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A FRAMEWORK FOR CATEGORISING MOBILE APPLICATIONS IN MATHEMATICS EDUCATION

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ABSTRACT

New learning technologies have brought fresh challenges to teachers in selecting appropriate educational resources, particularly in regards to mobile devices. There are an impressive number of educational mobile learning applications, more commonly known as apps, that teachers need to understand before integrating them into the classroom. To make this process more effective, educational apps can be categorised according to their specific role in the teaching and learning of mathematics along with their media richness. A framework of nine distinct categories of apps emerged, grouped into three main clusters, namely, investigative, productivity and instructive tools. The framework was validated with examples from a K-12 context.

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THE CASE FOR MOBILE LEARNING

The advent of mobile learning technologies into teaching and learning has brought both new possibilities and challenges to teachers. Mobile learning represents a technology that is ubiquitous in nature, wireless, highly portable and endowed with multimedia capabilities bringing a new dimension to curriculum delivery (Littlejohn & Pegler, 2007; Melhuish & Falloon, 2010).

The last few years have witnessed an impressive increase in the use of mobile learning technologies in schools. Although created for a non-educational environment, tools like tablets and smartphones soon made their way into the classroom. These devices have attracted interest from the educational community mainly due to their versatile gaming capabilities. They are very appealing, particularly for the current *e-Generation*, also coined “digital natives”, who were born into a society already using mobile devices and online technology (Al-khamaysah, Zmijewska, Lawrence, & Culjak, 2007). According to the 2012 Horizon Report (Johnson, Adams, & Cummins, 2012, p. 4):

> Mobile devices and apps are increasingly valued as important learning tools in K-12. Apps in particular are the fastest growing dimension of the mobile space in the K-12 sector right now, with impacts on virtually every aspect of informal life, and increasingly, potential in almost every academic discipline.

Mobile learning allows students to engage in problem-solving based learning activities. Mobile technology, like tablets and smartphones, also permit students to work on tasks that are goal oriented and open-ended with a strong gaming component (Murray & Olcese, 2011). In addition, mobile learning devices empower students to develop their own understanding through active involvement and sense-making. This is achieved through investigations and practical and hands-on activities within authentic contexts. Furthermore, learning experiences like digital simulations or manipulations have the capacity to bring interactivity thus enhancing cognitive and affective processes (Schuck, Aubusson, Kearney & Burden, 2010). One of the great advantages of mobile devices is that students learn to learn in new contexts such as visiting “virtual” museums or galleries or within different cultural and social settings. Opportunities for collaborative learning also emerge as students share their work through online media (Taylor, 2003).

MOBILE LEARNING IN MATHEMATICS EDUCATION

There is evidence that teachers in New South Wales schools are not confident using mobile devices (e.g., tablets, smartphone) in teaching mathematics (Handal, Campbell, Cavanagh, Petocz, & Kelly, 2012). Lack of professional development and resources seem to be major factors in hindering the implementation of mobile technology in schools. In addition, doubts have been raised about the quality of some educational software programs, which seem to be developed from a commercial perspective with little pedagogical consideration (Shuler, 2012).

Application software, also known as an application or an app, is computer software designed to assist users in performing single or various related tasks many with the purpose of creating learning. By the end of 2012 there were over 75,000 applications for the iPad and iPhone market categorised under the “Education” heading. There are nearly one million apps approved for the US Apple iTunes App Store (148AppsBiz, 2012).

Various models for categorising mathematics education software according to task orientation have been published. Among them, Handal and Herrington (2003) published a taxonomy categorising computer-based learning resources into six groups namely, drills, tutorials, games, simulations, hypermedia tools and open learning environments. Placed on a cognitive engagement continuum, drill and practice resources were situated at the lowest end of the spectrum because they are mostly rote learning based. However, those tools with an orientation towards simulations and open tools represented a transition towards more constructivist-oriented pedagogies. Similarly, Kurz, Middleton, and Yanik (2005) published a taxonomy for school mathematics software based on a tool-purpose conception. It included four main clusters, namely, review and practice, general, specific, environment and communication. These taxonomies were designed to organise educational technology which did not include mobile learning applications. The former add new perspectives to the learning experience such as touch screen affordances (Watlington, 2001) highlighting the need for re-examining current frameworks.

In a more general note, Goodwin (2012) conceptualised a framework to understand educational apps for the type of task embedded in the application. Apps were categorised according to their instructional design depending on any of three instructional roles addressed (explorative, productivity and instructive roles). According to Goodwin, constructive apps are useful for creating their own resources and/or content while manipulative apps assist in guided discovery and experimentation with a fixed framework. In turn, instructive apps engage learners in ‘drill-and-practice’ game-based activities.

The concept of media richness is associated to the degree of ambiguity and uncertainty that the software conveys to make the learning task structure more open-ended than rigidly structured (Shepherd & Martz, 2006). Richer rather than leaner media, when elements of high equivocality are embedded, are believed to efficiently facilitate problem solving and higher order thinking. Kerres and Witt (2003, p. 107) define equivocal tasks as those “with multiple and possibly conflicting interpretations to the available information” and argue that unequivocal tasks like ‘learning facts’ do not necessarily require ‘media’ because of their low ambiguity. Some criteria are offered in the literature as to what rich media means in the design of educationally software within a constructivist perspective of teaching and learning (Roblyer, 2003).

In their taxonomies Handal and Herrington (2003) and Kurz, Middleton and Yanik (2005) suggest a number of criteria in the areas of pedagogical affordances and operational affordances that emerge as critical features of media richness in digital tasks. Such criteria represent the competence of the app to facilitate increasing levels of problem solving and cognitive open-endedness. These task richness features would include: (a) levels of interactivity between the learner and the app, (b) capacity to foster deep learning, (c) learner’s control, engagement and feedback, (d) underpinning instructional models, (e) recognition of different learning styles and (f) opportunities for collaboration (Handal & Herrington, 2003).

In general, the above features define the app media richness and the learner’s role characterising the level of command that the student has over the application. Some applications afford the learner most of the learning control, while in others the student’s role is more passive. A proactive interaction fosters learner construction and autonomous learning while a reactive interaction provides response opportunities to specific stimuli or given questions. An open-ended task structure encourages discovery and greater interactivity rather than offering rigid pathways such as menus and branches (Handal & Herrington, 2003).

METHODOLOGY

The authors evaluated over 100 apps for mathematics at the primary and secondary level available from the Apple iTunes Store. The apps were selected from a previous study evaluating mobile learning resources (Learning Exchange, 2011) and from the highest rated apps showcased in the mathematics education category at the Apple Store.

The chosen apps were categorised identifying mainly their instructional role. The categorisation was guided by the taxonomies for appraising mathematics education related software by their intended instructional role as outlined by Handal and Herrington (2003) and Kurz, Middleton and Yanik (2005) including Goodwin (2012). As the examination unfolded using content analysis (Neuendor, 2001) more specific categories began to emerge as detailed in the next section. Questions guiding this analysis focused on the kind of learning activities associated with the app, the instructional experiences supported by the app and their media richness, the anticipated levels of cognitive involvement and users' control over their learning. Importantly, the apps were appraised in terms of how they contribute to learning experiences that cannot be provided otherwise in the standard curriculum. For instance, a simulator was found to be related to students' exploration of mathematical content by having an open-ended structure that allows them to predict physical outcomes that cannot be replicated in class.

As a result the authors identified nine categories within primary and secondary education contexts. These categories are not mutually exclusive but overlap in various dimensions. For example, a guided discovery app may have tutorial elements or an observing app might embed simulation characteristics. Similarly, a drawing/graph made by a student can be shared with peers through apps resembling a productivity tool.

CATEGORIES OF MATHEMATICS APPS BY INSTRUCTIONAL ROLE

The nine specific categories are outlined below and described along with examples in the next pages. These categories are as follows: (1) emulation, (2) simulation, (3) guided discovery, (4) measurement, (5) drawing/graphing, (6) composing, (7) informative, (8) drill and practice, and (9) tutorial apps.

SIMULATION APPS

Simulation apps allow students to study complex mathematical ideas by modelling real-life situations. This software is usually employed for specific learning areas where the physical environment cannot be replicated due to complexity, lengthy timeframes or hazardous conditions. Learners are able to link abstract mathematical concepts to concrete situations by modifying variables and observing changes in the general model in a non-static environment. Some of these apps present a gallery of shapes, objects and emulators where students can explore mathematical ideas with pre-determined content. Examples of simulation apps in the Apple store include ChanceLab and Projectile Calculator Vectors.

EMULATOR APPS

Emulators are similar to simulators except that the former replicate a mathematical system in real-time. Emulators are also different from simulators because the former are more conceptually oriented rather than procedural emphasizing how a system works as a whole mechanism. Because they are self-contained within an overall structure, emulators are used to model systems with higher fidelity and approximation. Emulators differ from simulators because manipulation is restricted to data and to a fixed structure supplied by the software. Examples of those apps in the Apple store include iCross, Trigonir, Observatory and Think 3D.

GUIDED DISCOVERY APPS

Guided discovery apps allow for exploration and experimentation within a pre-determined framework. Students visually investigate mathematical concepts and relationships usually by dragging and moving objects as they look for patterns and make generalizations. Manipulating apps demands high cognitive involvement from the learner while the teacher can assume a more facilitator-oriented role. Some guided discovery apps permit higher cognitive engagement by allowing students to create their own pictorial constructs on the touch screen. Students solve problems and develop their own unique structures by strategically planning solutions, or through intuition and the use of trial-and-error strategies. Examples of guided discovery apps in the Apple store include Sketchpad Explorer, Virtual Manipulatives, Move the Turtle, Think 3D and Weighing.

MEASUREMENT APPS
Measurement apps empower learners to make accurate measurements through multimedia tools. These might include dynamic rulers, protractors, clinometers, theodolites, stopwatches, among others. Examples of these apps in the Apple store include Protractor, Stopwatch, iRuler HD and Clinometer.

DRAWING/GRAPHING APPS
These apps provide open tools to represent mathematical ideas graphically. Drawing/graphing apps are open tools that allow learners to create their own visual content, facilitating pictorial illustration of geometrical concepts. Drawing/calculating apps permit students to accurately represent complex mathematical constructs at fast speed, freeing them from tedious calculation, tabulation and drawing. There is an immediate association among graphical, numerical and/or tabular data in representing graphs, curves, shapes and objects. Examples of those apps in the Apple store include GeoBoard, iDraw, Graphic Calculator HD and Quick Graph.

COMPOSING APPS
Composing apps allow users to create their own digital content or multimedia artifacts. Composing apps are not always specific to mathematics as they relate to teaching and learning across the curriculum. For instance, students can create an animated slideshow with sound, document a real-life investigation audio-visually, create a video tutorial, or produce a field-based research project rather than submitting assignments on paper format. Some of these apps allow users to create collaborative resources with audio, images and motion that can be shared online. Examples of composing apps in the Apple store include iMovies, SurveyBoy, ShowMe Interactive Whiteboard and SonicPics.

INFORMATIVE APPS
Informative apps present one-way mathematical information through a combination of text, images, video, sound and/or animations. Examples include dictionaries and lists of formulas. Rich information can be presented via a structured format and linear sequence, or through hyper linkable nodes of information enhancing the navigation experience. Real-time observational data can also be retrieved on attributes such as temperature, longitude, latitude and atmospheric pressure of a selected geographical location. Examples of informative apps in the Apple store include Math Dictionary, iFormulas, Google Earth and Maths Your Teacher.

DRILL AND PRACTICE APPS
Drill and practice apps provide routine exercises to reinforce basic skills and procedures within a very structured format. Most of the problems provided on these apps are quite homogenous and repetitive in nature. Summative feedback is usually provided through short phrases or audio/visual elements. The feedback focuses on the correctness of the answer rather than on the problem-solving process itself. Drill and practice apps can be used for individualized and differentiated instruction. Examples of drill and practice apps in the Apple store include Fractions, Algebra, Math Paradise and Math Hero.

TUTORIAL APPS
Tutorial apps are similar to drill and practice apps but they also provide formative feedback about the problem-solving process to the user. The application assesses the student's response, and feedback or remediation is given accordingly. Ideally, a tutorial would sequentially present information, provide problem-solving guidance, allow for practice by the students and assess performance. Examples of tutorial apps in the Apple store include Mathemagics, Wolfram Algebra, Algebra Touch and Hands-On Equations.

THE FRAMEWORK
The nine categories were further classified into three main clusters namely, explorative, productivity and instructive tools. This classification reflects the general arrangement of constructive, manipulative and instructive apps proposed by Goodwin (2012). Similarly, the proposed framework incorporated the concept of media richness for each instructional role as described earlier. Table 1 outlines the new framework in a mathematics education context.

Table 1: Taxonomy of Mathematics Education Apps by Instructional Role and Media Richness

Instructional Role	Media Richness		
	Lower	In between	Higher
Explorative	Emulators	Simulators	Guided discovery
Productivity	Measurement	Drawing/graphing	Composing
Instructive	Informative	Drill and practice	Tutorials

Media richness is represented horizontally on a continuum from lower to higher intellectual involvement. It represents the capacity of the app to provide high problem solving and low level of prescription. For example, guided discovery apps reflect increasing levels of reflection and problem solving. Likewise, the instructional role continuum runs vertically from higher to higher open-endedness, e.g., instructive apps are more prescriptive than their explorative counterparts.

Reading throughout the framework permits us to look at the two constructs (namely, app instructional role and media richness) in an integrated manner. For instance, *informative* apps show the lowest level of task richness as opposed to *Guided Discovery* tools, which situate themselves at the highest level of the spectrum. At that level students operate more as creators rather than consumers of knowledge. In general, the explorative cluster seems to provide the highest cognitive involvement and open-endedness.

CONCLUSIONS

Nine specific categories of educational mathematics apps were identified according to their instructional role and examples were provided. The nine categories were further classified into three main clusters namely, explorative, productivity and instructive tools. This classification reflects the general arrangement of constructive, manipulative and instructive apps proposed by Goodwin (2012). According to Goodwin, constructive apps are useful for creating their own resources and/or content while manipulative apps assist in guided discovery and experimentation with a fixed framework. In turn, instructive apps engage learners in ‘drill-and-practice’ game-based activities.

In the present study Goodwin’s categorization was amplified to embed the concept of instructional role in the context of mathematics education and media richness. The three modified categories were labeled explorative, productivity and instructive apps clusters. As described in the paper, explorative apps characterized apps to explore and demonstrate mathematical models or concepts through manipulating objects that mimic complex physical situations (e.g., animations, simulations). Productivity apps allow students to measure and graphically represent objects or concepts in two or three dimensions, collect data, make complex calculations, or create multimedia materials. Finally, productivity apps engages students in practicing content through drill exercises, acquiring new skills through questions and answers (tutorials), or retrieving factual information only.

Each of the three clusters informs its own mobile learning pedagogy. Explorative tools require teachers to act in a more facilitating role while ensuring that students do not construct inaccuracies while pursuing their own investigations. Teaching about product design will be a pedagogical skill required with working with higher order productivity tools. Given the mostly routine nature of drill and tutorials teachers might need to emphasize not only teaching about problem solving but also teaching for problem solving. Furthermore, engaging students in critically analyzing feedback and information provided by mobile learning apps as text, image or animations stands as a significant digital literacy teaching commitment in the modern mathematics curriculum.

In the process of selecting apps, teachers would find the process of understanding their instructional role very valuable, within the context of the above taxonomy, as a number of these applications bear little instructional value. With the ever increasing number of apps entering the market it is important to keep a watchful but enthusiastic eye on new developments for mobile learning in mathematics learning and teaching.

The proposed framework is also useful for demonstrating the relevance of mobile learning throughout a broad range of learning processes. In a way, the framework can be extended to understand the role of digital resources in mathematics education in general. Like the categorisations proposed by Handal and Herrington (2003), Kurz, Middleton and Yanik (2005) and Goodwin (2012), all frameworks are mutating constructs due to the transient nature of their theoretical assumptions (Kuhn, 1962).

Educational technologies are also subject to constant theoretical transformations and are more prone to inherit more instability. It is therefore highly predictable that futures advances in human-machine relations, as Facer and Sandford (2010) have proposed, let alone their synergies with mathematics education pedagogies, will undoubtedly keep re-arranging conceptualizations on a continual basis.

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