teacher-centred and high student-centred practices suggests that participants in this study use a range of practices in their regular teaching.

An expected factor in understanding effective teaching was teacher experience. It seemed reasonable to expect that longer serving teachers would have gained expertise and thereby have honed their preferred teaching practices. Participants in this study were not consistent in their scores of teacher and student-centred practices when filtered by teaching experience (Table 4.15). The graduate group, with up to five years' experience, had a similar score in both teacher and student-centred practices. Some pre-service teacher educators may find this result disappointing if the focus on training had been student-centred, collaborative practice; potentially reinforcing the influence of cultural pressures exerted in school organisations to adjust or alter practices (Geiger et al., 2017; Proffitt-White, 2017). The senior teacher (ST) group (more than ten years teaching experience) also showed little preference in style. This may well reflect the length of service of the senior teacher group who have likely worked in a transmission oriented pedagogical era for some time. It is interesting to note that the ST group showed an equal orientation to student-centred practices, which may reflect the emphasis of the rationale and the actions of the Proficiency Strands since incorporating the curriculum from 2012. The group with the greatest student-centred orientation were the classroom teacher group, with between five and ten years' experience, who also displayed a negative correlation between the two orientations, suggesting that accepting student-centred practice limited their use of teacher-centred orientations. Classroom practices, when classified by teacher experience, indicated that participants' classroom practices are, to a small degree, influenced by their length of service. However, there is not a sufficient spread or distinction in data to indicate that length of service is significant. In this analysis the small sample sizes must be considered before making generalisations beyond this participant group.

Overall the whole participant group considered, there was a strong focus on student-centred classroom practices as important features in their classrooms (Table 4.17). However, the participants also used teacher-centred practices regularly which is reinforced by the earlier findings of effective teaching and beliefs for the group (Tables 4.4 & 4.12). It is interesting to note that in Swan's (2006) findings only one teacher in his top fourteen responses, sorted by highest mean score, were student-centred in their orientation. This compares to the WA group where six of the top ten responses were student-centred. This can be viewed as a positive finding demonstrating teachers are adopting classroom practices that better align with the Proficiency Strands and General Capabilities of the curriculum. However, any conclusions drawn from this study need to be mindful of the lack of a clear understanding of participants' beliefs about what learning is.

Three questions in the scale garnered strong responses greater than four out of five (Table 4.16), where five indicated almost always in reference to classroom practices. The strongest response was elicited when respondents were asked about students making mistakes (Table 4.16, Q. 20). The Swan (2006) study had a mean response much lower than the WA participant group indicating that the WA participants were strong advocates of expecting students to make mistakes and using those misunderstandings as significant teaching points in a lesson. Using mistakes to improve understanding as a practice is reinforced by research evidence as effective teaching (Boaler, 2016; Ryan & Williams, 2007). These researchers posited that the inclusion of mistakes allows discussion of misunderstanding, leading to positive learning. Similarly, the current participant group indicated moving back and forth between topics (Table 4.16, Q11) as an important feature in their classroom practice. Such practices align with the views of Sullivan (2011) who claims that fluency and transfer are among best practices in mathematics. When participants were asked to respond to teaching the method of solution in topics (Table 4.16, Q3 & Q17) the WA participant group indicated that they were more likely to discuss more than one method of answering questions, whereas in the Swan (2006) study it was less likely to happen. Effective teaching strategies incorporating multiple solutions would also reflect best practices for both Sullivan (2011) and Boaler (2016).

Swan (2006) found that teachers who had a transmission orientation also displayed more teacher-centred practices. This finding was supported by teachers in the current study. Swan noted that teachers who had a connectionist orientation were more student-centred in their practices. The participant group in this study had only two teachers who identified as connectionist but they, notably, had the highest student-centred score in classroom practices across all belief orientations (Table 4.17). The findings have been triangulated against Research Question Two where those two teachers identified the connectionist lesson scenario as the most effective lesson (Table 4.21). Askew et al. (1997b) stated "it was clear that those teachers with a strongly connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strongly discovery or transmission orientations" (p. 28). Using the Swan (2006) scale allowed triangulation between teacher beliefs and classroom practices to establish teacher belief orientations. It was significant that for participants in this study classroom practice responses could be triangulated against effective teaching through lesson scenarios, and that such triangulation indicated consensus between measures (Table 4.21). Based on the above, it would be appropriate for other researchers to consider using the Swan scale when researching teacher beliefs and classroom practices.

Participants in this study were slightly more student-centred than teacher-centred in their classroom practices. Classroom practices were marginally influenced by the length of teaching experience. There was evidence that the participants' classroom practices were aligning with contemporary classroom practices involving the Proficiency Strands and the General Capabilities, including using student mistakes as discussion points and transferring knowledge between skills. It is accepted that the perceptions of effective teaching amongst this group of participants is thought to influence their teaching practices.

5.5 To What Extent Does Teacher Experience and Background Influence the Curricular Impact of Teacher Professional Learning?

To gain further insight, participants were categorised by years of teaching experience in secondary schools; background information about the level of acquired tertiary mathematical experience; and comments gathered concerning professional learning and how effective that learning had been in developing or shaping their current classroom practices (Figure 4.3). Graduate teachers in this study had less confidence in teaching higher level Year 11 and Year 12 courses, particularly the Mathematics Specialist course in Western Australia which offers the most advanced mathematical content of all courses (Table 4.18). Those teachers with the least academic tertiary experience also happened to be the teachers with shortest length of service, making it unreasonable to assert which factor primarily influenced their uncertainty in teaching courses at that level. There was no uncertainty amongst those teachers about teaching content at any other level. Interestingly, nine teachers thought themselves capable to teach Year 1 to Year 8 content, although only one participant identified themselves as having primary training. Such assertions may well be worth exploring further as it seems to suggest, for this group of participants, that mathematical content knowledge is more important than pedagogical content knowledge. The mathematical conceptual development required in teaching early years and primary students has been highlighted by research over many years (Ball et al., 2005; Ball et al., 2008; Hill & Ball, 2009) and appears to have been discounted or assumed by this participant group.

Western Australia recently (2015) moved students commencing Year Seven from primary into the secondary school sector, which exacerbated a shortage of secondary teachers of mathematics. To alleviate that shortage the Department of Education enabled a 'Switch' program (Department of Education, 2016) allowing teachers to retrain into subjects with staffing shortages, including mathematics. Research has indicated that the mathematical knowledge for teaching has an impact on effective teaching (Ball & Bass, 2000). For this reason, it was hoped that this study could explore how teacher background affected perceptions of effective teaching and classroom practice by interviewing 'Switch' teachers as part of its sample. Unfortunately, this was not possible as relevant teachers did not volunteer to be interviewed and as a result it is impossible to comment further. There is still work to be done to better understand the impact of teachers teaching out of subject area or without higher levels of advanced mathematics and the impact that has on Western Australian mathematics education. This topic remains one of interest in Western Australian mathematics education and should be researched further.

Professional learning (PL) is mandatory for Western Australian teachers. It is concerning to note that for this group of WA teachers there was a low response to how effective formal mathematics PL had been (Table 4.19). Participants engaged in less PL focussed on specific formal mathematics learning, compared with PL on classroom practices or whole school developments (Figure 4.4). When attended, participants classified PL as barely effective (Figure 4.5). Without collecting additional data, it is impossible to ascertain whether the PL was poor, or the mindsets of the participants were poor. It may not be surprising then that those same teachers struggled with the Proficiency Strand activity earlier (Table 4.2). It is reasonable to think that few of these participants have had detailed professional learning in that area, but as participants were not directly asked that specific question responses cannot be assumed. Upon reflection, it would have been prudent to identify proficiency strand professional learning specifically as one of the categories in the PL question. Without a planned and coordinated PL approach by the authorities and universities it will remain likely that practising teachers will have difficulty in gaining the support needed to successfully implement curriculum change, particularly if that change also embraces fundamental changes to pedagogical practices. As noted by Star (2016), effective change in education occurs through incremental changes to practice when given support and time for development. When filtered by teaching background (Table 4.21) there is some indication that teachers with a qualification of a Diploma in Education display some congruency when identifying what they consider to be an effective lesson. This will be discussed in the following section.

The data indicated that teaching experience does not appear to influence teacher beliefs and practices, though teacher background does appear to influence some beliefs but only at some levels. The results, however, are inconsistent. It would be prudent to conclude that teacher background would appear to influence teacher beliefs and practices but may not be a major factor. This will be considered in the following section.

5.6 Triangulation of Data

Triangulation of data is defined by Bryman (2012) as "the use of more than one method or source of data in the study of a social phenomenon so that findings may be cross checked" (p. 717). The purpose of triangulation is to establish greater confidence in the findings of research by using more than one method to gather data. Such an approach allows researchers to validate findings across instruments and participant responses. Triangulation was used in this research to validate participant responses relating to effective teaching and classroom practices gathered by the questionnaire (Appendix C) when compared to qualitative responses on effective teaching practices (Appendix G).

When analysed by teaching experience, the Proficiency Strand activity showed that graduate teachers fared poorly and that the most experienced teachers fared best (Table 4.20). One possible explanation for this is that experience brings with it a better understanding of what curriculum language might imply. A surprise and a potential concern is that the Heads of Learning Area (HOLA) group fared much lower than the other experienced teachers. It is a concern in that, in many schools, a HOLA is considered a leader in curriculum development and pedagogy. Again, the very small sample size needs to be recognised and means that results may not be generalisable. Teacher beliefs were also reflected in the success rates in the activity of matching the researcher's categorisation of the Proficiency Strands. Those teachers favouring a discovery orientation scored the highest matching response rate of all groups, closely followed by the transmission orientation. This may be because those teachers use a higher proportion of task-based activity, requiring the use of actions which are articulated in the language used to express the Proficiency Strands. Alternatively, it may simply relate to the number of teachers in the categories. When aggregating for the mean response rate, 'discovery teachers', defined as those participants who had the highest mean score as discovery orientation in Table 4.21, were still dominant (Table 4.20, 8.4 mean) in the matching of Proficiency Strand actions. This result suggests that discovery orientation teachers use task-based activities and are more familiar with the language and actions of the Proficiency Strands. The single participant with no demonstrated favoured orientation was ignored in that calculation. Participants with a transmission orientation had the lowest average group response (6.17 mean) although that was comparable to connectionist orientation participants (6.50 mean). It would appear that those orientations still affiliate with the actions of the Proficiency Strands, if not to the same degree as discovery orientation participants.

All four participants with a Diploma of Education selected Scenario A (discovery lesson) as their most effective lesson. This may indicate some commonality in training paradigms for educators with a less discrete subject focussed qualification. It may be possible to show that the pedagogical instruction undertaken with teachers following the Graduate Diploma courses may be impacting on their perceptions of effective teaching and could be worth further study. Literature suggests, though, that discovery lessons are not regarded as the most effective lesson style (Adler & Davis, 2006; Back et al., 2012; Swan, 2006). There were also common effective lesson choices for those with only 'some mathematics' in their background, selecting Scenario D, the connectionist lesson. The 'some mathematics' category was indicative of up to two years of tertiary mathematics content. Literature suggests the connectionist lesson would be most effective in contemporary pedagogy (Askew et al., 1997b; Back et al., 2012; Hattie, 2012; Swan, 2006; 2007). Conversely, there was no common assertion of effective lessons for participants with high or pure mathematics backgrounds. This may indicate that the mathematical knowledge required for teaching (Ball & Bass, 2000; Hill & Ball,

2009) does not influence the choice of teaching strategy used by highly mathematically qualified teachers, as described in the literature for Pedagogical Content Knowledge (Ball et al., 2008; Shulman, 1986). Lesson or task selection measured against teacher qualifications may be a valuable area of continued study in WA. Three of the four participants selecting the connectionist orientation as their belief orientation (Table 4.21) also selected scenario A as their most effective lesson. Similarly, the three teachers with the highest student-centred practices scores (Table 4.21) selected Scenario A as their effective lesson. This observation must take into account the association between the connectionist orientation and student-centred practices which means that the same teachers would have been counted in each finding. Nonetheless, the discovery lesson proved a popular lesson selection across beliefs, practices and effective teaching question data.

There was strong agreement across all measures that the least effective lesson was a transmission lesson (Scenario B). Comments about a lack of effective teaching in that lesson indicated that it did not allow for collaboration, had poor student engagement and did not allow for reasoning and problem solving (Table 4.7). The emphasis on fluency did not encourage thinking across topic areas and help build relational concepts in students, as commented by one respondent: "It doesn't sit with fluency and reasoning" (Participant F). This indicates that basic transmission lessons are lacking some of the six key principles outlined by Sullivan (2011) for contemporary education as they fail to include collaboration, critical and creative thinking, mathematical proficiencies or preparing students for 21st Century living (American Association of School Librarians, 2009; Hemmi & Ryve, 2015). The ineffective lesson would not align well with student-centred practices or discovery or connectionist orientations, reflected by more than half of the participant group. When considered against brain research for early adolescents

(Dweck, 2010) supporting learning, transmission lessons were not seen as effective by this group of participants.

Six participants identified themselves as transmission orientation in question two. It would have been reasonable to expect many of those participants to favour the transmission lesson. It is interesting to note that only one of those six teachers selected scenario B as an effective lesson, and that participant had the highest transmission and teacher-centred practices scores. This may well indicate that even those participants with a marginal transmission orientation are aware of student-centred practices, and hence, perhaps sub-consciously, look to accommodate them in their teaching practices. This, if it is accurate, would support the findings in Swan's (2006) study where teachers reported that they felt that they were teaching in ways which were uncomfortable to them.

In summary, this research found that the participants' beliefs and practices did help to determine their perceptions of effective teaching but that mathematical proficiencies were less important in that perception. Participants were not consistent in their understanding and interpretation of mathematical proficiencies, as described in the curriculum rationale Proficiency Strands. There is no evidence that teaching experience affected participants' understanding of the Proficiency Strands. Participants' lesson planning focussed on classroom management, pedagogical and lesson content, and less on the mathematical goals of the lesson. There was also evidence that mathematical proficiencies were regarded in an hierarchical sense of importance, from Fluency and Understanding through to Reasoning and Problem Solving. Such an interpretation would seem to be contrary to the curriculum rationale. The findings of the current research support the findings of studies by Swan (2006) and Stols et al. (2015). Teachers in this study mirror a range of beliefs, employ a range of effective practices and consider basic transmission lessons as the least effective lesson strategy employed in classrooms. However, they nominated that same transmission strategy as the most common lesson strategy used in WA. The reasons attributed to a transmission lesson as a common lesson included pressures placed on teachers by a busy curriculum, time difficulties in lesson planning and it being a useful strategy in behaviour management.

Chapter 6 Conclusion

The research question was: What are Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the Proficiency Strands? This was a broad but topical question relating to how well embedded the actions of the mathematical Proficiency Strands are in teaching, and whether those processes have an influence on regular practice. It was known from research that factors influencing teachers' regular practices include their personal beliefs (Swan, 2006), their own teaching experience and mathematical background (Ball et al., 2008) as well as the curriculum focus for them as individuals and the school as an organisation (Sullivan, 2011). It was prudent, therefore, to gather a range of information to better understand the regular practices teachers use on a consistent basis.

The study was conducted using quantitative and qualitative data gained from questionnaire and interview. It used a pragmatist epistemology using an interpretive case study theoretical perspective (Crotty, 1998) as well as a positivist approach to quantitative data, resulting in a mixed methods study. This was an appropriate epistemology as it was thought that the use of instrumental case study as its theoretical perspective for qualitative data would see participants match their expectations and actions (Evans, 2007), considered as a pertinent aspect of teacher practices within the model of schools and school organisations. The non-experimental surveys used to gather participants' background information, as well as to gather evidence of their beliefs and classroom practices, were used to triangulate data between theoretical perspectives whilst holding the positivist orientation that the actual response was as important as the number of responses.

The use of thematic analysis of semi-structured interviews allowed participants' opinions to be related to beliefs and practices gathered by questionnaire. Gathering background information allowed triangulation of teaching experience and mathematical background linked against espoused beliefs and declared practices. This information was housed under the umbrella of mathematical proficiency, as suggested by Sullivan (2011), considered by the researcher to be a core paradigm of the Western Australian mathematics curriculum (School Curriculum and Standards Authority, 2018b).

Relevant literature showed that the curriculum had been developed from the work of Kilpatrick et al. (2001) in the early 2000s. Kilpatrick et al.'s work had been subject to reform and development before Sullivan (2011) and his committee were tasked to complete the writing of a new curriculum for Australian mathematics appropriate for schools in 2011 (ACARA, 2012). The philosophy of the curriculum in mathematics centred around the use of mathematical proficiencies, a term developed by Kilpatrick et al. (2001). Five proficiencies were defined by Kilpatrick et al. as describing the actions of doing mathematics to allow students and teachers to embrace research-based teaching strategies suited to a 21st Century classroom. Sullivan (2011) adapted that work to use four Proficiency Strands, Understanding, Fluency, Reasoning and Problem Solving, to sit at the core of effective teaching of the content strands (School Curriculum and Standards Authority, 2018b).

At the same time research on learning showed the effects of brain plasticity on developing knowledge and understanding (Dweck, 2010). This led to developments in classroom practices which reinforced Kilpatrick et al. (2001) and Sullivan's (2011) earlier work (Boaler, 2016; Boaler et al., 2000). At the heart of those developments lay the concept of teachers of mathematics using challenging tasks as the basis for developing reasoning and conceptual development in students (Clarke et al., 2012b; Gerrard et al., 2013). The introduction of research-based classroom teaching strategies was known to require training and professional development in teachers (Day &

Hurrell, 2013) and Sullivan (2011) acknowledged teachers would require support in this development. As a teaching strategy, the use of tasks raised other issues for teachers. Much had already been written of the mathematical knowledge required by teachers to be effective educators (Ball et al., 2008; Shulman, 1986). Other researchers had embarked on programs of producing appropriate tasks (Back et al., 2012; Liljedahl et al., 2007; Sullivan et al., 2014) or developing skills to embed such tasks into practice (Boaler, 2016; Smith & Stein, 2011; Sullivan et al., 2014). This left questions about the effectiveness of the implementation of the new curriculum. Australian education had not fared well in comparative international studies and questions remained as to the best route towards improvement of standards and the engagement of students (Thomson et al., 2017).

Effective teaching was known to be subject to other influences on classroom teachers. The teachers' own beliefs of the purpose of mathematics and how to teach mathematics would influence each teacher's classroom practices. Research on those beliefs and influences (Askew et al., 1997b; Swan, 2006) and what teachers considered to be effective teaching (Perry et al., 2012; Stols et al., 2015; Zhang & Stephens, 2013) had already been conducted. However, little research with that emphasis had been completed in Western Australia and so it was thought appropriate that this research would use previously developed methods and scales by Swan (2006) to determine how local teachers perceived effective mathematics teaching. This was done by attempting to establish links between teachers' espoused beliefs and classroom practices relating to their teaching background and experience within the umbrella of the use of the Proficiency Strands as a core model of teaching practice. It was then triangulated against perceptions of effective classroom practices using material inspired by Stols et al. (2015).

In general, this group of Western Australian teachers have common views about what mathematics is and, in many cases, how to best teach it. They share common beliefs about what is not effective when teaching mathematics, which researchers would characterise as a transmission, teacher-centred lesson (Askew et al., 1997a; Swan, 2007). It is of concern that the teachers share a view that such teaching is the common methodology employed in schools today when it falls far from the principles set out by Sullivan et al. (2014) when supporting curriculum implementation. The actions of the Proficiency Strands of the curriculum are not well understood and the Proficiency Strands themselves may indeed be misunderstood as implying a hierarchical system of content knowledge acquisition. Teachers, when commenting upon effective teaching, often commented on using Proficiency Strands as a method of differentiation in teaching and student goal setting as well as using textbooks where Proficiency Strand names are used as a delineation of grading question difficulty. A programme of professional learning linked to clear goal setting of mathematical conceptual development using the actions of the Proficiency Strands would be particularly useful in developing mathematical proficiencies as designed and advocated by Kilpatrick (2001a).

This study has reinforced the work of Swan (2006) and Stols et al. (2015) in agreeing on what teachers accept to be effective teaching when based on their personal beliefs and classroom practices. This is interesting and cannot be undervalued in terms of professional learning. According to this sample at least, it would seem that length of teaching experience has a minor impact on the perceptions of effective teaching compared to personal beliefs and declared practices. It also appears that professional learning opportunities are not effective in developing a better understanding of curriculum philosophy about what effective mathematics teaching and learning looks like. There is no agreement in this group of teachers about what effective learning of mathematics is. This is in keeping with the Swan (2006) study. Developing a better scale or better refinement of the existing Swan (2006) scale may yield clearer results. This would benefit the mathematics teaching community and would be useful further study.

Similarly, there was no agreement over what constituted an effective lesson. There were common aspects of effectiveness characterised by student collaboration, meeting student needs, teacher modelling and explanation, as well as classroom control and management. Ineffective lessons included highly teacher-centred modes of delivery, employed what was described by many as direct instruction, had a lack of student engagement and required little reasoning, collaboration or activity by students. Many participants put this delivery method down to pressures of time, teacher knowledge and lesson content. More than one research participant commented "that such transmission lessons are simple to construct and when teaching a five-period day that is a critical consideration". If transmission lessons are indeed the norm in Western Australian government schools, then it would be beneficial if further research were undertaken to address concerns over lesson planning time and content overload which teachers feel are real and need to be addressed.

There is a need to review the quality and opportunity of professional learning sessions in formal mathematics development. Curriculum designers or professional educators would do well to create some opportunities to clearly delineate their own understanding of what content descriptors mean for teachers in both content and the actions of effective teaching and provide evidence of successful learning. It will then mean that mathematical proficiencies can become an important factor in the day-to-day teaching of students. As Stols et al. (2015) pointed out in their study, Japanese teachers spend preparation time discussing effective teaching of mathematics content towards clear learning goals whereas teachers in Stols et al.'s study group spent time planning effective classroom strategies linked to a new curriculum. That scenario is supported in other studies by higher performing nations than Australia in comparative international statistical measures (De Bortoli & Thomson, 2010; Gerrard et al., 2013; Mikk et al., 2016; Thomson et al., 2017). There is evidence in this research that this group of Western Australian teachers also regarded effective teaching in terms of classroom management and engagement of students. The participants often commented about embracing challenging tasks and reasoning activities, but did so without employing a clear focus on the actual learning goal of the lesson, or how those activities help lead towards a clearer understanding of the desired goal.

The research, therefore, found that participants' beliefs and practices did help determine their perceptions of effective teaching but that mathematical proficiencies were less important in that perception. Participants were not consistent in their understanding and interpretation of the Proficiency Strands, as described in the curriculum rationale. There was no evidence that teaching experience or mathematical background affected participants' understanding of mathematical proficiencies. Participants' lesson planning centred on classroom management and lesson content and less on the mathematical proficiencies a lesson may possibly extend. There was also evidence that the Proficiency Strands are regarded in a hierarchical sense from Fluency and Understanding through to Reasoning and Problem Solving, an interpretation which seems contrary to the curriculum rationale.

Five years after the adoption of the Australian Curriculum: Mathematics, this group of Western Australian teachers have, for the most part, not yet fully adopted the ethos laid out by Sullivan (2011) when suggesting how the curriculum could be effectively delivered. More work is required by stakeholders in mathematics education to better educate teachers as to the benefits and gains for students when teachers embrace the complete rationale behind the curriculum. Teachers have legitimate concerns over the volume of content prescribed in curriculum documentation. A review of that content, tasked to streamline the 'key ideas', as described by Sullivan (2011) would allow for a robust discussion as to how such ideas can be effectively taught and understood by learners. Other nations with higher comparative performance than Australia would appear to employ a stronger focus on the mathematical content for a lesson, in contrast to this group of Western Australian teachers who emphasised a greater behavioural focus in lesson planning. Professional learning could provide a suitable platform for this change but needs to be structured to keep a focus on Sullivan et al.'s (2013) six key skills. Researchers could support that development by focussing on improving and supporting teachers of mathematics. They could do this through encouraging the development of better material resources which are directed at the actions described by the Proficiency Strands and focussed on the conceptual learning goals of the curriculum, rather than only deeming to be a rich or challenging tasks. Education authorities might consider establishing key learning content for the curriculum to direct teachers who are struggling with perceptions of curriculum overload. Teachers would do well to better develop their understanding of the rationale of the Proficiency Strands in future professional development opportunities.

Recommendations

- A prominent feature in this research is what constitutes effective teaching and learning. Findings indicate that participants expressed beliefs regarding effective teaching and learning are not reflected in their ascribed classroom practices. Further research into what constitutes effective learning linked to teacher beliefs would improve understandings of effective teaching and learning in mathematics.
- Literature points to an uneven uptake by teachers in adherence to curriculum rationale without a planned program of teacher professional learning being undertaken. For this group of teachers there is uncertainty in their understanding of Proficiency Strands. More work is required by stakeholders in mathematics education to better involve teachers in conversations as to the benefits and gains for students when teachers embrace the complete curriculum rationale, incorporating the Proficiency Strands.
- Emphasis on professional learning for teachers of mathematics would include a focus on the range of teacher understanding of the 'fluency' and 'understanding' strands. There is evidence in this study that the 'fluency' strand is subject to misunderstanding by participants.
- The WAC:M (School Curriculum and Standards Authority, 2018b) is very closely related to the ACARA (2012) curriculum. WA teachers, in this study, have legitimate concerns over the volume of content prescribed in curriculum documentation. A review of that content, tasked to streamline the 'key ideas' would be welcome.
- Existing teachers of mathematics in WA would benefit by undertaking targeted and detailed professional learning centred around the Proficiency Strands, with

an emphasis on how those proficiencies underpin the curriculum. The teaching and learning goals such an emphasis might inspire could allow growth of the ideas underpinning mathematical development, or as Sullivan et al. (2013) describes "the important ideas" (p. 464). Designing professional learning to support such a structure will be a challenge for curriculum designers and others delivering professional learning opportunities, whilst meeting Australian Institute for Teaching and School Leadership Standards (Australian Institute for Teaching and School Leadership, 2018), especially standards 1,2,3 and 5. The work of Day and Hurrell (2013) could offer support in the planning of such events.

- Professional learning needs to be structured to make a stronger focus on conceptual understanding and incorporate Sullivan et al.'s (2013) six key skills.
- Many participants in this research incorrectly positioned the Proficiency Strands into a hierarchical structure of difficulty, similar to Bloom's taxonomy (Krathwohl, 2002). It appears they expect students to progress through the proficiencies in some order towards completion, in a hierarchical model. More work needs to be done to offer exemplification of the use and scope of Proficiency Strands in daily lesson planning.
- In general, this group of Western Australian teachers have common views about what mathematics is and, in many cases, how to best teach it. They share common beliefs about what is not effective when teaching mathematics, which researchers would characterise as a transmission, teacher-centred lesson (Askew et al., 1997a; Swan, 2007). It is of concern that the teachers share a view that such teaching is the common methodology employed in schools today when it falls far from the principles set out by Sullivan et al. (2014) when supporting

curriculum implementation. Further detailed studies would be welcome to elaborate this finding.

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Appendix A

Participant background information questionnaire

Participant Number

Research Project Title

Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands.

Round One Data - Questionnaire

A research study by James O'Neill

Master of Philosophy student

School of Education

University of Notre Dame Australia

Fremantle

Section A Information about your educational background and teaching experience.

This survey will help to better understand your own educational background and your teaching experience so far. This information is completely confidential and will remain anonymous.

Please indicate by placing a tick (✓) in the relevant box for each answer.

1. How long have you been teaching mathematics in secondary schools?

Up to 2 years	
Between 2 and 5 years	
Between 5 and 10 years	
More than 10 years	

2. Please indicate other education levels where you have taught mathematics. If appropriate, please note the time (in years) spent in each school level.

School Level	Yes/No	Length of service in years
Primary school		
District high school		
Secondary/ Senior high school		
Tertiary level (Please specify)		

3. How would you describe your formal mathematics qualifications?

Pure Mathematics degree	
Other degree with high mathematics content	
Other degree with some mathematics content	
Diploma in education/teaching	
No formal mathematics education beyond high school	

4. Please indicate how confident you feel in teaching mathematics at different stages?

	Highly confident	**Very confident**	**Mostly confident**	**Some confidence**	**Not confident**
Y11/12 Specialist					
Y11/12 Methods					
Y11/12 Applications					
Y11/12 General courses (Essential)					
Year 10A					
Years 7 – 10					
Years 1 – 8					

5. Please indicate your current school role:

Role (✓)		**Length of time in that role in years**
Head of Learning area or equivalent		
Level 3 Classroom teacher		
Senior teacher		
Classroom teacher		
Graduate teacher		

6. Please indicate the Professional Learning you have undertaken in the last 5 years by giving each of the statements a percentage so that *the sum* of the three percentages is 100%.

Activity	**% of total Professional Learning**	**Please rate each category out of 100 as to its effectiveness in improving your classroom teaching**
Formal mathematical training		
Classroom practices		
Other school development		
Total	100%	

Appendix B

Participant beliefs questionnaire

Section B Information about your beliefs about what mathematics is, how to learn it and how to teach it.

This information will help us understand what you feel is important about mathematics and how to learn mathematics.

Give each of the three statements a percentage so that ***<u>the sum of</u>*** the three percentages in each section is 100%.

Statement 1

Mathematics is:

A given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to students.	
A creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	
An interconnected body of ideas which the teacher and the student create together through discussion	
Total	100%

Statement 2

Learning is:

An individual activity based on watching, listening and imitating until fluency is attained.	
An individual activity based on practical exploration and reflection.	
An interpersonal activity in which the students are challenged and arrive at an understanding through discussion.	
Total	100%

Statement 3

Teaching is:

Structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.	
Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	
A non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	
Total	100%

Source: Swan (2006)

Appendix C

Participant classroom practices questionnaire

Section C Information about how you like your classroom to function on a regular basis.

This information will help us understand how you like your classroom to function when teaching mathematics.

Please 'describe the frequency' of the following twenty-five classroom behaviours on a five-point scale.

Please indicate by placing a tick (✓) in the relevant box for each answer.

5 – Almost always, 4 – Most of the time, 3 – Half the time, 2 – Occasionally, 1 – Almost never.

Number	Question	Almost always	Most of the time	Half of the time	Occasionally	Almost never
1	Students learn through doing exercises.					
2	Students work on their own, consulting a neighbour from time to time.					
3	Students use only the methods I teach them.					
4	Students start with easy questions and work up to harder questions.					
5	Students choose which questions they tackle.					
6	I encourage students to work more slowly.					
7	Students compare different methods for doing questions.					
8	I teach each topic from the beginning, assuming they know nothing.					
9	I teach the whole class at once.					

Number	Question	Almost always	Most of the time	Half of the time	Occasionally	Almost never
10	I try to cover everything in a topic.					
11	I draw links between topics and move back and forth between topics.					
12	I am surprised by the ideas that come up in a lesson.					
13	I avoid students making mistakes by explaining things carefully first.					
14	I tend to follow the textbook or worksheets closely.					
15	Students learn through discussing their ideas.					
16	Students work collaboratively in pairs or small groups.					
17	Students invent their own methods.					
18	I jump between topics as the need arises.					
19	I tell students which questions to tackle.					
20	I encourage students to make and discuss mistakes.					
21	I only go through one method for doing each question.					
22	I find out which parts students already understand and don't teach those parts.					

Number	**Question**	**Almost always**	**Most of the time**	**Half of the time**	**Occasionally**	**Almost never**
23	I teach each student differently according to individual needs.					
24	I know exactly what mathematics the lesson will contain.					
25	I tend to teach each topic separately.					

Source: Swan (2006)

Thank you.

This is the end of the data collection for Round One.

Thank you for your participation. As there is no method to link your name to this data please ensure that you have the same participant number for the information we collect in Round Two.

Appendix D

Proficiency Strands comprehension activity

THE UNIVERSITY OF NOTRE DAME AUSTRALIA

Participant Number

Research Project Title

Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands.

Round Two Data – Teaching Scenarios & Comprehension Activity

A research study by James O'Neill

Master of Philosophy student

School of Education

University of Notre Dame Australia

Fremantle

Teaching Scenarios

You will be given four different classroom teaching scenarios for you to consider. The chosen topic is from the Australian Curriculum: Mathematics content strand from Year 8 Patterns and Algebra – ACMNA194

"Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution."

You will be asked some questions about each scenario. Your responses will be recorded and transcribed later. This will allow you to freely express your opinions without the inevitable hesitation when writing or typing responses. Please feel free to offer your opinions or to seek clarification from the researcher.

You should assume that all students are well behaved and attentive and that no other factors affect the different teaching scenarios.

Comprehension Activity

This activity will help us to better understand your familiarity with the proficiency strands and how they apply to typical scenarios taken from the Australian Curriculum: Mathematics content strands.

Card Shuffle Instructions

You have been given a set of numbered phrases. Write the phrase number into the space you feel is the most appropriate proficiency strand. If you feel any phrase might be better placed into more than one strand please do so.

1. Understanding	2. Fluency
3. Problem Solving	4. Reasoning

Thank you.

This is the end of the data collection for Round Two.

Thank you for your participation. You have now completed the research study and your information will be collected and analysed along with other research participants.

This analysis is expected to be completed and available in the period October – December 2018.

You will be contacted by email when that process is complete allowing you to request a copy of the research findings.

James O'Neill

Researcher

Card Shuffle

1. Can you represent or calculate in different ways?	2. In what ways can you prove …?	3. Can you work flexibly with a concept?
4. How can you test your idea?	5. Can you choose a suitable algorithm?	6. How reasonable is your answer?
7. Can you rearrange this formula?	8. Use mathematical language to describe …	9. What is the same about …?
10. Is there a rule we can use to describe …?	11. In what ways can you model and plan …?	12. What patterns, connections, relationships can you see?
13. What can you recall?	14. What can you infer?	

Appendix E

Interview guide for questionnaires and semi-structured interview

Researcher Sheets Only – Not to be given to participants

Offer consent form and collect from participant.

Offer random number sheet and allow participant to select a number (they score it off the list)

Round 1

Issue Round 1 Questionnaire – Describe the activity as gathering background information to help filter information gained over the course of the survey. Ask for clarification and assist as required.

Repeat for each activity.

Collect responses and ensure participant number is written on cover.

Round 2

All interviews must be recorded with a preamble …

"This is a recording of Round 2 survey data. The interview is conducted with participant number XX. In this interview you will be given four different teaching scenarios depicting the teaching of a year 8 topic from pattern & algebra. After you have had time to read each scenario you will be asked some questions. Your answers will be recorded and transcribed at a later date. Are you willing to continue?"

Topic

Year 8 Patterns and Algebra – ACMNA194 "Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution."

Background to Scenarios

Scenario A – use of manipulatives, student activity, student interaction, students in groups of 4, discourse involving different pattern leading to different equations, discussion of each pattern and features of pattern, use of solve and substitute within the lesson to find further pattern values.

Scenario B – Whiteboard lesson using worked examples with strict 'rules' to be followed. Starts by reviewing 'collecting like terms' and expands into rearranging formula using change of side and sign. Uses a textbook for working where examples and exercise are structured in difficulty easier to harder. No mention of practical use or relevance to life. Students sit in single rows.

Scenario C – Students work in pairs. Teacher uses recall questioning of previous content. Teacher writes structured examples following the 'I do – We do – You do' philosophy. Students use 'show-me' boards to answer quick fire question/answer session before being set a book task. The book exercise has sections covering U/F/R and students are directed to start at particular questions based on prior knowledge and feedback from the show-me activity.

Scenario D – AV intro sets up real life context, use of manipulatives through a worksheet, establishes inverse operations and thinking to rearrange equations, simple discussion of correct method and answers, final example of 'correct method and layout', book exercise finished for h/work. Students sit in pairs.

Scenario A, B, C & D Feedback

- *What were the most effective aspects of this lesson? Explain why you chose those aspects?*
- *What were the least effective aspects of this lesson? Explain why you chose those aspects?*

Concluding Questions

After all scenarios have been read, pose three further questions:

Lay all scenarios out for the participant to review:

- *Pick one scenario which you regard to be the most effective in terms of pupils learning about algebra and explain your choice.*
- *Pick one scenario that you regard as the least effective in terms of pupils learning about algebra and explain your choice.*
- *Pick one scenario that you think most resembles the teaching of mathematics in your school and explain what that lesson has in common with school practice*.

Comprehension activity

Explain comprehension activity which is to match each of 14 statements to the most relevant Proficiency Strand. Make sure participant is aware they can attach a statement to more than one strand if necessary.

Ensure participant number is written onto the collection page.

Appendix F

Teaching scenarios rationale

Rationale used in developing the Teaching Scenario activity

Each of the four scenarios was used to illustrate a belief orientation as described by Askew et al. (1997b) and inspired by the vignettes used by Stols et al. (2015). The responses were triangulated against participant beliefs and practices as described in other instruments (Appendices A, B, C & D).

Scenario A – was deemed to be a discovery lesson involving the use of manipulative materials with clear evidence of student activity and directed student interaction within groups of four. This 'student-centred' lesson depicted discourse involving patterns leading to equations, with discussion of the patterns culminating in the use of a 'solve and substitute' strategy to find further pattern values.

Scenario B – was used to depict a transmission lesson. This 'teacher-centred' whiteboard lesson used worked examples with strict 'rules' to be followed. The lesson started by reviewing 'collecting like terms' and expanded into rearranging formulae using change of side and sign. This technique was chosen as it was expected to polarise participants in this study, in its depiction of both pedagogy and practice. The lesson also used a textbook for student practise where examples and exercise were structured in difficulty, ostensibly moving from easier to harder examples. It did not refer to practical use of the topic or relate the topic to real-life situations. Students sat in single rows, which was expected to prompt participants in this study to discuss collaboration and peer involvement as elements of the teaching and learning environment.

Scenario C – was essentially a second transmission lesson using commercial small whiteboards for questioning. This was a teacher-centred lesson, albeit with strong student involvement, and was expected to attract the attention of the research participants who employ both student and teacher-centred practices. Students worked in pairs and the teacher used recall questioning of previous content to establish prior understanding. The teacher then wrote structured examples following the 'I do – We do – You do' explicit teaching philosophy of Fisher and Frey (2008). Students used individual whiteboards to engage in a quick-fire question/answer session, before being set a book task. The book exercise had sections covering the Proficiency Strands of Understanding, Fluency and Reasoning and students were directed to start at specific questions, particular to them, which were based on feedback from the interactive activity.

Scenario D – was a connectionist lesson which aimed to link different concepts in the development of a patterns and equations lesson. It used a video introduction to introduce a real-life context, then used manipulative materials through a teacher directed interaction which aimed to establish inverse operations and reasoning to rearrange equations. It highlighted discussion of correct methods and answers leading to a final example of 'correct method and layout' from the teacher. This was reinforced with a textbook exercise finished for homework. This lesson had a strong student-centred learning style with the teacher challenging and directing student understanding and development and was designed to attract teachers with a connectionist orientation.

Appendix G

Teaching scenarios

Scenario A

Students are seated in groups of 3 or 4. The teacher uses a lesson scenario describing a central plant bed surrounded by tiles. The teacher demonstrates the layout using students as plants and surrounds them with A4 paper 'tiles'. Students are encouraged to ask clarifying questions.

The teacher then offers each group a set of coloured tiles and asks them to investigate the situation further, suggesting there are patterns in the number of tiles used and that the students will be asked questions after completing the activity. Through group discussion students advance word sentences and are then encouraged to develop algebraic representation of their patterns.

The teacher then takes feedback from a number of groups which result in different patterns and representative equations. Groups with incorrect patterns are encouraged to demonstrate their thinking with other students suggesting suitable corrections to their pattern. The teacher then invites each group to consider how many tiles would be needed for 100 plants and how many plants would be needed if you had 1000 tiles. Students are challenged to find answers in a different way.

The teacher then invites students to present their findings visually using any suitable type of graph they think appropriate.

Students end the lesson preparing a report of findings so far. H/work is to find at least one different pattern and show that it is the same as their own pattern. They are then to give a visual display of their findings so far.

Scenario B

Students are organised in single rows. The teacher reviews a previous lesson on 'collecting like terms' to check for understanding.

The teacher then uses the whiteboard to work through an example of solving a simple linear equation. The teacher takes time and care to detail each step in the process. The students make detailed notes from the board. The teacher clearly models the idea of changing the sign of the variable or number when it changes side of the equation. Students are questioned regularly for understanding of the process. The teacher asks them to check their answer by substitution, although this is not modelled.

The teacher then sets an exercise from the course text which has graded questions. All students start at question 1. The teacher circulates the room and helps those students who require assistance. The homework set is a sheet of further graded questions.

Scenario C

Students are organised in pairs. The teacher uses recall questioning to review understanding of variables and collecting terms in algebra.

The teacher writes a few structured examples on the board for the students to copy. Students then use small personal 'show-me' whiteboards to answer a series of quick fire questions on the examples used in the lesson. The teacher invites a few students to write answers to some further examples on the main whiteboard. Each selected student answers the question correctly, some with prompting from the teacher.

The teacher then models checking of correct answers by substituting the answer back into the given equation. This is again modelled on the board and students use show me boards to check understanding.

The class are then set an exercise from the textbook. The selected textbook has questions grouped into headings of Understanding / Fluency / Reasoning. The teacher directs certain students to start at a particular question based on prior student knowledge and feedback from the 'show-me' activity earlier.

Scenario D

The students are organised in pairs. The teacher uses a selected video clip to set the context of solving equations in real life situations. Students are then given a set of a commercial foam tiles to be used as a concrete representation of numbers and variables. They have used this material before when gathering terms and when working on integers.

The teacher gives each pair a structured worksheet of problems to be solved using the tiles. The students are encouraged to use inverse operations as part of their thinking. The teacher takes volunteers to describe their solutions to particular questions. Each student demonstrates a correct solution, with prompting from partners or other students.

The teacher then invites students to justify their answer as being correct. They quickly gain an understanding of using the answer to substitute back into the question to justify a correct response.

The teacher then displays the 'expected' layout for working and answers, as well as checking of the answer by substitution, before setting a textbook exercise for students to complete. Students are expected to complete the exercise for homework.

Appendix H

Ethical information

Government of **Western Australia**
Department of **Education**

Your ref :
Our ref : D17/0306072
Enquiries :

Mr James O'Neill
14 Matuka Mews
MAIDA VALE WA 6057

Dear Mr O'Neill

Thank you for your application received 14 December 2016 to conduct research on Department of Education sites.

The focus and outcomes of your research project, *Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands,* are of interest to the Department. I give permission for you to approach principals to invite their participation in the project as outlined in your application. It is a condition of approval that upon conclusion the results of this study are forwarded to the Department at the email address below.

Consistent with Department policy, participation in your research project will be the decision of the schools invited to participate and individual staff members in those schools. A copy of this letter must be provided to principals when requesting their participation in the research. Researchers are required to sign a confidential declaration upon arrival at Department of Education schools.

Responsibility for quality control of ethics and methodology of the proposed research resides with the institution supervising the research. The Department notes a copy of a letter confirming that you have received ethical approval of your research protocol from the University of Notre Dame Human Research Ethics Committee.

Any proposed changes to the research project will need to be submitted for Department approval prior to implementation.

Please contact Dr Alesya Drozdova, A/Coordinator Research Applications, on (08)9264 5512 or researchandpolicy@education.wa.edu.au if you have further enquiries.

Very best wishes for the successful completion of your project.

Yours sincerely

ALAN DODSON
DIRECTOR
EVALUATION AND ACCOUNTABILITY

21 July 2017

151 Royal Street, East Perth Western Australia 6004

Information to Principals

James O'Neill
Research Student
School of Education, Mouat Street, Fremantle
University of Notre Dame Australia, Fremantle
James.oneill2@my.nd.edu.au

Dear *Principal Name,*

Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands.

My name is James O'Neill and I am writing to you on behalf of the School of Education, University of Notre Dame Australia, Fremantle. I am conducting a research project that aims to examine how secondary mathematics teachers perceive effective mathematics teaching. This will be investigated through the actions of teaching mathematics which the Australian Curriculum: Mathematics (AC:M) describes as the proficiency strands. This project is being conducted as part of a Master of Philosophy Degree by research at University of Notre Dame Australia, Fremantle.

I would like to invite members of the mathematics department in your school to take part in this project. I have gained in principle agreement from your Head of Department to conduct the research with members of staff willing to volunteer. Your school is one of a number of schools approached to allow the participant population of approximately 20 Western Australian teachers to be reached.

As a practising Head of Department in a large secondary school I am well aware of the variety of approaches taken by teaching staff in efforts to engage and improve mathematics education for our students. This project will hope to offer some evidence-based information which may help engage better informed discussion about the issues we face in improving mathematical education in schools.

What are the benefits of the research project?

This research is significant as there is little existing research evidence in Western Australia to compare the perceptions of effective teaching practices of mathematics teachers to the AC:M proficiency strands. The decline in student uptake in senior mathematics courses has been highlighted as a concern (Holton et al., 2009) and more recently in an opinion piece written by Peter Klinken, Chief Scientist WA (Office of the Chief Scientist WA, 2016). Adding to the understanding of effective teaching practices may offer further insights into the factors influencing student uptake in relation to the quality of mathematical instruction. Also, it is important to better understand the professional development needs of new and practicing mathematics teachers. This research may offer information relevant to those training teachers, graduate teachers, practicing teachers and providers of professional development.

What does participation in the research project involve?

I seek access to teachers of mathematics in your school. Teachers will be invited to participate in the research which gathers information in two rounds, each of around 30 minutes.

- It involves - questionnaire, semi-structured audio taped interview, and a comprehension activity.
- In round one a questionnaire with three (3) sections will ask about participants teaching experience and their beliefs and perceptions of mathematics teaching. All items will be de-identified for anonymity.
- In round two participants will be asked to comment on four (4) textually depicted teaching scenarios. Their responses will be audio taped for transcription thereafter. Recordings will then be destroyed for anonymity with the transcriptions stored for analysis. The final activity is a card shuffle comprehension activity on the AC:M proficiency strands.
- Each round of activities is estimated to take 30 minutes to complete. Participants will have the option of conducting both rounds in one sitting or over two sittings.
- The study will take place at your school, with permission from you, or at a mutually convenient library venue at a time of the participants choosing.
- Should the participant encounter any out of pocket expenses these will be reimbursed in full through bank-transfer.

I will keep the school's involvement in the administration of the research procedures to a minimum. However, it will be necessary for the school to allow access to a small room allowing surveys and questionnaires to be conducted confidentially. If a number of staff agree to participate then each participant will be interviewed separately. This will mean multiple visits to school. Interviews are expected to be conducted at the end of the school day and should not impact on teaching and learning of students.

To what extent is participation voluntary, and what are the implications of withdrawing that participation?

Participation in the research project is entirely voluntary.

If any mathematics teacher decides to participate and then later changes their mind, they are able to withdraw their participation at any time. If a participant withdraws, all information they have provided will be destroyed. Participants cannot withdraw after they submit survey/questionnaire responses, as surveys are non-identifiable.

There will be no consequences relating to any decision by an individual or the school regarding participation, other than those already described in this letter. Decisions made will not affect the relationship with the researcher, research supervisors or University of Notre Dame, Australia.

What will happen to the information collected, and is privacy and confidentiality assured?

Information that identifies any participant will be removed from the data collected at the earliest opportunity. Data will be stored on a password protected computer during the analysis phase. Once complete data is then stored securely at the School of Education, University of Notre Dame Australia, Fremantle and can only be accessed by the researcher, supervisors and thesis examiners. The data will be stored for a period of 5 years, after which it will be destroyed. This will be done in accordance with the procedures of the University of Notre Dame Australia, Fremantle.

The identity of participants and the school will not be disclosed at any time, except in circumstances that require reporting under the Department of Education *Child Protection* policy, or where the research team is legally required to disclose that information.

Participant privacy, and the confidentiality of information disclosed by participants, is assured at all other times. The data will only be used for this project and will not be used in any extended or future research without first obtaining explicit written consent from participants. Consistent with Department of Education policy, a summary of the research findings will be made available to the participants, the participating school and the Department. You can expect this to be available in October – December 2018.

Is this research approved?

The study has been approved by the Human Research Ethics Committee at The University of Notre Dame Australia (*approval number 017019F*). The research has met the policy requirements of the Department of Education as indicated in the attached letter.

Working with Children clearance is not appropriate for this research as there is to be no contact with students at your school. The researcher does have current Working with Children clearance which is attached to this letter for information purposes.

Who do I contact if I wish to discuss the project further?

If you have any questions about this project please feel free to contact either myself at the number below (james.oneill2@my.nd.edu.au) or my supervisors, Dr. Derek Hurrell or Lorraine Day at (+61 8) 9433 0555 (derek.hurrell@nd.edu.au / lorraine.day@nd.edu.au). My supervisors and I are happy to discuss with you any concerns you may have about this study.

If you have a concern or complaint regarding the ethical conduct of this research project and would like to speak to an independent person, please contact Notre Dame's Ethics Officer at (+61 8) 9433 0943 or research@nd.edu.au. Any complaint or concern will be treated in confidence and fully investigated. You will be informed of the outcome.

How do I indicate my willingness for the school to be involved?

If you have had all questions about the project answered to your satisfaction, and are willing for you school to participate, please complete the **Consent Form** on the following page.

This information letter is for you to keep.

James O'Neill
Research Student
University of Notre Dame Australia, Fremantle
James.oneill2@my.nd.edu.au
T: (08) 9293 6400 M: 0458 157 219

References

Holton, D., Muller, E., Oikkonen, J., Valenzuelad, O. A. S., & Zizhaoe, R., (2009) Some reasons for change in undergraduate mathematics enrolments. International Journal of Mathematical Education in Science and Technology, 40(1), 3–15

Office of the Chief Scientist of WA (2016) Science the key to prosperity. Opinion piece published on Tuesday 24 May 2016 in The West Australian. Retrieved from https://www.dpc.wa.gov.au/science/Documents/TheWestAust%20-Peter%20Klinken.pdf

CONSENT FORM for School Principals

THE UNIVERSITY OF NOTRE DAME AUSTRALIA

This project is titled:

Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands.

- I have read this document and understand the aims, procedures, and risks of this project as described within it.
- For any questions I may have had, I have taken up the invitation to ask those questions, and I am satisfied with the answers I received.
- I am willing for this school to become involved in the research project, as described.
- I understand that that participation in the project is entirely voluntary.
- I understand that the school is free to withdraw its participation at any time, without affecting the relationship with the research team or the University of Notre Dame Australia, Fremantle.
- I understand that once surveys have been de-identified then participants cannot withdraw from the study.
- I understand that this research may be reported as a thesis, as a journal article or form part of a conference paper.
- I understand that the school will be provided with a copy of the findings from this research upon its completion.

Name of Site Manager (printed):	________________________________
Signature of Site Manager:	________________________ Date ____________

PARTICIPANT INFORMATION SHEET

THE UNIVERSITY OF NOTRE DAME AUSTRALIA

Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands.

Dear Colleague,

You are invited to participate in the research project described below.

What is the project about?

This research aims to examine how secondary mathematics teachers perceive effective mathematics teaching through the actions of teaching mathematics which the Australian Curriculum: Mathematics (AC:M) describes as the proficiency strands. This will be done through an analysis of participant perceptions of effective teaching practice as noted through responses to questionnaire and textually depicted teaching scenarios. This research will aim to establish potential linkage between the actions of teaching mathematics in secondary school, the understanding of the rationale of the proficiency strands and participant beliefs, classroom practices and professional experience in an effort to establish Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands.

Who is undertaking the project?

This project is being conducted by James O'Neill and will form the basis for the degree by research of Master of Philosophy at The University of Notre Dame Australia, under the supervision of Dr. Derek Hurrell and Lorraine Day.

What will I be asked to do?

If you consent to take part in this research study, it is important that you understand the purpose of the study and the tasks you will be asked to complete. Please make sure that you ask any questions you may have, and that all your questions have been answered to your satisfaction before you agree to participate.

This project will gather data in two rounds, each lasting around 30 minutes:

- It involves - questionnaire, semi-structured audio taped interview, and a comprehension activity.
- In round one a questionnaire with three (3) sections will ask about your teaching experience and your beliefs and perceptions of mathematics teaching. All items will be de-identified for anonymity.
- In round two you will be asked to comment on four (4) textually depicted teaching scenarios. Your responses will be audio taped for transcription thereafter. Recordings will then be destroyed for anonymity with the transcriptions stored for analysis. The final activity is a card shuffle comprehension activity on the AC:M proficiency strands.
- Each round of activities is estimated to take 30 minutes to complete. You will have the option of conducting both rounds in one sitting or over two sittings.

The study will take place at your school, with permission from the principal, or at a mutually convenient venue at a time of your choosing.

Should you encounter any out of pocket expenses these will be reimbursed in full through bank-transfer.

Are there any risks associated with participating in this project?

There is no foreseeable risk in you participating in this research project.

What are the benefits of the research project?

This research is significant as there is little existing research evidence in Western Australia to compare the perceptions of effective teaching practices of mathematics teachers to the AC:M proficiency strands. The decline in student uptake in senior mathematics courses has been highlighted as a concern (Holton et al., 2009) and more recently in an opinion piece written by Peter Klinken, Chief Scientist WA (Office of the Chief Scientist WA, 2016). Adding to the understanding of effective teaching practices may offer further insights into the factors influencing student uptake in relation to the quality of mathematical instruction. Also, it is important to better understand the professional development needs of new and practicing mathematics teachers. This research may offer information relevant to those training teachers, graduate teachers, practicing teachers and providers of professional development.

What if I change my mind?

Participation in this study is completely voluntary. Even if you agree to participate, you can withdraw from the study at any time without discrimination or prejudice. If you withdraw, all information you have provided will be destroyed. You cannot withdraw after you submit your survey/questionnaire, as surveys are non-identifiable. These decisions will not affect your relationship with your Principal or Head of Department.

Will anyone else know the results of the project?

Information gathered about you will be held in strict confidence. This confidence will only be broken if required by law.

Once collected from you the data will be de-identified for analysis. Once the study is completed, the data will be stored securely in the School of Education at The University of Notre Dame Australia for a period of at least five years. The results of the study will be published as a thesis and potentially as a journal article or conference paper. After this period the data will be destroyed in accordance with the procedures of the University of Notre Dame Australia, Fremantle.

Participant privacy, and the confidentiality of information disclosed by participants, is assured at all times, except in circumstances where the research team is legally required to disclose that information.

The data will be used only for this project and will not be used in any extended or future research without first obtaining explicit written consent from you.

Will I be able to find out the results of the project?

Once the information from this study has been analysed it will be reported as a thesis to satisfy the requirements of the examining body of the University of Notre Dame Australia. A summary of the research findings will be made available upon completion of the project. You will be contacted by email at the completion of the project offering access to the findings. You can expect to receive this feedback in October – December 2018.

Who do I contact if I have questions about the project?

If you have any questions about this project please feel free to contact either myself at james.oneill2@my.nd.edu.au or my supervisors, Dr. Derek Hurrell or Lorraine Day at (+618) 9433 0555 or derek.hurrell@nd.edu.au / lorraine.day@nd.edu.au. My supervisors and I are happy to discuss with you any concerns you may have about this study.

What if I have a concern or complaint?

The study has been approved by the Human Research Ethics Committee at The University of Notre Dame Australia (approval number 017019F). If you have a concern or complaint regarding the ethical conduct of this research project and would like to speak to an independent person, please contact Notre Dame's Ethics Officer at (+61 8) 9433 0943 or research@nd.edu.au. Any complaint or concern will be treated in confidence and fully investigated. You will be informed of the outcome. The research has met the policy requirements of the Department of Education.

How do I become involved?

If you have had all of your questions about the project answered to your satisfaction, and are willing to become involved, please complete the Consent Form on the next page.

This information letter is for you to keep.

James O'Neill

Research Student

University of Notre Dame Australia, Fremantle

James.oneill2@my.nd.edu.au

References

Holton, D., Muller, E., Oikkonen, J., Valenzuelad, O. A. S., & Zizhaoe, R., (2009) Some reasons for change in undergraduate mathematics enrolments. *International Journal of Mathematical Education in Science and Technology*, *40*(1), 3–15

Office of the Chief Scientist of WA (2016) *Science the key to prosperity.* Opinion piece published on Tuesday 24 May 2016 in The West Australian. Retrieved from https://www.dpc.wa.gov.au/science/Documents/TheWestAust%20-Peter%20Klinken.pdf

PARTICIPANT CONSENT FORM

This project is titled:

Western Australian teachers' perceptions of effective secondary mathematics teaching through the lens of the 'actions' of mathematics - the proficiency strands.

- I agree to take part in this research project.
- I have read the Information Sheet provided and been given a full explanation of the purpose of this study, the procedures involved and of what is expected of me explained in language I understand.
- I understand that I will be asked to complete a questionnaire, give verbal responses to written classroom teaching scenarios and undertake a comprehension activity related to the Australian Curriculum: Mathematics.
- The researcher has answered all my questions and has explained possible problems that may arise as a result of my participation in this study.
- I understand that that participation in the project is entirely voluntary and that I may withdraw from participating in the project at any time without prejudice.
- I understand that once surveys and data have been de-identified then I cannot withdraw from the study.
- I understand that all information provided by me is treated as confidential and will not be released by the researcher to a third party unless required to do so by law.
- I agree that any research data gathered for the study may be published provided my name or other identifying information is not disclosed.
- I understand that I can request a summary of findings once the research has been completed.

Name of participant	
Signature of participant	Date

- I confirm that I have provided the Information Sheet concerning this research project to the above participant, explained what participating involves and have answered all questions asked of me.

Signature of Researcher	Date