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Lorraine Day
University of Notre Dame Australia, lorraine.day@nd.edu.au

Derek Hurrell
University of Notre Dame Australia, derek.hurrell@nd.edu.au

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An explanation for the use of arrays

to promote the understanding of mental strategies for multiplication


Lorraine Day
University of Notre Dame
<lorraine.day@nd.edu.au>

Derek Hurrell
University of Notre Dame
<derek.hurrell@nd.edu.au>

Lorraine Day and Derek Hurrell provide a convincing argument for using arrays to promote students' understandings of mental computation strategies for multiplication. They also provide a range of different examples that illustrate the benefits of arrays in the primary classroom.

As part of *Understanding proficiency,* the *Australian Curriculum: Mathematics* requires that "They (students) develop an understanding of the relationship between the 'why' and the 'how' of mathematics" (ACARA, 2014, p. 2). In this paper we would like to address the issue of how we develop the 'why' around the 'how' of multiplication, leading to students being fluent users of multiplication for mental and written computation.

In order to add meaning rather than relying on memorised procedures for multiplication and division, students need to be able to think about multiplication in a number of different ways. As early as Year 2 the *Australian Curriculum: Mathematics* states that students should "recognise and represent multiplication as repeated addition, groups and arrays" (ACARA, 2014). In particular, arrays and regions assist to support the shift from additive thinking ('groups of' model) to multiplicative thinking ('factor–factor–product' model) (Siemon, 2013) and eventually to proportional and algebraic reasoning.

When asked to create four groups/lots of three, students will often create a model that looks something like Figure 1. While this is correct, it is not necessarily the most helpful or even efficient representation, if we want to move the students from additive to multiplicative thinking. Whilst Figure 1 does show four lots of three it can encourage students to use repeated addition which does not "address all situations in which multiplication is helpful" (Willis et al., 2008, p. 28). Whereas Figure 2, a region or array model, not only shows the strategy of repeated addition it also encourages other understandings.

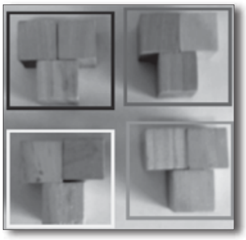
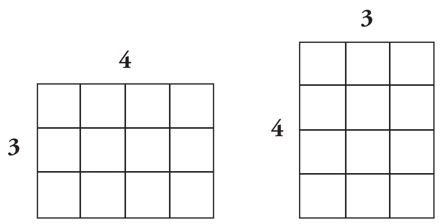


Figure 1

Figure 2

One such understanding is the identification and naming of factors and multiples, an important and often undervalued piece of mathematical understanding. Giving students a strong understanding of factors and multiples through manipulating the materials to come to a shared definition of these terms is extremely empowering. Another important mathematical understanding that arrays can help to develop is that of commutativity (Figure 3). As a mental and written computation strategy, commutativity, the understanding that for multiplication (and addition), it does not matter which order you use the numbers, the result will be the same, is vital, and the capacity to rotate an array to show that four lots of three gives the same total as three lots of four is illustrative of this.



Introduce the language of factors. Three groups of 4 and the same as 4 groups of 3 (commutativity). Are these the same, or do they give the same total?

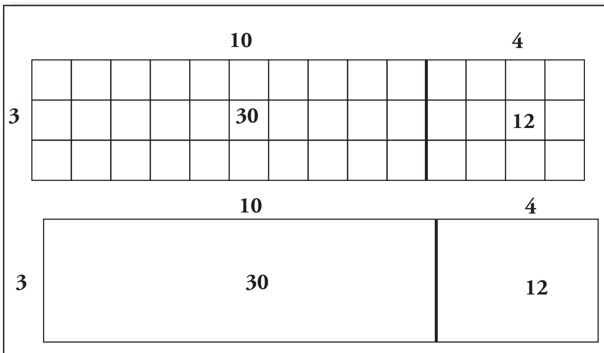
Figure 3

Another understanding that can be developed through the use of arrays, and should not be underestimated, is 'number families'. Often students are told that if you know 3 × 4 you know the associated facts of 4 × 3, 12 ÷ 3 and 12 ÷ 4, and many accept this as being the case without ever seeing why it is so. The array model created with materials and then manipulated to discuss and illustrate these 'number families' is a very visual and powerful model. This same model then readily lends itself to being represented on square grid paper, and then later still, having the associated facts represented abstractly though numbers. This movement from the concrete to the abstract can all happen whilst keeping the materials handy to maintain a clear illustration of the connection between the associated facts, and therefore, developing fluency through, and with, understanding.

Another strength of the array model is that it can be extended into two-digit by one-digit multiplication (Figure 4). Representing multiplication in this way provides a meaningful illustration of partitioning of numbers and encourages an understanding of the magnitude of numbers in a very visual manner. It also encourages the development of the distributive property (that is that 14 × 3 is equivalent to 10 × 3 + 4 × 3, another important mental computation strategy) and the link between multiplication and area.

Once the model has been used to develop two-by one-digit multiplication, it is then a reasonable step to move to a representation of two-by two-digit multiplication (Figure 5).

Students who have not had the benefit of using an array model may assume that 13 × 12 can be calculated by 10 × 10 + 2 × 3.



3 x 14 = 3 x 10 + 3 x 4 = 30 + 12 = 42

Figure 4

Using the array model and identifying the areas assists students to see why this is not the case. Once again the array proves to be an efficient construct which supports the *Australian Curriculum: Mathematics* Year 3 content descriptor which encourages students to "represent and solve problems involving multiplication using efficient mental and written strategies..." (ACARA, 2014).

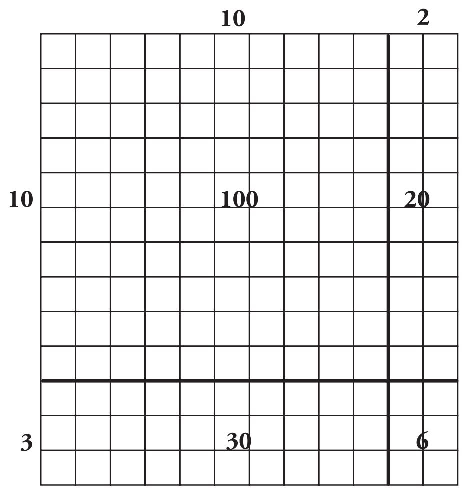


Figure 5

At this point it is not unreasonable to start representing the algorithm in a more abstract manner. By having the array representation at hand (Figure 5), a direct comparison can be made between this representation and a more abstract representation (Figure 6). The use of a non-standard algorithm rather than a standard algorithm may better serve to bridge the understanding between the eventual use of the 'abbreviated notation' employed in the standard algorithm and to what the 'abbreviated notation' actually refers (for example, the five in 156 refers to five-tens).

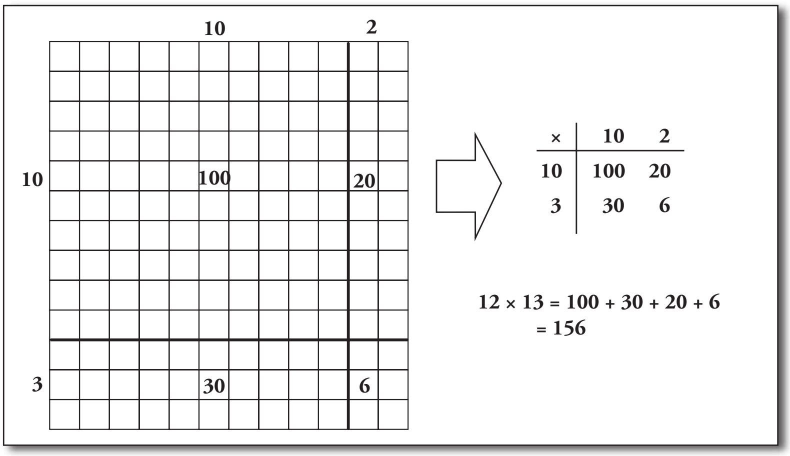


Figure 6

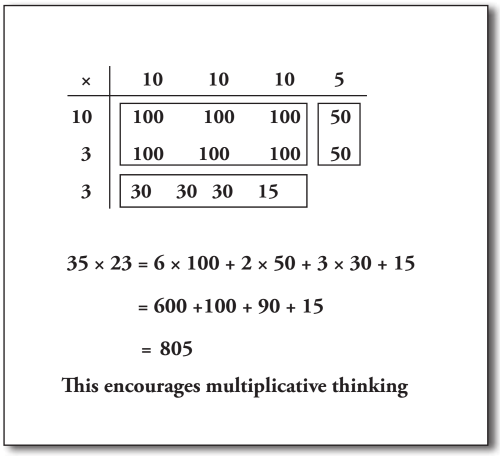


Figure 8

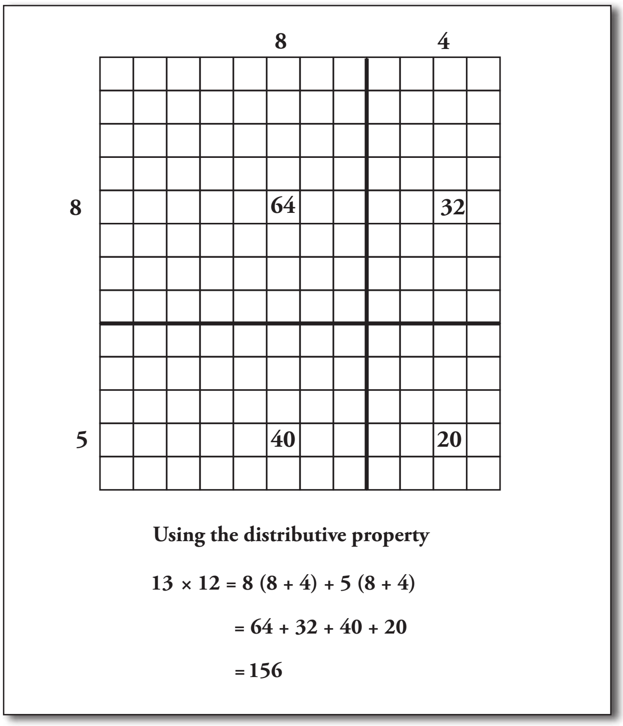


Figure 7

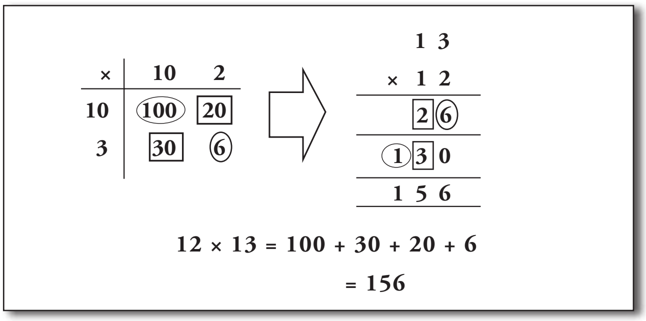


Figure 9

This model can also be used to demonstrate that it does not matter how the numbers are partitioned—we use ten because it makes the calculations easier, but the distributive property is not restricted to partitions involving tens (Figure 7). This knowledge provides students with more flexible mental computation methods.

The Australian Curriculum: Mathematics Year 5 states that students should "solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies" (ACARA, 2014). This model lends itself to further exploration of these larger numbers (Figure 8).

A standard, written algorithm is a useful tool. A standard, written algorithm, which is constructed with understanding, is an exceptionally powerful tool. Once the non-standard algorithm in Figure 6 has been established, discussed and understood it can then be used to develop an understanding of the components of the standard written algorithm (Figure 9) if this is seen as necessary.

Although not a focus of this article (and there is probably another article which can be written further unpacking the move into numbers other than whole numbers) the array model can also be utilised for providing a compelling visual model of multiplying decimals (Figure 10) and fractions (Figure 11). The *Australian Curriculum: Mathematics* suggests Year 6 students should"multiply decimals by whole numbers" and in Year 7 "multiply and divide fractions and decimals using efficient written strategies and digital technologies" (ACARA, 2014).

By forming the foundation of array or region models in the primary setting teachers are not only providing powerful understandings to aid mental and written computation, they are also paving the way for easier connections to be made in secondary mathematics. In Year 7 according to the *Australian Curriculum: Mathematics*, students should "apply the associative, commutative and distributive laws to aid mental and written computation" and "extend and apply the laws and properties of arithmetic to algebraic terms and expressions".

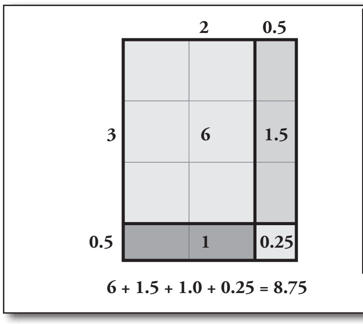


Figure 10

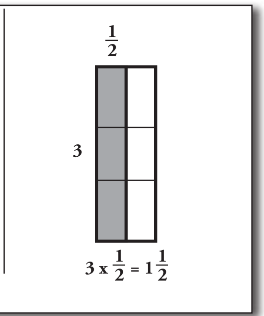


Figure 11

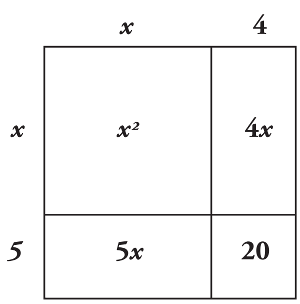
> The ability to 'see' how 0.5 x 0.5 is equivalent to 0.25 is such an important visualisation for students especially for those who believe that multiplication always makes the answer larger. It also assists with the understanding associated with division of decimals and fractions, as the idea of the inverse relationship has already been developed.

Furthermore in Year 8 students should "extend and apply the distributive law to the expansion of algebraic expressions" and "factorise algebraic expressions by identifying numerical factors" (ACARA, 2014). Another great strength of the region or array model is that is can easily be extended into algebraic reasoning. It may be a good idea at this stage to re-introduce a concrete model here (such as *Algebra Tiles Australia* (Day, 2014)).

$3\ (x + 4) = 3x + 12$

Figure 12

The model can be used for expanding quadratic functions:



×	x	4
x	x^2	$4x$
5	$5x$	20

$$\begin{aligned}(x + 4)(x + 5) &= x\,(x + 5) + 4\,(x + 5)\\ &= x^2 + 5x + 4x + 20\\ &= x^2 + 9x + 20\end{aligned}$$

Figure 13

There are many reasons, to use an array or region model in the teaching of multiplication and relating it seamlessly to division. The most compelling of these reasons is to "support the shift from an additive groups of model to a factor–factor–product model which is needed to support fraction representation, the multiplication and division of larger whole numbers, fractions and decimals, and algebra" (Siemon, 2013, para. 4). We believe that the array model is a very powerful way in which to take students to a robust understanding of not only the 'how' of multiplication but the 'why' as well. It supports students in a way which simply teaching the mechanics of the algorithm cannot.

To summarise the array method:

- Is visual
- Makes sense
- Supports a strong instructional practice of moving from concrete to representational to abstract
- Encourages multiplicative thinking
- Links multiplication to area
- Demonstrates the distributive property
- Models commutativity
- Makes the transition to algebraic reasoning easier.

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