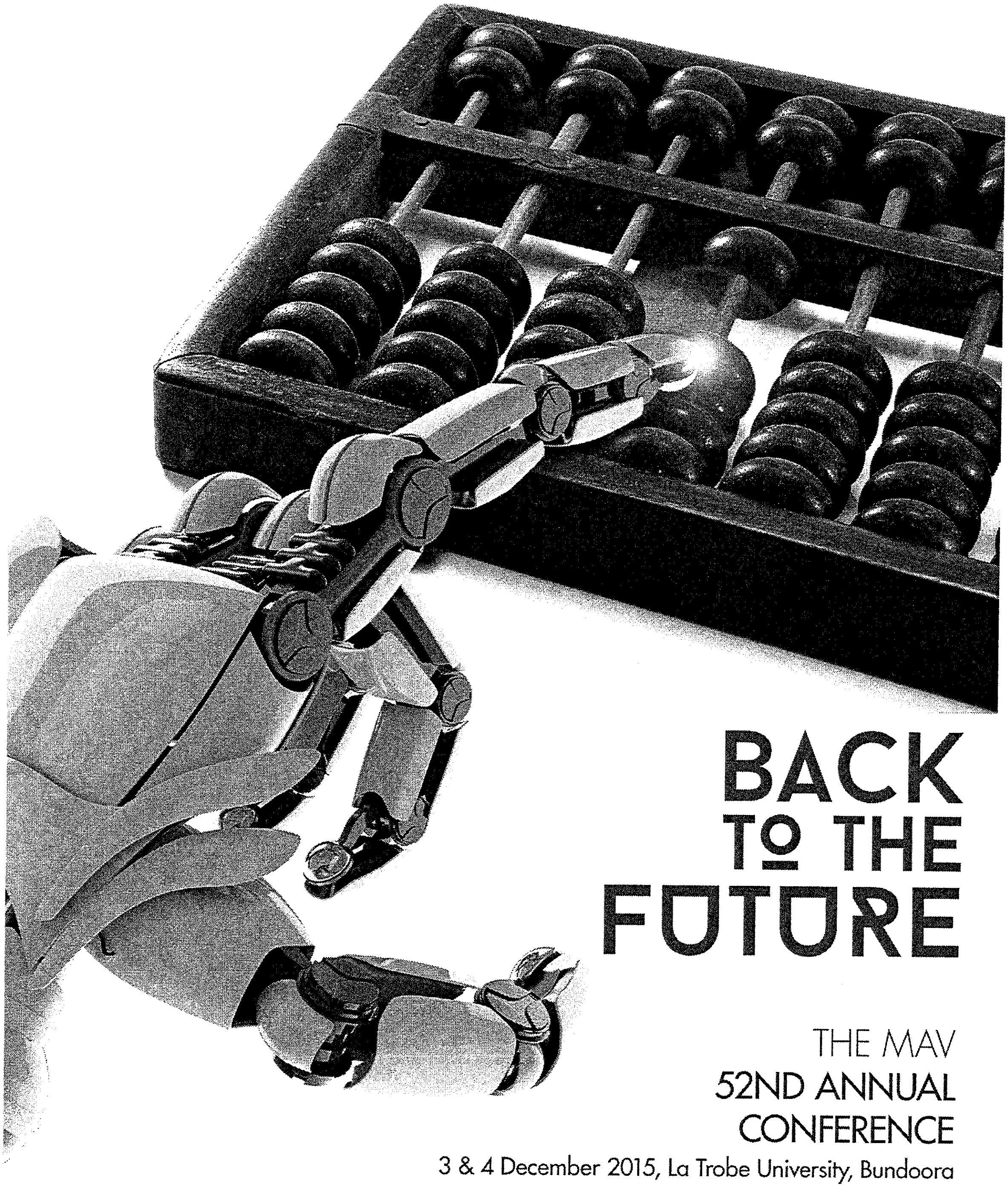
2015

Developing proportional reasoning

Derek Hurrell
University of Notre Dame Australia, derek.hurrell@nd.edu.au

Lorraine Day
University of Notre Dame Australia, lorraine.day@nd.edu.au



BACK TO THE FUTURE

THE MAV
52ND ANNUAL
CONFERENCE

3 & 4 December 2015, La Trobe University, Bundoora

www.mav.vic.edu.au

THE MATHEMATICAL
ASSOCIATION OF VICTORIA

Editorial team:
Dr Dona Martin, Ms Tina Fitzpatrick, Dr Simon Smith, Dr Christopher Lenard, Dr Sharyn Livy, Emeritus Professor Terence Mills, Dr Jonathon MacLellan, Ms Diane Itter, Dr Katherine Seaton, Ms Kym Barbary, Ms Cindy Tassone

Published December 2015 by
The Mathematical Association of Victoria
"Cliveden"
61 Blyth Street
Brunswick VIC 3056
Designed by Idaho Design & Communication

National Library of Australia Cataloguing-in-Publication Data:

ISBN: 978-1-876949-59-4

Should readers wish to contact any of the authors, full addresses are available from the Professional Officer, the Mathematical Association of Victoria.

CONTENTS

***** Double blind reviewed paper**
**** Peer reviewed paper**
*** Summary paper**

DEVELOPING PROPORTIONAL REASONING

Derek Hurrell and Lorraine Day

University of Notre Dame Australia

Proportional reasoning is one of the big mathematical ideas students will encounter. It applies to a wide range of contexts across all of the content strands and is considered a critical concept for success in secondary mathematics. It requires an ability to think multiplicatively and relationally, and is often problematic for students.

Introduction

Proportional reasoning is difficult (Weinberg, 2002). The ability to reason using proportional relationships is a complex form of reasoning that depends on many interconnected ideas and strategies (see *Figure 1.*) developed over an extended period of time. It takes many varied physical experiences to develop an understanding of what a proportional relationship is and then even more time to be able to deal with it abstractly (Cordel & Mason, 2000; Siemon, 2015; Van de Walle, Karp, & Bay-Williams, 2010).

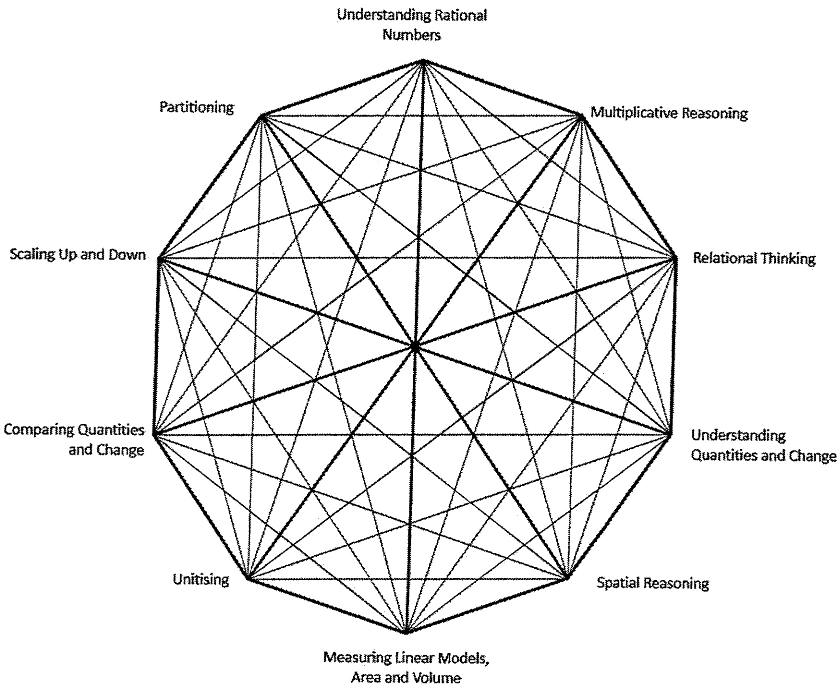


Figure 1. Interconnected concepts of proportional reasoning.
Adapted from Ontario Ministry of Education (2012).

To develop proportional reasoning is not the same as being able to complete an algorithm to solve a proportional problem. "It is important that students learn to solve proportional-reasoning problems using their own intuitive strategies before they are taught the cross-multiplication algorithm. In fact, even after students learn the algorithm, teachers should continue to talk about the students' informal reasoning strategies and how they result in the same answer as the algorithm." (Fazio & Siegler, 2011, p.19). Students use proportional reasoning as soon as they start to look at equivalent fractions and there is much literature which supports the position that students can often 'do' equivalent fractions without necessarily understanding what they are doing and why. In the parlance of the Australian Curriculum, we seem to be able to foster a degree of Fluency but may sacrifice Understanding, Problem Solving and Reasoning in its execution.

When working with proportional reasoning it is not unusual for teachers to teach 'cross multiplication' as a manner of finding a solution. What we would like to propose is that there are ways of developing and understanding concepts of proportionality, so that when cross multiplication is introduced, it is based on understanding.

What is Proportional Reasoning?

According to the work of researchers (Behr, Harel, Post, & Lesh, 1992; Lamon, 2006; Wright, 2005) proportional reasoning is the ability to understand (recognise, explain, make conjectures about, graph etc.) the multiplicative relationship inherent in situations of comparison. Dole (2010) explained that the research has shown that both students and teachers generally have a poor understanding of proportional reasoning.

The capacity to reason proportionally requires careful development (*Figure 2.*). Whilst we do not have the time and space to pursue the issue, we would like to suggest that without the careful development of mathematical thinking, it would be hard to imagine that a student would find the development of proportional reasoning to be possible.

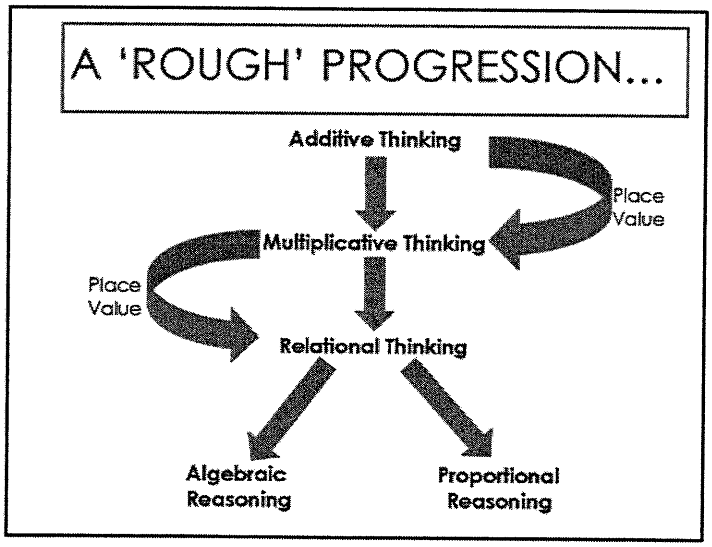


Figure 2. A 'rough' progression of mathematical thinking.

Is Proportional Reasoning Important?

According to Dole, Clarke, Wright and Hilton (2012), proportional reasoning is seen as being fundamental for success in the areas of mathematics and science. Kilpatrick, Swafford and Findell (2001) considered proportional reasoning as being a gateway to higher levels of mathematical success. Both Charles (2005) and Siemon, Bleckly & Neal (2012) included proportional reasoning amongst their 'big ideas'. Charles (2005, p. 10) defined a 'big idea' as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole".

From a perspective beyond school mathematics, proportional reasoning is important for such things as; calculating which of two items represents the best value; adjusting amounts in recipes; working with maps and diagrams; currency conversions and concentrations of mixtures, for example the oil to petrol ratio for two-stroke motors.

Developing Proportional Reasoning

While it is necessary to be cognisant of the fact that proportional reasoning is not an easy mathematical understanding to develop, teachers should celebrate that it is possible to do so. What is needed is a considered way of exposing students to proportional reasoning. In order to develop proportional reasoning we need to:

- Provide students with proportional situations that span a wide range of contexts and relate to their world.
- Offer problems that are both qualitative and quantitative in nature. Qualitative problems encourage students to engage in proportional reasoning without having to manipulate the numbers.
- Help students distinguish between proportional and non-proportional situations.
- Encourage discussion and experimentation in predicting and comparing ratios.
- Help students relate proportional reasoning to what they already know. E.g. connect how unit fractions and unit rates are very similar.
- Recognise that mechanical procedures for solving proportions do not develop proportional reasoning and that students need to be flexible in their thinking and acquire many strategies.

(Ontario Ministry of Education, 2012; Siemon, 2015; Van de Walle, Karp, & Bay-Williams, 2010)

Activities for Developing Proportional Reasoning

Baby in the Car

A very rich activity which can be used to develop proportional reasoning can be found in maths300 (http://maths300.com/). The lesson is called "Baby in the Car" (Lesson 111).

In this lesson the students are invited to consider a scenario which is a 'real world' context which has drastic ramifications, the context of babies being left unattended in poorly ventilated cars. It is a topic which unfortunately is raised too often in light of a baby being left in a car for a period of time and the consequences it can have on the health of that baby. The students are asked to consider why it is that if a baby and an adult are left in a poorly ventilated car, that the adult will perhaps suffer very minor effects whereas for the baby it may be catastrophic.

The students are then invited to work as mathematicians do and create some models for the situation. The implied message here (which should be made overt) is the purpose and power of mathematical modelling. The first model is quite simple and is used to make sure that all students understand the concept of surface area. The students are given one wooden cube and asked about the number of faces and therefore the surface area.

Once it is established that the volume (the number of cubes) is one and that the surface area (the number of exposed faces) is six, there is a discussion generated as to how this is representative of skin-area through which moisture can escape through evaporation, and that this single block represents a baby. The students are then invited to construct a cube which is twice as big in all directions to represent an adult. This then leads to recording the volume and the surface area of the baby and the adult and the surface area to volume ratio is introduced (Table 1).

Table 1 *Relationship between Volume and Surface Area*

	Volume	Surface Area	SA : Vol.
Baby	1	6	6 : 1
Adult	8	24	3 : 1

From this simple model and from this initial set of data, a more complex model is then constructed (model A required a deal of fraction work, which is mitigated in model B) and investigated for surface area (*Figure 3*). The students 'build' a baby and then an adult which is twice as big in every direction. Once again the volume and the surface area of both models is scrutinised.

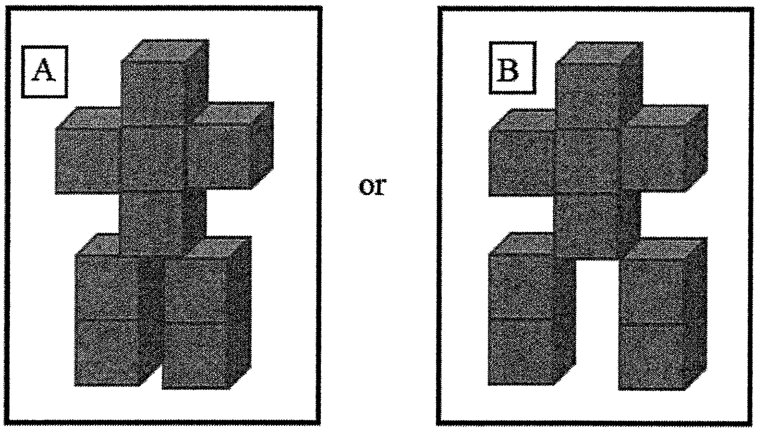


Figure 3. 'Complex' models of a baby constructed from blocks.

Eventually, and if required, an even more realistic model, using cylinders and spheres can be constructed and the data can be further refined.

This activity was employed in a professional learning session conducted for a group of twelve Secondary mathematics teachers, some of whom were teaching in-field (their area of specialisation) and some who were teaching out of field. What may have been of surprise was that on the first activity in building the simple model (one cube baby, eight cube adult) the notion of twice as big, caused more than one of the teachers to pause and consider what it meant. In fact, the first model constructed by some of the participants for the 'adult' was made from four blocks and was twice as big in only two of the three possible dimensions. This was soon amended by the other members of the group, and the need to provide correction and instruction came thick and fast, not unlike the occasions when this activity has been used with school aged students.

The rich discussion that accompanied the construction and calculation of the surface area of the 'complex' adult models highlighted the language of reasoning, problem solving and fluency as groups endeavoured to find the most efficient method in which to calculate the surface area. Several groups checked their calculations by trying a second method, just as a mathematician would.

Orange Soda

In this same professional learning session an activity was conducted that was adopted from the Mathematics Assessment Resource Centre (MARS) (available from http://mathshell.org/ba_mars.htm). This activity was based upon a scenario often used in the teaching of proportion but with the significant 'twist' of introducing manipulative materials in the form of unifx blocks, a consideration that improves accessibility to the task and to an understanding of proportions. The aim of this activity is to determine the relative strength of drinks. The participants were given a series of cards with different representations of proportions (*Figure 4*) and asked to arrange these cards in order from the 'least orangey' to the 'most orangey'.

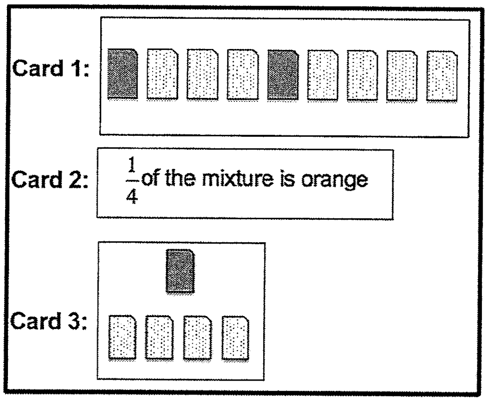


Figure 4. Cards with different representations for proportional reasoning.

In order to make comparisons as to the strength of the different drinks, it was necessary to interpret the various representations, meaning that the task was quickly elevated from a fluency task, to one much richer. Concrete materials, in the form of unifix blocks, helped participants to make sense of the problem which made it more accessible (*Figure 5*). The discussion that arose from the use and comparison of the different representations gave a meaningful and powerful insight to the thinking and reasoning of the participants.

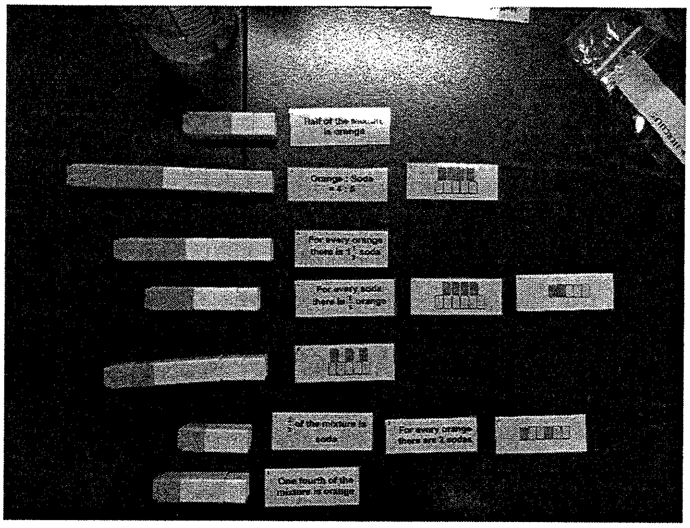


Figure 5. Using concrete materials to make sense of the problem.

This activity generated quite a deal of discussion with regard to the representation which best supported the understanding of proportional reasoning. Whilst some found that using the illustrations (Card A, *Figure 6*) on which to base all of their decisions, others preferred the cards with words and symbols (Card L, *Figure 6*), and others still, found the introduction of the unifix blocks as being the most useful representation. Although the direct question was not asked, no-one incidentally offered the ratio in symbolic form (Card H, *Figure 6*) as being the most 'helpful'.

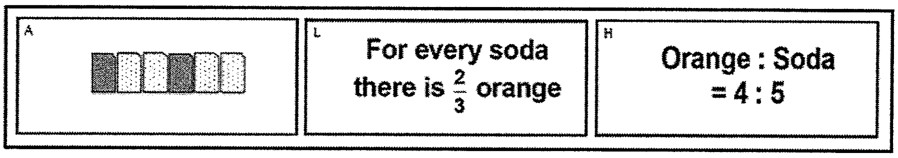


Figure 6. Multiple representations for proportional reasoning activity, Orange Soda.

Conclusion

What the two brief illustrations of activities in this article show is that, long before there is an expectation of 'calculating' proportions, there are opportunities to get students to engage with and develop proportional reasoning in contextually significant ways. The power of the physical act of manipulating the blocks or the unifix and cards and the ensuing conversations cannot and should not be underestimated. Activities such as these allow both the teacher and the students to employ informal and intuitive strategies which can be used to support understanding and reasoning for proportional reasoning which can be applied later when constructing algorithms.

If we relate the two highly accessible activities back to what is needed to be considered to develop proportional reasoning, four of the six features that the research (Ontario Ministry of Education, 2012; Siemon, 2015; Van de Walle, Karp, & Bay-Williams, 2010) indicates as important, are quite apparent. These four features are: providing students with proportional situations that span a wide range of contexts and that relate to the students' world; providing problems which are both qualitative and quantitative in nature; encouraging discussion and experimentation in predicting and comparing ratios; and helping students relate proportional reasoning to mathematics with which they are already familiar.

References

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio and proportion. In D. Grouws (Ed.), *Handbook on research of teaching and learning* (pp. 296-333). New York: McMillan.

Charles, R.I. (2005). Big ideas and understandings as the foundation for early and middle school mathematics. *NCSM Journal of Educational Leadership*, 8(1), 9-24.

Cordel, B., & Mason, R. (2000). *Proportional reasoning*. Fresno, CA: Aims Education Foundation.

Dole, S. (2010). *Making connections to the big ideas in mathematics: Promoting proportional reasoning*. Retrieved from http://research.acer.edu.au/research_conference/RC2010/17august/5/

Dole, S., Clarke, D., Wright, T., & Hilton, G. (2012). Students' proportional reasoning in mathematics and science. In T.Y. Tso (Ed.), *Proceedings of the 36th conference of the International Group for the Psychology of Mathematics Education*, (Vol. 2, pp. 195-202). Taipei, Taiwan: PME.

Fazio, L., & Siegler, R. (2011). *Teaching fractions: Educational practices series – 22*. Belley, France: International Academy of Education.

Kilpatrick, J., Swafford, J., & Findel, B. (Eds.) (2001). *Adding it up*. Washington, DC: National Academy Press.

Lamon, S.J. (2006). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

Mathematics assessment resource service (n.d.). Retrieved from http://mathshell.org/ba_mars.htm

Ontario Ministry of Education. (2012). *Paying attention to proportional reasoning K-12.* Ontario: Queens Printer for Ontario.

Siemon, D. (2015). *Common misunderstandings – levels 5-9*: Proportional reasoning. Retrieved from http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/lvl5prop.aspx

Siemon, D., Bleckly, J., & Neal, D. (2012). Working with the big ideas in number and the Australian curriculum: Mathematics. In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.). *Engaging the Australian national curriculum: Mathematics – Perspectives from the field* (pp. 19-45). Online Publication: Mathematics Education Research Group of Australasia.

The Task Centre Collective. (2015). *maths300.* Retrieved from http//:maths300.com/

Van de Walle, J.A., Karp, K.S., & Bay-Williams, J. M. (2010). *Elementary & middle school mathematics: Teaching developmentally* (7th ed.). Boston: Pearson Education Inc.

Weinberg, S.L. (2002). Proportional reasoning: One problem, many solutions! In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions: 2002 NCTM yearbook* (pp. 138-152). Reston, VA: National Council of Teachers of Mathematics

Wright, V. (2005). Proportional reasoning: Some rational thoughts. In J. Mousley, L. Bragg, & C. Campbell (Eds.), *Mathematics: Celebrating achievement* (pp. 363-375). Melbourne: Mathematical Association of Victoria.