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ACCESSING MATHEMATICAL CONTENT THROUGH THE PROFICIENCY STRANDS

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The Australian Curriculum: Mathematics differs from previous state versions of curricula by the inclusion of four Proficiency Strands alongside the three Content Strands. The Proficiency Strands are the power behind the curriculum as they shape the way in which the content should be accessed. Mathematically rich tasks that use a problem solving or inquiry approach allow students to develop deep understanding of mathematical concepts through reasoning and communication that will lead to greater fluency.

The Proficiency Strands

The Australian Curriculum: Mathematics (ACM) (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016) is constructed around three Content Strands; Number and Algebra, Measurement and Geometry and Statistics and Probability. These strands provide teachers with guidelines about what to teach. Sullivan (2012) called these the ‘nouns’ of the curriculum. The ACM does more than inform teachers about the content areas that need to be addressed. Through the four Proficiency Strands; Understanding, Fluency, Problem Solving and Reasoning, the ACM shapes the way in which the mathematical content should be accessed, providing some direction on how this may be achieved. Askew (2012) explained that these Proficiency Strands should be enacted during the learning of mathematical content rather than just when applying it. Maintaining the grammar analogy, Sullivan (2012) referred to the proficiencies as the ‘verbs’ of the curriculum. The importance of these ‘verbs’ was endorsed by Burns (2012) who affirmed that the Standards for Mathematical Practice, the equivalent to the ACM Proficiency Strands in the US, should be front and centre of mathematics teaching and learning.

The Proficiency Strands in the ACM were adapted from Kilpatrick, Swafford and Findell (National Research Council, 2001) who conveyed five strands:

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition (p. 5)

They saw the strands as being mutually dependent and intertwined like a rope, demonstrating that the individual strands should not and cannot be viewed in isolation from each other, or from the content they are acting upon. Kilpatrick et al. (2001) argued that problem solving is where all of the mathematical proficiencies come together, to provide a way for students to weave all of the strands together in such a way that allows teachers to assess student performance on all of the proficiency strands. Problem solving, after all, is what mathematicians do (Holton & Lovitt, 2013; Rigelman, 2013).

Lappan and Phillips (1998) in the Connected Mathematics Program developed criteria for a good mathematics problem:

- The problem must have important, useful mathematics embedded in it.

- Students must be able to approach the problem in multiple ways, using different solution strategies.
- The problem should allow various solution strategies or lead to alternative decisions that can be taken and defended.
- The problem should engage students and encourage classroom discourse.
- Solution of the problem should require higher-level thinking and problem solving.
- Investigation of the problem should contribute to students' conceptual development.
- The mathematical content of the problem should connect other important mathematical ideas.
- Work on the problem should promote skilful use of mathematics and opportunities to practise important skills.
- The problem should create opportunities for the teacher to assess what students are learning and where they are experiencing difficulty. (pp. 87-88)

Task Selection

These criteria for good problems look very similar to the qualities of mathematically rich, investigative tasks (e.g. Day, 2012, Flewelling & Higginson, 2003; Lovitt & Clarke, 2011). Both have all of the Proficiency Strands embedded within them as well as stressing the importance of mathematical content. The choice of mathematically rich inquiry tasks is critical, although the tasks themselves need to be driven by quality pedagogy and teacher decision making (Aubusson, Burke, Schick, Kearney, & Frischknecht, 2014). As Boaler (2016) pointed out "Teachers are the most important resource for students." (p. 57). Sullivan and Davidson (2014) identified key considerations teachers need to make when selecting tasks as having multiple entry and exit points, encouraging deep and sustained thinking as well as argumentation. Hunter (2014) suggested that rich mathematical reasoning ensues when teachers play the vital role of demanding that students justify results and encourage generalisation.

Boaler (2016) identified six questions teachers should ask themselves when adapting or designing tasks for better mathematical learning:

1. Can you open the task to encourage multiple methods, pathways, and representations?
2. Can you make it an inquiry task?
3. Can you ask the problem before teaching the method?
4. Can you add a visual component?
5. Can you make it low floor and high ceiling?
6. Can you add the requirement to convince and reason? (pp. 77-86)

Lovitt (personal communication, 2012) would add: Can you make it kinaesthetic? When selecting tasks, teachers should consider whether these criteria have been addressed.

It is important when selecting, adapting or designing tasks that teachers consider both the mathematical content as well as the pedagogical considerations of how the mathematics is to be 'mined' from the task in such a way that develops deep conceptual understanding. Mason (2015) maintains that in order for mathematical thinking to take place a conjecturing atmosphere needs to be developed in classrooms. Carefully crafted tasks, in the hands of skilful teachers, "offer students more and deeper learning opportunities" (Boaler, 2016, p. 90).

A Sample Task

'Greedy Pig' is a well-known dice game that is used in many classrooms (see *Figure 1*). There are several versions and adaptations of this game (e.g. www.maths300.com, www.nzmaths.co.nz/resource/greedy-pig-0, <https://nrich.maths.org/1258>, http://www.curriculumsupport.education.nsw.gov.au/digital_rev/mathematics/assets/stage4/g_pig.pdf)

The class stands up, the teacher rolls a dice and students score whatever is rolled. This is repeated, with students aggregating their scores. They decide when to sit down and hence keep their score to that stage. However, if the number 2 is rolled while they are standing up their entire score for that round is zero. A game is made up of 5 rounds and the first 2 rolls of each round are free, so even a 2 can be scored. The investigative question becomes *When is the best time to sit down?*

Figure 1. How to play 'Greedy Pig' (Lesson 5, maths300)

When I first saw this activity, I thought it was a great game and I took it straight back to my classroom. The students loved playing 'Greedy Pig' and often asked if they could play the game. What I had neglected to consider was what mathematics I hoped the students would learn as a result of playing this game, or how I could engineer the task to ensure that there was rich mathematical learning occurring rather than the students just having fun. I have nothing against students having fun in mathematics lessons, as long as they are not just having fun, but are also learning important mathematical concepts and skills. Just playing the game does have some incidental learning associated with it, such as being too 'greedy' provides feedback about how likely it is that a two is rolled. Similarly by playing too safe students soon see that they never amass a large score. However there is so much more mathematics that can be 'mined' from this simple game and by employing the Proficiency Strands the quite sophisticated mathematical content can become accessible to the students.

The game setting of 'Greedy Pig' allows all students to enter the task in a non-threatening environment. By physically playing the game, the context for making sense of the mathematics is set and, as students tend to enjoy this game, they want to know more about how to determine a successful strategy for winning. When investigating the potential for this task and aligning it to the Australian Curriculum: Mathematics (ACARA, 2016) content descriptors were identified from Years 3-12 that could be accessed through this task. In Years 5-9 there were eleven possible Statistics and Probability content descriptors that could relate to this task, depending on the path the teacher chooses to take (see Appendix 1). This fulfils the criterion that good tasks are easy to begin and have multiple exit points as well as having rigorous mathematical content within it (Boaler, 2016; Lappan & Phillips, 1998; Sullivan & Davidson, 2014).

The collection of class data after playing and totalling the points from five rounds begins the students' mathematical journey with this task. The use of student-generated data is much more relevant to students and this data is best represented with a stem and leaf plot. Students tend to find stem and leaf plots quite intuitive and, if they are new to this representation of data, I find that I only have to show a few students how the plot works and then the others demonstrate their understanding of the process by watching and learning from others. This to me is a much more powerful approach than running a formal lesson on how to generate stem and leaf plots. It is another example of incidental learning within this task. The data collection provides the springboard to using statistical inference to start thinking about when is the best time to sit down when playing Greedy Pig, which in turn leads to other aspects of understanding and fluency in statistics and probability.

The idea of a probability distribution is visually apparent within the stem and leaf graph, demonstrating the likelihood of scores falling within a particular range. The use of statistical measures such as range, median, quartiles and other comparative statistics can easily be addressed once a stem and leaf graph is constructed. By finding patterns within the graph and suggesting reasons for those patterns begins the process of statistical inference and allows students to start to generate hypotheses for when the best time is to sit down. It is also important that students recognise that the graph represents a limited number of trials which is an indicator of a general population, as this will lead to the big probability idea of long run frequency (The Law of Large Numbers).

The problem solving and reasoning aspects of this task are introduced by the question "When is the best time to sit down and how do you know?". Once students have generated some hypotheses it is time to test those hypotheses. I always get students to work in pairs for this part of the task, as the communication and reasoning aspect is heightened when the students have someone other than me to convince. This is when the introduction of technology for students to gather data about the performance

of different strategies is useful. In doing this students are beginning to model what mathematicians do and the role technology plays in their investigations. By using the maths300 software students are able to play a hundred or more games and keep statistics in the form of stem and leaf graphs, mean, median, upper and lower quartiles as well as highest and lowest scores for each strategy. The software also introduces the visual box and whisker display which changes as each game is played, so students can see how it is formed and the effect of large numbers of trials of stabilising the statistics. When students have narrowed their preferred strategies down to two, they can use the software to compare the strategies. For older students this could entail an investigation of the levels of confidence needed to be convinced whether one strategy is superior to another.

Once students have decided on their preferred strategy, it is important to put their strategy to work by playing five more rounds. There is an opportunity for discussion about expected frequencies versus observed frequencies at this stage, as well as the introduction of back-to-back stem and leaf plots so that before-and-after strategy development comparisons can be made with the class data. Other challenges, such as investigating the effect of changing the 'killer' number can be introduced as can calculating the theoretical results. A full explanation of a method to calculate the theoretical probabilities may be found in Holton and Lovitt (2013). Finally, students can write a mathematical report about how they developed their strategy, including the reasoning they used when testing strategies, the content that they learnt and how they used statistical analysis to predict probability outcomes.

Conclusion

This is just one example of a task in which employment of the Proficiency Strands allows teachers and students to access a range of interconnected and important content through a mathematically rich inquiry task. Rather than just playing the game and hoping that some mathematics will be incidentally learnt by students, carefully designed learning experiences based on problem solving, reasoning, fluency and the development of concepts that promote deep understanding mean teachers can help students to access important mathematical content in an interconnected way. My analogy for the Content Strands and the Proficiency Strands is a car. I see the Content Strands as the passengers in the car, or the 'nouns' as Sullivan (2012) referred to them. Passengers are important, we want to get our passengers from point A to point B safely and securely, albeit sometimes with some detours on the way. I see the Proficiency Strands as the engine of the car, or the 'verbs' as Sullivan described them, as they are the power behind the ACM. In order to move the passengers to their destination, I need the engine of the car to enable the car to transport the passengers safely and securely.

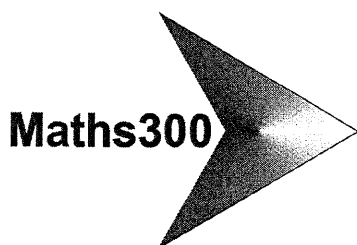
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Appendix 1

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Connecting with the Australian Curriculum: Mathematics

Greedy Pig

Year Levels: 3-12

Proficiency Strands: Problem Solving, Reasoning, Understanding, Fluency

General Capabilities: Literacy, Numeracy, Critical and Creative Thinking, ICT Capability

Content Strands: Number and Algebra, Statistics and Probability

Number and Algebra

Year 3: Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation ([ACMNA055](#))

Statistics and Probability

Year 3: Conduct chance experiments, identify and describe possible outcomes and recognise variation in results ([ACMSP067](#))

Year 4: Identify events where the chance of one will not be affected by the occurrence of the other ([ACMSP094](#))
Select and trial methods for data collection, including survey questions and recording sheets ([ACMSP095](#))

Year 5: List outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions ([ACMSP116](#))

- Pose questions and collect categorical or numerical data by observation or survey (ACMSP118)
- Year 6: Describe probabilities using fractions, decimals and percentages (ACMSP144)
 Conduct chance experiments with both small and large numbers of trials using appropriate digital technologies (ACMSP145)
 Compare observed frequencies across experiments with expected frequencies (ACMSP146)
- Year 7: Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)
 Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)
 Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171)
 Describe and interpret data displays using median, mean and range (ACMSP172)
- Year 8: Investigate techniques for collecting data, including census, sampling and observation (ACMSP284)
 Explore the variation of means and proportions of random samples drawn from the same population (ACMSP293)
- Year 9: Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282)
 Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283)
- Year 10: Determine quartiles and interquartile range (ACMSP248)
 Construct and interpret box plots and use them to compare data sets (ACMSP249)
- Year 10A: Calculate and interpret the mean and standard deviation of data and use these to compare data sets (ACMSP278)
- Years 11/12: Calculate measures of central tendency, the arithmetic mean and the median (ACMEM050)
 Perform simulations of experiments using technology (ACMEM150)
 Recognise that the repetition of chance events is likely to produce different results (ACMEM151)
 Identify relative frequency as probability (ACMEM152)
 Determine the probabilities associated with simple games (ACMEM157)
 Use relative frequencies obtained from data as point estimates of probabilities. (ACMMM055)
 Recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases (ACMMM140)