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Where we were…where we are heading: One multiplicative journey

C Hurst

D Hurrell
The University of Notre Dame Australia, derek.hurrell@nd.edu.au

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Where we were . . . where we are heading: One multiplicative journey


Chris Hurst
Curtin University, WA
<c.hurst@curtin.edu.au>

Derek Hurrell
University of Notre Dame, WA
<Derek.Hurrell@nd.edu.au>

A journey into multiplicative thinking by three teachers in a primary school is reported. A description of how the teachers learned to identify gaps in student knowledge is described along with how the teachers assisted students to connect multiplicative ideas in ways that make sense.

> "We had to focus on understanding it ourselves and our school focus was multiplication and division. It needed to be more than an approach based on filling the gaps . . . we had to focus on what we knew and start from there."

To introduce this 'journey', children in this Year Five class were working on a task that required them to match number sentences to word stories by cutting and pasting, and then to represent the story—this is an example of Bessie's work:

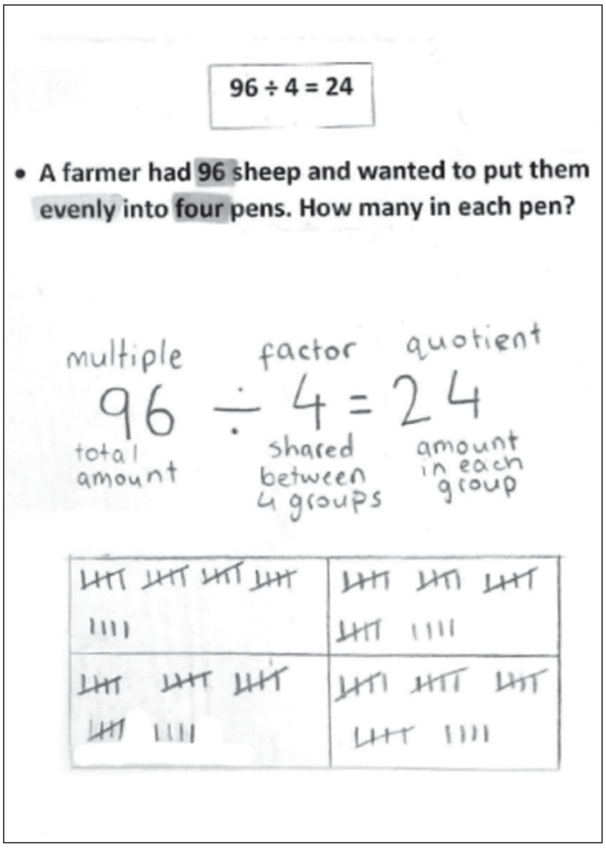


Figure 1. Bessie's story.

This may not necessarily be considered an amazing sample of a child's work but it represents some important development that took place in three classrooms in a primary school situated south of Perth.

This article documents part of the professional learning journey of three teachers at the school—Abbie, Carl, and Dan—and the extent to which their learning is reflected in the work of some of their students. It describes how these teachers have begun to develop into genuine 'connectionist teachers' who are well aware of how the 'big ideas' of mathematics are structured and related.

Introduction

In 1997 Askew, Brown, Rhodes, Johnson and Wiliam wrote what has become a seminal text with regards to what characterises effective teachers of numeracy. They came up with three categories of teachers, with the most effective category given the title of connectionist teachers. One of the criteria that distinguished connectionist teachers was that they made rich connections between mathematical ideas. Unfortunately, the classroom being the busy place it is, is not always an environment which permits the time and space to reflect on how the 'bits' of mathematics fit together, and thereby allow teachers to develop the capacity to become connectionist. In fact, with the partitioned curricula that operate in most parts of the world it is a task beyond the majority of us. One way of connecting the mathematics is not to consider the atomised curriculum as individual bits of content but rather to consider what underpins mathematics—the 'big ideas'. Some authors (Charles, 2005; Hurst, 2014; Siemon, Bleckley & Neal, 2012) have taken the opportunity to reflect on this and become part of a conversation about what constitutes the big ideas of mathematics. All consider multiplicative thinking to qualify as a big idea as it underpins important mathematical concepts such as place value, division, fractions, measurement, statistical sampling, proportional reasoning, rates and ratios, and algebraic reasoning (Siemon, Beswick, Brady, Clark, Faragher, & Warren, 2011). Amongst other things, multiplicative thinking (MT) is about having a flexible understanding of a range of numbers and relationships between them, recognising and working with a range of multiplication and division situations, and communicating and understanding of these ideas in a variety of ways (Siemon, Breed, Dole, Izard, & Virgona, 2006).

Background

In 2015, Abbie, Carl, and Dan's school undertook some professional learning (PL) regarding multiplicative thinking. This stimulated their interest to seek guidance as to how they could better develop MT with their students and how they could judge what elements of MT were already being well taught and learned in their school and which elements required further consolidation. A Multiplicative Thinking Quiz (MTQ) was administered with children in Years 4, 5, and 6 classes and results shared with the teachers. Teacher Abbie describes how this developed:

> This initial training and testing resulted in an increased awareness and interest by us into how these skills and understandings had an impact on the broader range of mathematical concepts taught through the primary school years. Our journey initially focused on the data produced from the MTQ and whether these skills were being effectively taught to students as they progressed through [the school]. Analysis of the data suggested that [the school] was typical of most primary schools tested, in that the multiplicative thinking skills required by students to effectively progress into harder mathematical concepts were not being consistently developed and practised by students, resulting in students not being able to explain their reasoning or knowledge when completing a variety of multiplication tasks.

Abbie, Dan, and Carl established their own Professional Learning Community (PLC) to enhance their own understanding of multiplicative thinking skills so as to develop a learning program focused on improving MT amongst their students. This common focus allowed the teachers to develop an action plan that began with developing their own professional knowledge and pedagogy with plans to later broaden the focus to include the remainder of the school staff. The action plan involved the trialing of resources developed through a research project *Children Thinking Multiplicatively* (Hurst & Hurrell, 2016) and these resources provided deeper insight into student understanding and reasoning. Specific focus areas were students' ability to reason and explain their knowledge and their understanding and use of arrays, the commutative property, the distributive property, and the inverse relationship between multiplication and division. Associated with these focus points was the development of flexible mental computation strategies with specific emphasis on students explain ing their thinking conceptually, as opposed to following procedures.

The journey at the school

This article was written twelve months after the initial assessment was completed using the Multiplicative Thinking Quiz. Given that this was a relatively short amount of time, there are clear indications of very strong growth in mathematical and pedagogical content knowledge of the three teachers, and that this journey has involved significant learning for not only their students but themselves. When interviewed after twelve months working on multiplicative thinking, Carl stated that "One thing was us actually learning what these words meant, defining progressions to see where it actually fitted into their learning" and Abbie supported that with her comment, "It's just as much a learning curve for us as it was for the kids . . . it was more about what are we doing as teachers and what can we do to improve". It was evident that the teachers had developed a clearer view of how and why multiplicative thinking was important. When asked about that, Dan and Abbie responded with the following comments respectively

Dan Well, it kind of underpins everything doesn't it . . . those higher concepts, ratios, fractions . . . if those concepts are not embedded at an early age, it's going to be difficult for them to understand them at a later age, especially when they get to high school.

Abbie So they can calculate efficiently . . . kids can calculate things without multiplicative thinking but they might not be able to work

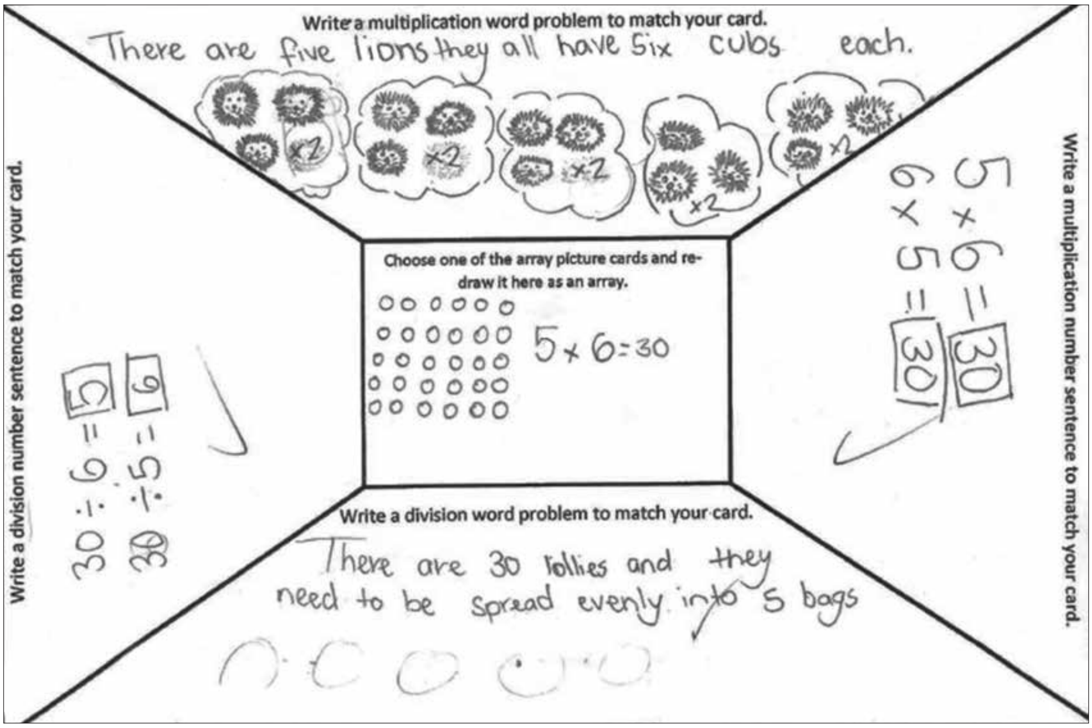


Figure 2. Marlon's Think Board.

out the answer efficiently or as accurately because multiplicative thinking opens them up to more areas along the way.

Abbie and Dan were seeking to work from a more informed position and to develop a rich conceptual understanding of multiplicative thinking in their students. In particular, different tasks were chosen in order to present particular concepts in different ways, such as those tasks about the distributive property (a critically important understanding that numbers can be partitioned to make operating with them easier, for example 34 × 7 can be considered as [30 × 7] + [4 × 7]) and inverse relationship (another important understanding that sometimes it is easier and more efficient to use division even though the problem appears to be a multiplication one, for example, 24 × ? = 264 may be better thought of as 246 ÷ 24 = ?). The following examples provide specific evidence of how the thinking of the teachers changed, how that had an impact on the tasks they chose for their students, and how student understanding was enhanced.

Explicit teaching—arrays and connections

The teachers decided to begin with some explicit teaching around the multiplicative array as a representation of the multiplicative situation (Hurst, 2014) and then link this to the properties of multiplication and division and the inverse relationship. In the interview, Abbie stated that, "We started off with arrays, because that was the basic foundation of being able to visualise a multiplication problem and they should be able to visualise it before they go on." Carl added, "And we linked it with concrete materials". Abbie continued to say that, "Once we felt that they had a solid knowledge of the properties and arrays, we linked the properties to mental multiplication and division strategies". Clearly there was a focused approach being used here which Dan explained as, "There was a lot of explicit teaching of strategies based on properties and arrays". The connectionist nature of the teaching taking place is underlined by Carl's comment: "We're teaching arrays as representations of the commutative property and it needs to have a whole focus to understand the connections". Dan supported when discussing how arrays were taught in the early years and saying, "But we see in upper primary that they're not using that knowledge so where those concepts might have been taught in isolation, we've brought it into a more explicit program where they can see how they all link together".

One of the tasks used was the Array Think Board in which children had to complete a Think Board based on a picture array card. Marlon's Think Board (Figure 2) indicates an understanding of how the array can represent the multiplicative situation in terms of writing number sentences and word stories for both multiplication and division, and reflects, and may be a consequence of the explicit teaching referred to by Dan. A task called Ice Cream Arrays was also used and students were asked to write a number statement and draw an array for each of three situations.

> There are eight different types of ice creams and four different types of cones. How many different ice cream and cone combinations can you have?
>
> There are some different types of ice creams and four different cones. Altogether you can make up 32 different ice cream and cone combinations. How many different ice cream flavours are there?
>
> There are 32 different ice cream and cone combinations and eight different types of ice creams. How many different types of cone are there?

Figure 3 shows a work sample done by Li Mei.

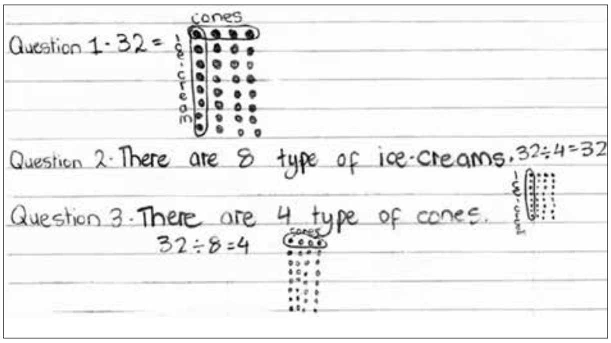


Figure 3. Li Mei's ice cream arrays.

Mei's work shows how the array may be used to understand combination problems with two variables. It is also interesting to note that she has chosen division sentences to show what is happening in the second and third situations.

This ability to make connections between the array, a combination problem, and division sentences is encouraging. This type of connectivity is also evident in Ben's work. His sample (Figure 4) is based on the same task as shown in Figure 1 (Bessie's Story Sample). Here, Ben seems to have recognised the connection between the word 'quarter', the division construct for fractions, and the inverse relationship between multiplication and division. That is, instead of trying to work with the unknown in the middle of the algorithm $\frac{1}{4}$ of $x = 24$, he has chosen to use the inverse operation and have the unknown in the answer $4 \times 24 = x$.

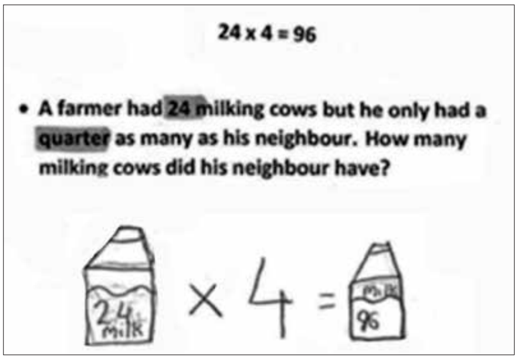


Figure 4. Ben's story.

Abbie also noted the importance of using the language of the multiplicative situation saying that, "A lot of the kids had a lot of the knowledge there but they didn't know how to articulate it" and "We've gone back a step or two and talked about what we label each number in a multiplication or division sentence—what are the factors, multiple, product and quotient, which has really helped". The use of such language is evident in Bessie's sample (Figure 1).

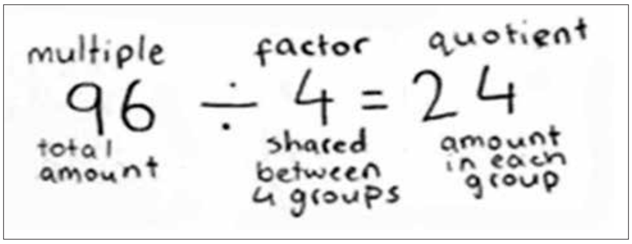


Figure 5. Excerpt from Figure 1—Bessie's sample.

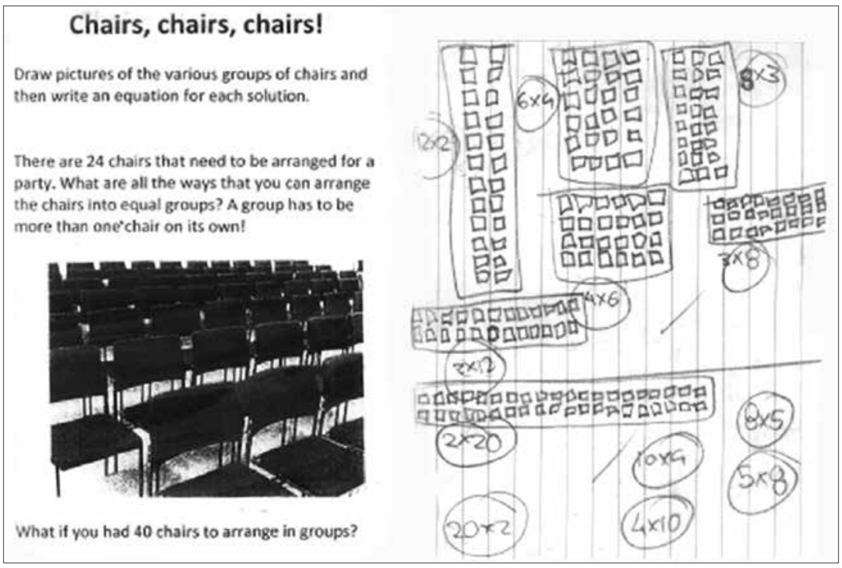
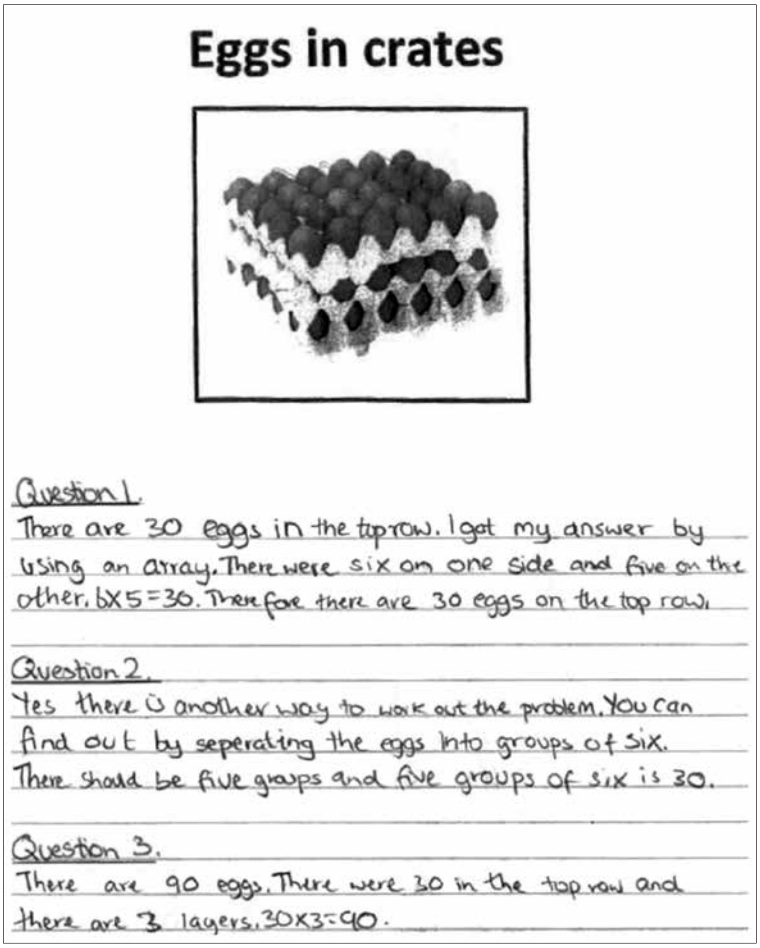


Figure 6. Charlie's chairs, chairs, chairs sample.


Eggs in crates

Question 1.
There are 30 eggs in the toprow. I got my answer by using an array. There were six on one side and five on the other. 6X5=30. There fore there are 30 eggs on the top row.

Question 2.
Yes there is another way to work out the problem. You can find out by seperating the eggs into groups of six. There should be five groups and five groups of six is 30.

Question 3.
There are 90 eggs. There were 30 in the top row and there are 3 layers. 30X3=90.

Figure 7. Jack's eggs in crates sample.

Further examples of the use of arrays to understand situations in different tasks is provided in Figures 6 and 7 which contain samples by Charlie and Jack. Note how Charlie has shown multiple arrays and has demonstrated an understanding of the commutative property (the vital understanding that for multiplication [and addition] the order of the factors does not affect the product, for example 5 × 4 gives the same product as 4 × 5)in showing both a 6 × 4 and 4 × 6 array as well as other pairings. This also links to the knowledge of factor pairs and seems to be further evidence of some explicit teaching around arrays. Jack has explicitly stated that he used an array to solve the Eggs in Crates task (Figure 7), which again demonstrates how the mathematical language is becoming embedded through explicit teaching.

Explicit teaching—the distributive property

Another aspect of multiplicative thinking that has been explicitly taught is the distributive property and this has been done by exposing students to a variety of strategies and representations. Abbie described it in this way:

> With a focus on so many different strategies, they find what works for them and they have something that they can rely on and when they have to show/explain how they solved a problem, most of them know the distributive property because that's something that they're comfortable with.

Even though they've learned the written strategy, that's what they like to use, so that's a good thing—we've given them a bit of a 'tool kit' to use in their life and they've found something that they're successful with.

Figures 8, 9, 10, and 11 illustrate some of specific and different ways in which the understanding of the distributive property has been developed.

Lily's sample (Figure 8) is interesting as it gives further credence to the success of the explicit and connected teaching that appears to have been occurring as the student seems to have made the connection between division and the distributive property. Kat's sample (Figure 9) suggests that there has been explicit linking between the work on the distributive property and men tal computation strategies such as estimating. Abbie's work (Figure 10) illustrates the importance of understanding the distributive property before use of a formal algorithm is attempted.

What did the teachers learn?

Perhaps the greatest learning was in terms of realising the complexity of the mathematics involved in teaching about multiplicative thinking, recognising that there were gaps in students' understanding, but most importantly, knowing how to go about remediating the situation. In discussing the importance of developing mental strategies before teaching algorithms, Dan said, "I think for us right now, we understand the underlying skills and strategies necessary before they get to that written stage. It's like 'backwards teaching'". Each of the three teachers candidly said that prior to their professional learning, they may not have been aware that there were gaps in their students' knowledge, let alone how to help, as this exchange from the interview demonstrates.

Abbie I guess . . . we may have even recognised the gaps but we may not have been able to give the kids the best support . . .

Dave Just random 'plugging of holes' . . .

Carl I think also some of the things that we covered, we wouldn't have realised in the past that there was a gap or that they didn't know something. For example, we focused on the written strategies and we wouldn't have even known that the mental strategies were a gap, and with arrays, that would have just 'gone through' and if we asked them what an array was, they wouldn't have known but we wouldn't have even recognised that as a gap. We wouldn't have been plugging those gaps in the past because we wouldn't have known that they existed.

Conclusion

Askew et al. (1997) identified 'connectionist' teachers as the most effective teachers of numeracy through their ability to understand the connections between mathematical ideas. There is plenty of evidence in the form of student samples presented here to suggest that Abbie, Carl, and Dan are developing into connectionist teachers. As well, in recognising gaps in their students' understanding, they are making connections between student misunderstandings, the specific mathematics needing to be learned, and the most effective tasks for achieving that learning. The success of the intervention described here prompted the three teachers to initiate some whole school professional learning in order to develop the underpinning concepts, knowledge, and skills across all years. They want to ensure that the foundations of multiplicative thinking are very sound to enable the development of higher order ideas such as ratio and proportional reasoning. The final words are from the teachers themselves:

Dan Our goals are to develop a scope and sequence identifying the key ideas, and secondly, consider

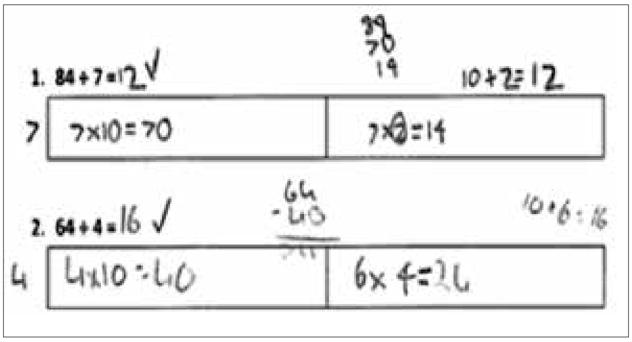


Figure 8. Lily's division example.

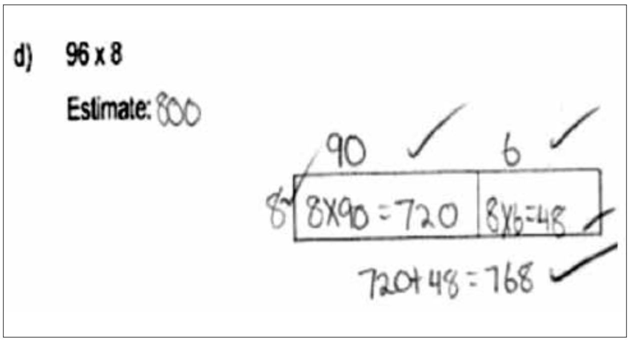


Figure 9. Kat's 2 digit multiplication.

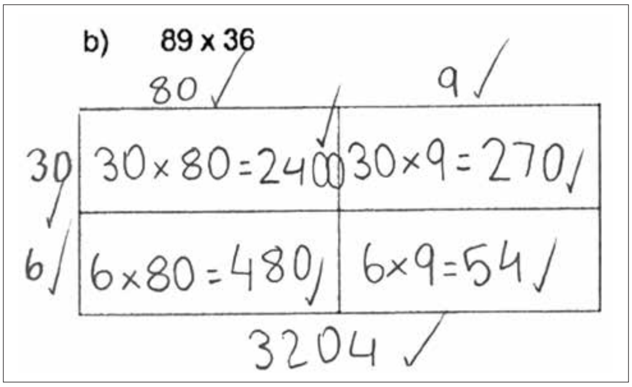


Figure 10. Abbie's 2 x 2 digit multiplication.

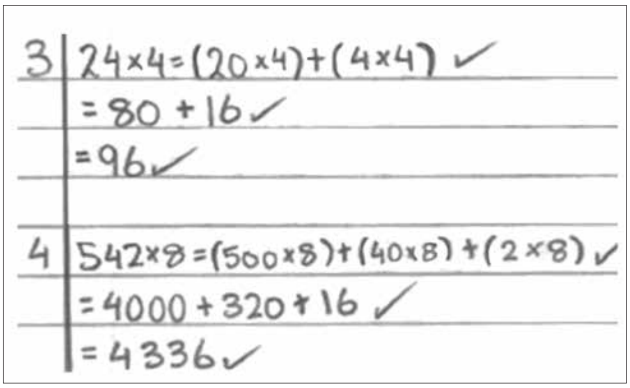


Figure 11. Xavier's distributive exercises.

how to go about teaching it to children. It would be useful to have some identification of key areas and tasks for each year level, but also emphasising the overlap between year levels.

Carl It's about building up the consistency across all classes.

Abbie We need to help people realise that they are already doing a lot of good things so 'this is not a whole new workload'—but they might need to make what they're doing a little more explicit or tweaking it a little bit. Also show them some resources and tasks we have used.

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