2014

The importance of fractions in being a successful mathematics student

Derek Hurrell
University of Notre Dame Australia, derek.hurrell@nd.edu.au

Lorraine Day
University of Notre Dame Australia, lorraine.day@nd.edu.au

Follow this and additional works at: https://researchonline.nd.edu.au/edu_conference

This conference paper was originally published as:
implementing the new curriculum. I believe, very strongly, that it most certainly is relevant into the future. As mentioned, the strength of the Interview is that it focuses on students' mathematical thinking and the strategies they choose to use, an integral component of the Australian Curriculum: Mathematics. Connections to the new curriculum are clear, so data from this interview will offer teachers support in their planning against the new curriculum and what should be included in their mathematics program for the mathematical growth of their students to continue.

References

THE IMPORTANCE OF FRACTIONS IN BEING A SUCCESSFUL MATHEMATICS STUDENT **

Derek Hurrell and Lorraine Day
University of Notre Dame Australia

Research and experience tells us that fractions are not easy to teach and learn and that a solid conceptual understanding of fractions has far reaching ramifications in the secondary mathematics classroom. A failure to acknowledge and remediate ‘inherited’ challenges to this understanding may result in the further success of students being severely impeded.

The Teaching and Learning of Fractions
The ideas associated with fractions are amongst some of the most complex, yet important, concepts that students encounter (Behr, Lesh, & Post, 1983). One of the reasons that fractions are so important is that they provide the foundations on which other number work, algebraic thinking, and proportional reasoning are built (Booth & Newton, 2012; Brown & Quinn, 2007; Chinappan, 2005; Wu, 2001). Siegler, Fazio, Bailey, and Zhou (2012) stated:
Poor fraction knowledge in elementary school predicts low mathematics achievement and algebra knowledge in high school, even after controlling for general cognitive abilities, knowledge of whole number arithmetic, and family education and income. High school algebra teachers recognize this relation; they rank students’ fraction knowledge as among the largest impediments to success in their course. (p.18)
Apart from these number based aspects of mathematical understanding, the topic of fractions also supports students to make critical conceptual links in such strands as geometry and measurement (Perchembly & Hunting 1996; Siemon, 2003). Other areas, such as probability, statistics, and rates of change require a solid conceptual understanding of fractions.
The Importance of Fractions in Being a Successful Mathematics Student

Fractions are Difficult

Fractions enjoy attention in curricula around the world but have long been documented to cause students difficulties (Anthony & Ding, 2011; Anthony & Wälshaw, 2003; Capraro, 2005; Nunes & Bryant, 2009; Usiskin, 2007; Wu, 2005). Indeed, Smith (2002, p. 3) asserted, “No area of school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios and proportion.” The National Assessment of Educational Progress Report, declared that fractions are “exceedingly difficult for children to master” (Braswell et al., 2001, p. 5). Chapin and Johnson (2000) concluded that “...this complex topic causes more trouble for elementary and middle school students than any other area of mathematics” (p. 73). Indeed Baba and Iwasaki (2003) found that some university students could not understand fractions.

Fractions and the Secondary School Student

In the content descriptors from The Australian Curriculum: Mathematics (ACARA, 2014) the writers stopped using the word fractions after the Year Seven entries. This provides a clear indication that the expectation is that students on reaching secondary school, are already well on their way to a sound understanding of fractions. This expectation would perhaps be reasonable if not for the fact, as previously established, that fractions are hard to teach and to learn.

Data from a study by Clarke, Mitchell, and Roche (2005) determined that when asked to identify what fraction part D was of a circle (see Figure 1) only 43% of the Year Sixes asked (n=323) could do so. When asked to identify the size of Part B of the circle, although 86% of the same students responded with the correct answer, the remaining students, that is, 14% of Year Six students, could not identify a fraction they had been working with for most of their schooling years. The Australian Curriculum: Mathematics (2014) asks Year Twos to “Recognise and interpret common uses of halves, quarters and eighths of shapes and collections (ACMNA033).”

In 1998, Carpenter, Kepner, Corbitt, Lindquist, and Reys reported that when estimating the answer to a problem (see Figure 2), and being asked to select the correct response from a bank of four responses, only 24 percent of 13 year olds chose correctly. From the authors' own experience in working with adults in 2013, when asked to identify an equivalent improper fraction for a mixed numeral (see Figure 3) just over 12% of 224 people could not do so.

Figure 2. Estimating a fraction sum.

![Figure 2](image1)

Figure 3. Equivalent improper fraction.

![Figure 3](image2)

Fractions in the ‘Real’ World

Elizabeth Green (2014) wrote an article for the New York Times which was adapted from her soon to be published book. In this article she wrote:

One of the most vivid arithmetic failings displayed by Americans occurred in the early 1980s, when the A&W restaurant chain released a new hamburger to rival the McDonald's Quarter Pounder. With a third-pound of beef, the A&W burger had more meat than the Quarter Pounder; in taste tests, customers preferred A&W's burger. And it was less expensive. A lavish A&W television and radio marketing campaign cited these benefits. Yet instead of leaping at the great value, customers snubbed it.

Only when the company held customer focus groups did it become clear why. The Third Pounder presented the American public with a test in fractions. And we failed. Misunderstanding the value of one-third, customers believed they were being overcharged. Why, they asked the researchers, should they pay the same amount for a third of a pound of
meat as they did for a quarter-pound of meat at McDonald's. The ‘$4$’ in
‘$\frac{4}{3}$’ larger than the ‘$3$’ in ‘$\frac{3}{3}$’ led them astray. (p. 5)

Quite clearly then, many students are not arriving into the secondary mathematics
classroom with a well-developed understanding of fractions, and neither are they
leaving with one. If we want the students to be a position to successfully work with
fractions then an acceptance of the need to determine what understandings the students
actually have, and then reach back into the curriculum and find the ‘big’ ideas which will help
the students to progress are necessary. Once the ‘big’ ideas are identified it is also necessary to
determine the pedagogies which are suitable for the secondary student.

Pedagogical Considerations

Walker (2004) wrote about understanding and recognising the nature of early adolescents
and how this should inform the school curriculum. It is a recognition that although the
mathematical content that needs to be covered is developmentally suitable for primary school
aged students, the pedagogy adopted needs to be cognisant of the psychological needs of the
adolescent. Carmichael and Hay (2008) argued that these psychological needs are competence,
autonomy (being able to have some choice in what they do), and social-relatedness (personal
involvement). Consequently, a guided discovery method (Dinham & Rowe, 2008) (which
these authors might argue as being a ‘true’ representation of social-constructivism) which
involves hands-on activities, discussion, cooperative learning, and co-construction of the
learning seems highly appropriate pedagogy.

One approach that can be used is based upon C.R.A. (Concrete, Representational,
Abstract) sequence of instruction. Although much of the research shows the efficacy
of this approach with students who have learning difficulties (e.g. Allsopp, Kyger, & Lovin,
2007; Flores, 2009), the work of Witzel, Mercer and Miller (2003) supported the use of
C.R.A. with ‘mainstream’ students when they showed that algebra students from non-
nspecial needs backgrounds benefited more through the use of C.R.A. than they did from a
more traditional approach.

Examples of the C.R.A. Approach with Fractions

A Chocolate Dilemma

One activity which has been used to demonstrate fractions as quotients (division) is an
activity we refer to as “A Chocolate Dilemma.” In this activity there are three chairs at the
front of the class and 10 students standing outside of the classroom. On chair one there is
one bar of chocolate, chair two has two bars of chocolate and chair three has three bars of
chocolate. The bars of chocolate are all the same size. The ten students are invited into the
room one at a time and choose to stand behind whichever chair they wish, knowing that
when all of the students are standing behind the chair of their choice, they will share the
chocolate on that chair. The aim is for the students to maximise the amount of chocolate they
can get.

For instance, if two students stand behind chair one they will each receive a half of the
bar. If five students stand behind the chair with two bars they will receive two-fifths of a
bar each. The remaining three students behind the chair with three bars will end up with
one bar each. In this example the people who stood behind the chair with three bars are the
happiest! The number of chairs, bars of chocolate and students can be varied to suit and to
extend the potential of the problem. Using the students themselves provides a kinesthetic
experience which we interpret as being as ‘concrete’ as you can get.

Once the students have experienced physically being part of the activity, if required, the
activity can then be replicated through modelling with three pieces of paper to
represent chairs, six counters of one colour to represent the chocolate bars and 10 counters
of another colour to represent the students, or as purely a diagrammatic representation.
The diagrammatic form can then be made abstract through the use of symbols, where the
denominator is the number of students and the numerator is the number of chocolate bars.

Fraction Estimation

Another activity which employs the C.R.A. approach effectively is a task about
estimating fractions. It is adapted from maths@300 (Education Services Australia, 2010). In
this activity the concrete element consists of ropes of different lengths and some household
pegs. Students are asked to estimate how far along the rope various fractions are situated
and they check their estimations by folding the rope into equal partitions. Different lengths
of rope are used so that the students recognise that they need to identify what constitutes
the whole before they divide it up into equal parts. An important part of this task is the
ability of the students to visualise the whole and mentally divide it up into equal parts, then
count the number of parts they need. They soon become quite adept at folding the rope
into equal parts to check their results.

The recording of this activity is done by representing the rope as a line segment and
marking on the line where the fraction is positioned. Different length ropes are represented
by shorter or longer line segments to reinforce the idea that the size of the partition is
dependent on the size of the whole that is being partitioned. Once students have had lots of
experience with the concrete activity and representing them on paper, it is time to move to
the excellent software that is available for this activity.

The Fractions Estimation software in the maths300 package (Education Services Australia, 2010) has three different representations for fraction estimations, strips (which relate to the rope), towers, and pies. There is an option to include different fractions depending on the level at which students are working. One of the beauties is that there is no pre-partitioning so the students have to visualise how to break the whole up into equal parts. Once students are happy with their choice, the computer then partitions the whole into equal parts so that the students can see how close they were to the actual fraction (see Figure 4). There is an option to work backwards which uses all three of the previous representations. For this option students need to enter either the numerator or the denominator of the fraction that is pictured (see Figure 5), which is quite abstract. Four other options that relate fractions to decimal and percentages are included to encourage students to be able to move between them flexibly.

![Fraction strip](image1)

**Figure 4.** Fraction strip.

![Fraction number](image2)

**Figure 5.** Fraction number.

These two examples demonstrate appropriate pedagogical approaches for use with students in the middle years of schooling to assist them to develop a conceptual understanding of fractions that is so important if they are to succeed in secondary school mathematics.

Some Concluding Remarks

The teaching and learning of fractions is not easy, and the ramifications of the students not developing a sound understanding can be far reaching. Simply assuming that the students who are entering the secondary mathematics classroom have the required understanding to allow them to employ their fraction knowledge in other areas of the mathematics curriculum is perhaps unwise. It could be the case that some time devoted to the conceptual development of fractions will prove to be a wise investment. There could truly be a solid argument that what could be considered as a step back in the curriculum, will actually mean that later the strides will be greater. We would very much like to suggest that in some instances a re-examination of the pedagogy employed to teach fractions may prove to be fruitful.

References


The Importance of Fractions in Being a Successful Mathematics Student


---


