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Captain Zero . . . hero or villain?

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Our current research into children's multiplicative thinking has shown that children have the capacity to think multiplicatively and that some aspects of multiplicative thinking are more thoroughly understood than others. We look at a data set obtained over three classes in the same year level and explore the considerable variation in responses to several key questions on a multiplicative thinking quiz. The questions relate to the 'times bigger' notion in comparing numbers, the ability to use standard place value partitioning when operating, the ability to articulate what happens when numbers are multiplied and divided by powers of ten, and the role of zero. It could reasonably be implied that the variation in understanding across the three classes may be due to pedagogical emphases.

Introduction

'Big idea' thinking in mathematics has been in evidence for some time, perhaps initiated by the connectedness of the work of Ma (1999) and the seminal paper by Charles (2005), and followed more recently by the work of Clarke, Clarke & Sullivan (2012). Multiplicative thinking has been identified as a 'big idea' (Hurst & Hurrell, 2015; Siemon, Bleckley & Neal, 2012) which underpins much of the mathematical learning that occurs in late primary years and beyond. Amongst other things, it provides important foundational understanding for fraction concepts, decimals and percentages, ratio and proportion, and algebraic reasoning (Siemon, Bleckley & Neal, 2012). Unfortunately, many students are not strong multiplicative thinkers and as many of 40% of them in Years 7 & 8 perform below expectations (Siemon, Breed, Dole, Izard, & Virgona, 2006). Over the past three years, we have conducted research into children's multiplicative thinking for several reasons. Firstly, we wanted to understand the specific mathematics that constituted multiplicative thinking. Secondly, we wanted to identify the aspects with which students and teachers experienced the most difficulty. Thirdly, we were keen to develop some tasks and pedagogies that would be of benefit to teachers and students.

What Constitutes Multiplicative Thinking?

Two instruments were used to gather data – a written quiz and a semi-structured interview. The quiz has been administered to over 1000 students with about fifty being interviewed. Our initial ideas about what comprised the component parts of multiplicative thinking were refined as the data were analyzed and six themes were established as follows:

1. The 'multiplicative situation', or the relationship between multiplication and division, the use of the multiplicative array, the language of factors and multiples, and the links with fraction, ratio, and proportion, with all of these points being expressed and described in a range of problem types, stories, and number sentences.
2. The notion of a number being '...times bigger' or '...times smaller' than another number. This is distinct from the additive notion that a number is '...more' than another number (e.g., 40 is 4 more than 36).
3. Multiplication and division by powers of ten and, what happens when a number is multiplied or divided by another number.

4. Use of a variety of materials such as bundling sticks, and MABs to develop an understanding of the multiplication and division algorithms.
5. Properties of multiplication and division and the relationships between them including the commutative property, distributive property, inverse relationship and extension of number facts.
6. Extension of multiplication and division beyond 2 digit by 1 digit or 2 digit ÷ 1 digit, and including the use of algorithms based on multiplication properties (e.g., distributive and extended facts).

- This paper reports on the use of aspects of the quiz with three unstreamed, heterogeneously grouped Year 4 classes at the same school. Interesting observations can be made about aspects of Themes 1, 2, 3, and to some extent, Theme 5. When taken on their own, the responses to each section of the quiz could be seen as 'unremarkable' but when considered together, they suggest some clear pedagogical differences across the three classes.

Results and Discussion

Theme 1 – Numbers of Equal Groups; Representation With Arrays

Students were asked the answer to the number fact 8 × 7, to explain what the numbers in the fact told them about, and to write a story about it. They were then asked to represent the number fact with a drawing. Responses are summarised in Table 1 and given as a percentage of the class total. Class 1 had 30 students, Class 2 had 23 students and Class 3 had 28 students.

Table 1. *Responses of students in three classes to Theme 1 questions*

Criterion	Class 1	Class 2	Class 3
Knows about group size and number of groups, and/or writes appropriate story about given number fact	43	32	11
Represents a number facts as a number of separate groups	37	36	18
Represents a number fact as a multiplicative array	43	25	42

Students in Class 3 were generally unable to articulate about number and size of equal groups in the multiplicative situation yet nearly half of them drew a multiplicative array, considered to be a powerful representation of it. In Class 1 a similar percentage of students drew an array and a greater proportion of them articulated about groups than students in Class 3. Of interest is the fact that twice as many students in Class 1 also depicted the situation with a drawing showing equal groups than did students in Class 3. Also less students in Class 2 drew an array than did students in Class 3 but more of them were able to talk about numbers of equal groups. What might this indicate?

It is problematical to draw a clear conclusion from the data in Table 1other than to suggest that it might be due to pedagogical influences. Perhaps the teacher of Class 3 had explicitly taught the use of arrays but had not explicitly made the connection with numbers of equal groups. Perhaps the teacher of Class 1 had explicitly taught the concept of numbers of equal groups and also linked it to representing with drawings, both of arrays <u>and</u> separate groups.

Theme 2 – The Role of 'Captain Zero', and the Notion of 'Times Bigger'

Data related to this section is presented in two tables and some figures. Students were asked to explain what happened to a number when it was multiplied by ten and then were asked to say 'how many times bigger' a number was than another number. These numbers can be seen in Table 2.

Table 2. *Summary of explanations for multiplying a number by ten*

Criterion	Class 1	Class 2	Class 3
Explanation based on 'adding a zero'	77	72	46
Explanation based on other ideas	17	14	11
No response given	6	7	0
Explanation based on digits moving a place, and/or moving to the left	0	7	43

There are some stark differences between the three classes with these data. The great majority of students in Classes 1 and 2 explained the result of multiplication by ten in terms of 'adding a zero', while only two students (7%) in Class 2 explained it conceptually in terms of the movement of digits to a higher value place. However, almost half the students in Class 3 explained it in that conceptual way. The difference is even more noteworthy when one considers the particular way in which students in Class 3 explained their thinking. Figure 1 contains five examples of their work.

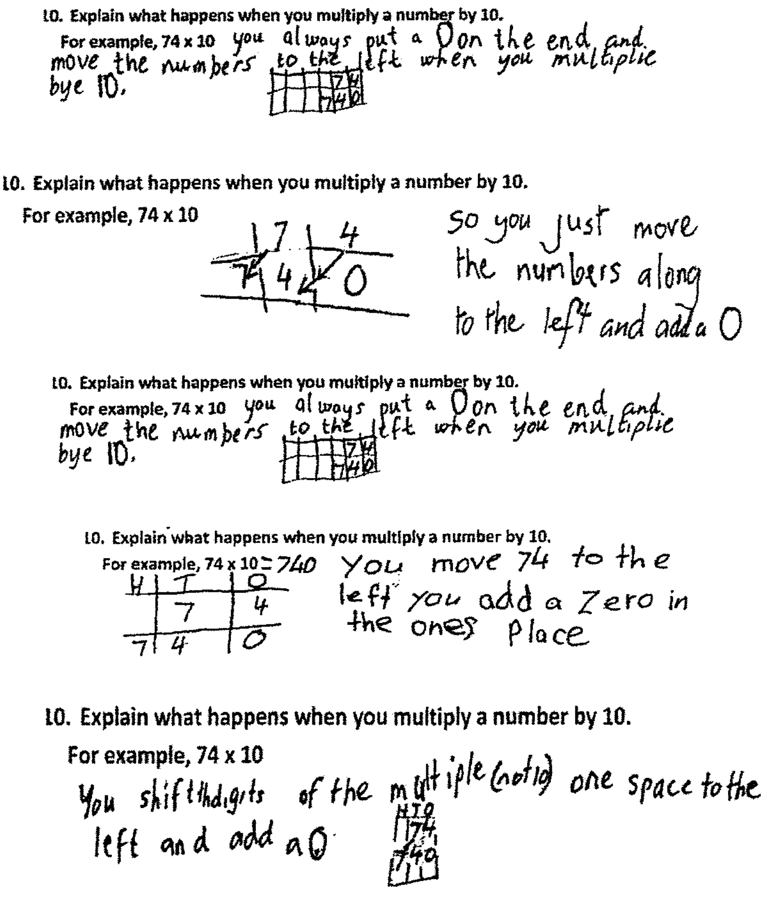

Figure 1. Examples of conceptual explanations of multiplication by ten

It seems likely, given the level of explanation in the Figure 1samples that, in Class 3, there has been some explicit teaching of what happens when numbers are multiplied by ten. Even though a similar percentage of students explained it in terms of the 'Captain Zero' phenomenon, the proportion of students demonstrating conceptual understanding is considerably higher in Class 3 than in Classes 1 or 2. The 'adding a zero' explanation is considered to be procedural and not likely to be underpinned by conceptual understanding. The difference is that, while students in Class 3 did talk about adding a zero, they stated the vital aspect about the digits moving to a place of higher value. The situation becomes even more intriguing when data from the 'how many times bigger than . . .' questions are considered.

Table 3. *Summary of responses to the 'how many times bigger' questions*

Criterion	Class 1	Class 2	Class 3
Identifies 40 as 10 times bigger than 4	70	71	32
Identifies 400 as 10 times bigger than 40	43	18	25
Identifies 4000 as 10 times bigger than 400	33	21	25
Identifies 400 as 100 times bigger than 4	53	43	32

The data in Table 3 are interesting in themselves as they show some clear differences between some of the classes and the responses of students. For instance, and taken at face value, students in Class 3 do not seem to have responded as well to these questions about 'times bigger' as they did to the previous question about multiplying by ten, yet the underpinning ideas in both are at least very similar. To clarify this situation, we need to consider the range of responses given by students to the four questions. These responses are contained in Table 4.

With some exceptions, students responded in one of three ways. Response Set 1 contains four correct responses demonstrating an understanding of the 'times bigger' notion. Response Set 2 indicates that students have likely considered only the size of the first number in each question with no consideration given to the 'times bigger' notion. They were able to give a correct response for the first and fourth questions, but likely for the wrong reason. Response Set 3 is purely an 'additive' response obtained by subtracting one number from the other, with no understanding of 'times bigger'.

Table 4. *Summary of sets of responses given to the 'times bigger' questions*

Criterion	Set 1	Set 2	Set 3
Identifies 40 as 10 times bigger than 4	10	10	36
Identifies 400 as 10 times bigger than 40	10	100	360
Identifies 4000 as 10 times bigger than 400	10	1000	3600
Identifies 400 as 100 times bigger than 4	100	100	396

We can now consider the results shown in Table 3 in a somewhat different light and present Table 5 showing only the results for the second and third questions (where understanding the 'times bigger' notion is essential in arriving at the correct answer), and including the percentage of students who gave the 'additive' responses.

Table 5. *Summary of sets of responses given to the 'times bigger' questions*

Criterion	Class 1	Class 2	Class 3
Identifies 400 as 10 times bigger than 40	43	18	25
Identifies 4000 as 10 times bigger than 400	33	21	25
Provided additive responses (shown in Set 3)	13	14	50

Two things stand out from Table 5. Firstly, even though nearly half of the students in Class 3 could explain conceptually what happened when a number is multiplied by ten , only a quarter of the class could articulate the 'times bigger' relationship in these two questions. Further interrogation of the data indicates that the students who did so were not the same students (with two exceptions) who provided a strong explanation of multiplication by ten. Secondly, a relatively high percentage of students in Class 3 provided an additive response to the 'times bigger' questions. On further interrogation of the data, three of those students did provide a conceptual explanation of multiplication by ten. As previously noted these data suggest that there may have been some explicit teaching around the concept of moving digits to places of higher value, and possibly of the 'times bigger' notion, but explicit connections between the two ideas have not been drawn.

Theme 3 – Properties of 2x1 Digit Multiplication

Students were given the example 6×17 and were asked to calculate it mentally and explain how they did it, and then to show a written method for working it out. We wanted to see if they used a standard place value partition either mentally or in their written method, that is, the answer was calculated by using $(6 \times 10) + (6 \times 7)$. The results are shown in Table 6.

Table 6. *Summary of responses given to the 6 × 17 questions*

Criterion	Class 1	Class 2	Class 3
Correctly calculated answer to 6×17 using mental computation	60	46	29
Explained mental computation for 6×17 based on place value partition	57	25	11
Correctly multiplies 6×17 using a written method	0	3	39
Correctly uses standard algorithm and shows place value partition	0	3	0

Once again, there are clear differences between responses of students in different classes. It would seem that students in Classes 1 and 2 were familiar and comfortable with mental computation of such examples with many of them in Class 1 able to explain their thinking in terms of the standard place value partition. Less students in Class 2 and even less in Class 3 were able to do so. However, further interrogation of the responses from students in Class 3 indicates that 50% of them used the 'lattice method' for their written calculation but only half of them did so correctly. A further four students in Class 3 used an alternative partition or non-standard method of calculation. It is also worthy of note that no student in Class 3 who used the 'lattice method' was able to calculate 6×17 mentally and provide an explanation in terms of the standard partition. The use of the lattice method was confined to Class 3 so it is likely that it has been explicitly taught to that class. However, the presented evidence suggests a lack of underpinning conceptual understanding to support its use.

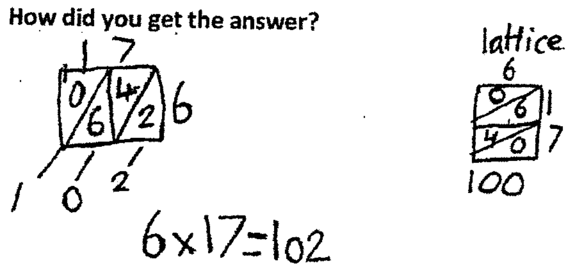

Figure 2. Example of correct and incorrect use of lattice method

Conclusions

When considering the data presented here, it seems reasonable to come to some tentative conclusions about some of the pedagogies used in the three classes. With regard to the first theme, it is likely that there has been some explicit teaching around the use of arrays to depict the multiplicative situation but at the same time, the connection to the idea of a number of equal groups has not been made clearly in each of the three classes. Data from the second theme suggests that students in all three classes had been taught a procedure based on 'adding a zero' when multiplying by ten/s although the very strong conceptual explanation provided by some students in Class 3 suggests that they had been explicitly taught that digits move to a place of higher value when multiplied by ten. Responses to the 'times bigger than' questions suggest that many students have difficulty understanding the difference between that concept and the idea of 'how many more than', as indicated by the high number of additive responses from one class at least. Also, many students interpret 'times bigger than' questions in terms of the size of the bigger number rather than as a comparison. Finally, data from the third theme suggests that explicit teaching of mental computation strategies has occurred in some classes while students in Class 3 have likely been explicitly taught the 'lattice method' for multiplication as a procedure without the associated conceptual understanding of partitioning. This paper reports on one aspect of our current research into children's multiplicative thinking. As part of the study it has been arranged to revisit the school and interview the three Year 4 teachers to provide a firmer base for drawing conclusions.

Overall, the presented data suggests that none of the students in the three classes has a broad understanding of multiplicative thinking but that they all have a partial knowledge. While this is reasonably expected at Year 4 level, explicit teaching based on connecting various aspects of multiplicative thinking as outlined here would be of benefit. Such teaching needs to be based on development of conceptual understanding as opposed to the use of procedures. The impact of 'Captain Zero' provides a good example of how the development of conceptual understanding can be hindered by the use of procedures which, in the long run will be found wanting.

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