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The reasoning proficiency

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If students are to reason mathematically, they need to be engaged in mathematically rich, investigative tasks that allow them to explain their thinking, justify the strategies they use and the conclusions they reach, and adapt the known to the unknown. The importance of contextualised learning should be highlighted so students may be encouraged to transfer their learning from one context to another, explain their choices within a context, and compare and contrast related ideas. The reasoning proficiency naturally interrelates with the understanding, problem solving and fluency proficiencies.

The Australian Curriculum: Mathematics (ACM) (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2013) is not a syllabus. It does more than tell teachers what to teach, it describes how the learning environment needs to be constructed. Through the proficiencies it articulates the constructivist basis that is expected, and gives direction on how this can be achieved. The ACM is a document about the holistic development of numeracy capabilities rather than just a set of essential skills and knowledge (ACARA, 2013).

In 2010, ACARA proposed that the ACM should be constructed around three content areas; Measurement and Geometry, Number and Algebra, and Statistics and Probability. Sullivan (2012) calls these the 'nouns' of the curriculum. ACARA further proposed there should be four proficiency strands; Understanding, Problem Solving, Fluency and Reasoning, and that these proficiencies should be enacted when learning mathematics not just when applying it (Askew, 2012b). Sullivan (2012) refers to the proficiencies as being the 'verbs' of the curriculum. The importance of these 'verbs' is echoed by Burns (2012), who states that the Standards for Mathematical Practice, the US equivalent of the proficiency strands, should be at the forefront of mathematics teaching and learning.

The proficiencies were adapted from the work of Kilpatrick, Swafford and Findell (National Research Council, 2001) who articulated five strands:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (p.5)

They saw the strands as being interdependent and interwoven and in fact illustrated it as such. This 'weave' was considered essential.

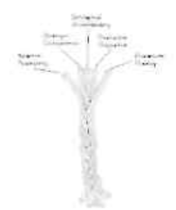

Figure 1. Intertwined strands of proficiency (National Research Council, 2001, p. 5)

Prior to the ACM, states such as Western Australia employed their own curricula and included something referred to as 'Working Mathematically' (Curriculum Council, 1998). Working Mathematically had the same essential elements as the proficiencies, but laboured under the issue of being seen in the same vein as the content strands. In fact rather than being something which was incorporated into all of the other strands as intended, it was often seen as another lesson and timetabled as 'problem solving'. Although this was not the intent, for many teachers it became the practice.

Figure 2. Working Mathematically (Curriculum Council, 1998, p. 182)

The proficiencies are positioned and emphasised to try to avoid the difficulties encountered in trying to get the Working Mathematically as a natural part of teachers' views when constructing every mathematics lesson. They are intended to be an integral part of the curriculum rather than added extras, as they form the foundation on which to build the content areas. This paper will investigate the Reasoning Proficiency in particular, although it should be noted that all four are naturally intertwined with each other with each one reliant on the others to become evident.

Reasoning

Lannin, Ellis and Elliot (2010) explain that mathematical reasoning is an evolving process. The development of a classroom culture where the quality of the mathematical justification that is provided determines the correctness of a response and students are encouraged to take risks by sharing their reasoning (Clarke, Roche & van der Schans, 2012). A climate where students feel safe to share correct and incorrect ideas allows students to develop an "increasingly sophisticated capacity for logical thought and actions" (ACARA, 2013, p. 6).

Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices (ACARA, 2013, p. 6)

The teaching and learning of mathematics is about interactions between the students, the teacher and the mathematics (National Research Council, 2001). In order to facilitate these interactions, explicit links between written and spoken mathematics, problem settings and students' solution methods should be made. To this end students and teachers should be encouraged to make conjectures, analyse, prove, investigate, explain, infer, justify, and develop and evaluate arguments (ACARA, 2013; Lannin et al., 2011).

Mathematical Discourse

Small and whole-group settings allow students to share and debate the reasons why they believe a statement to be true and hold all students accountable for understanding (Clarke et al., 2012). Askew (2012a) talks about private talk and public conversations, where the former allows students to engage with the mathematics being discussed and share their thinking in a safe setting. The small group setting provides students with an opportunity to rehearse and refine the ideas that they might share more widely. The public

conversation provides a platform for students and teachers to share and build on ideas, emphasise and model mathematical reasoning and problem solving (Kilpatrick et al., 2001).

In order for this to be achieved, mathematical discourse should be well planned to develop students' understanding of key ideas. It should not be confined just to answers, but should include discussion about connections to other problems, multiple representations and solution strategies, justification, argumentation and generalisation (Kilpatrick et al., 2001). It should not be restricted to 'show and tell' or only 'checking for understanding', although the latter may be one of the goals of the conversation.

Rich Investigative Mathematical Tasks

The two activities that assist to improve mathematical reasoning are the engagement in mathematical thinking by solving problems and investigating mathematical situations, and reflecting on the experience (Siemon et al., 2011). Mathematically rich investigative tasks provide the opportunity to transform school mathematics from a collection of rules without meaning to a vibrant, connected subject where there is the opportunity to understand, explore and reason about mathematical concepts. Kilpatrick et al., (2001) suggest that mathematical tasks are central to students' learning, as they shape the manner in which students view the mathematics as well as providing the opportunity for students to learn the mathematics. They go on to point out that the cognitive demand of tasks can vary significantly.

The best rich investigative tasks allow students to work like mathematicians, and see others working like mathematicians, by allowing them to get started and explore, while still providing opportunities for challenge, cognitive demand and extension. They cater for student diversity as a result of their openness, encourage a variety of learning styles and expect students to explain their thinking. Within meaningful or intriguing contexts they develop thinking, reasoning and communication skills while seeking deep understandings. They highlight the interdisciplinary connections both within and outside of mathematics and use information communication technologies effectively (Day, 2012; Lovitt & Clarke, 2011).

Kilpatrick et al. (2001) contend that problem solving should be where all of the mathematical proficiencies come together, to provide an avenue for students to weave all of the proficiencies together and allow teachers to assess student performance on all of the proficiency strands. The use of rich investigative tasks provides an opportunity for mathematical teaching and learning to become more concerned with thinking, reasoning, communication and justification rather than facts, skills and rules without meaning. This is the reason for the importance attached to the mathematical proficiencies within the Australian Curriculum: Mathematics.

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