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A teacher's journey through multiplicative thinking with early secondary students

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As part of the Reframing Mathematical Futures national project four WA secondary schools embarked on a journey to help students to become more able multiplicative thinkers, so that many areas of secondary mathematics would become more accessible to them. This involved targeted teaching within the regular mathematics classes. This paper is about the journey of one of these teachers.

Reframing Mathematical Futures

The Australian Department of Education and Training under the Mathematics and Science Partnership Program (AMSPP) awarded two national grants, a one-year (2013) Priority Project, *Reframing Mathematical Futures* (RMF), and a three-year (2014-2017) Competitive Grant Project, *Reframing Mathematical Futures II* (RMFII), to Prof. Dianne Siemon from RMIT University who worked with a team of researchers across Australia. The RMF Project had the aim "to improve student outcomes in relation to multiplicative thinking and proportional reasoning in Years 7 to 10" (Siemon, 2014, personal communication). Multiplicative thinking was selected for this Priority Project as it had previously been identified as the area most responsible for the eight-year range in mathematical understanding from Years 5 to 9 (Siemon, Virgona, & Corneille, 2001). *The Scaffolding Numeracy in the Middle Years Linkage Project* produced a research-based Learning and Assessment Framework for Multiplicative Thinking, two formative test options and teaching resources (Siemon, Breed, Dole, Izard, & Virgona, 2006) and it was decided that these resources would be used in the RMF Project. It was seen as important for schools who joined the project in the RMFII Project stage to complete the RMF component before moving onto the RMFII stage, as the second stage of the project built upon the multiplicative thinking component.

Background Information

Kalamunda Senior High School is a suburban school with 1100 students, in an area of Perth with a number of independent schools surrounding it. The school has a focus on Outdoor Education and the Arts, with a gifted and talented program in each of these areas. A nearby school has a gifted and talented program in Mathematics and Science. Kalamunda SHS has an Index of Community Socio-educational Advantage (ICSEA) value of 1005. Students seem content to achieve a pass mark. NAPLAN results were steady, but had not achieved the school's Business Plan target of exceeding the Australian mean in the past three years. It has long been as issue at the school that Year 10 grades in mathematics appear to fall without a well understood reason and there is a poor uptake of certified mathematics courses in Years 11 and 12.

Teachers at the school have identified that many students experience problems with mathematics and have had to cater for a large range of abilities for a number of years. The strategic approach to this was to stream students, typically starting in Year 9, into three ability groups following different courses of work. There is much research (e.g. Brophy, 1983; Hattie, 2012; Zevenbergen, 2003) to suggest that this is an unproductive strategy. Classroom management in the lower groups was generally an issue, with lower groups seldom provided with anything other than simplified worksheets, repetitive and memory based to try to improve the students' understanding. Teachers often felt powerless to know what to do but to go back to easier material. Learning Support staff were always willing to assist, but often endeavor to do so without adequate materials or development time.

The Journey

Part of the attraction of involvement in the RMF Project was access and training in using diagnostic assessments and teaching materials in a targeted approach. As the nominated teacher in charge, I was trained over three days in the issues and difficulties faced by students with conceptual misunderstandings in numeracy and how to move students from being additive to multiplicative thinkers. I had not understood that many students would never achieve multiplicative thinking without specific intervention (Breed, 2011). When I returned to school I updated the team on the information and options available and we agreed to use the RMF materials on half the cohort in Year 7 and in Year 8. Together we predicted that between 20 to 25% of students would have difficulty with multiplicative thinking and we expected a five-year range in our Year 8 groups.

Diagnostic Results

After the initial diagnostic assessment, we found that our school was normal! Our students had a range from zone 1 to zone 8 in Year 8 and ranged from zone 1 to zone 7 in Year 7 (see Figures 1 and 2). Around 44 % of our students were below the multiplicative thinking threshold (lower than zone 4), approximately 32% of the students were at the threshold (zones 4 and 5) and 23% were multiplicative thinkers (above zone 5).

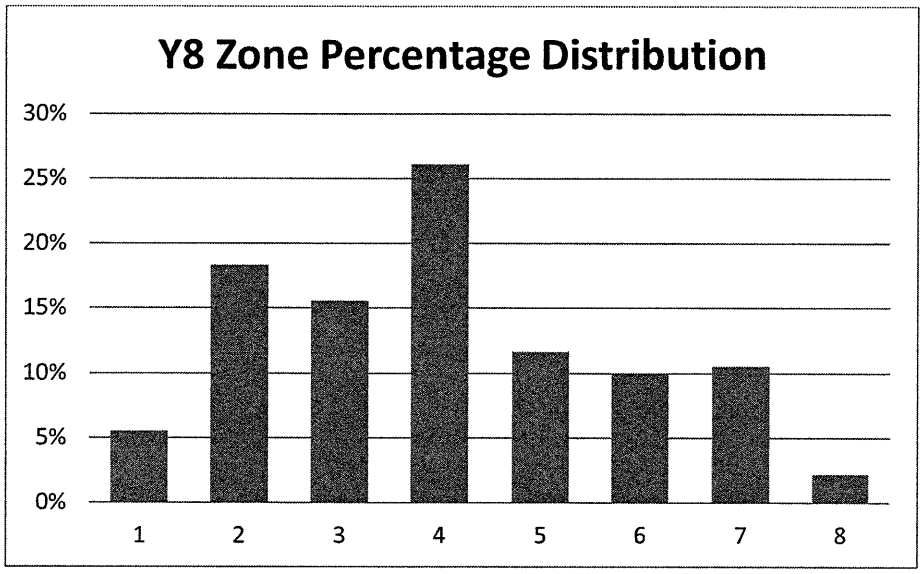

Figure 1. LAF zone distribution for Year 8 students.

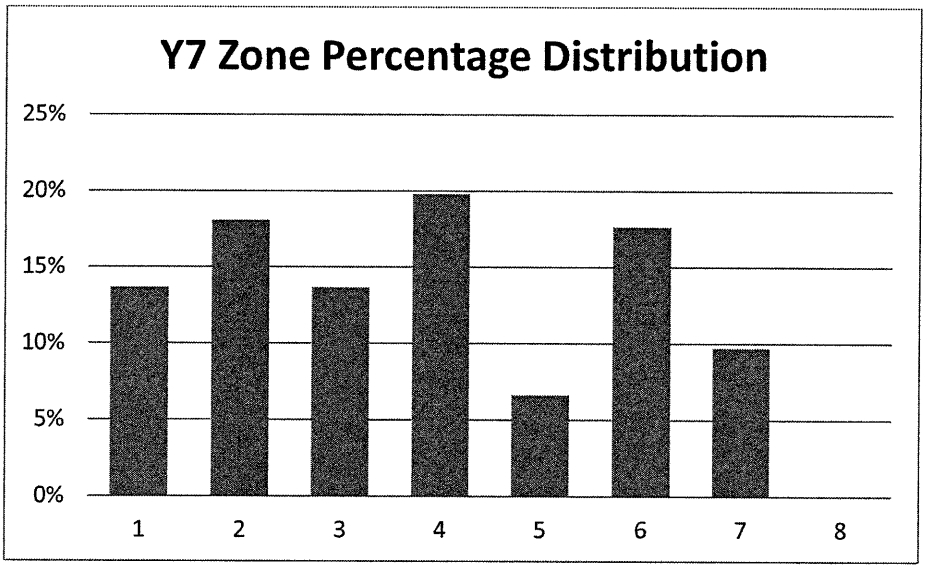

Figure 2. LAF zone distribution for Year 7 students.

These results were totally unexpected. The teachers had not predicted the depth of the lack of understanding of the students. We had to accept that the problem was bigger than we had expected or wanted to believe. The data did, however, offer an insight into the potential issues faced in our school. The challenge would be to take the information and use it to improve teaching and learning.

The Challenge and Intervention

Now we had a better understanding of the depth of the issues faced by our students, the next phase was to design an intervention to assist in specific ways for individual students. A big challenge! The development of multiplicative thinking was seen as a benchmark in predicting future success in Years 9 and 10. Kilpatrick, Swafford and Findell (2001) considered proportional reasoning as being a gateway to higher levels of mathematical success, and Dole, Clarke, Wright and Hilton (2012) saw proportional reasoning as fundamental to further success in mathematics and science. Knowing what the range of difficulties students faced offered a chance to start a targeted intervention program to assist all students to be able to access mathematics in future years.

As a team, we spent time discussing the strategies and materials we would need to approach RMF in a way that would suit our students. We had very few manipulatives and tasks to exploit the use of concrete materials. Over the years our school has spent thousands of dollars buying newer and better differentiated text materials and yet our results had never reflected that investment. It appeared that our problems were deeper than any standard text book could accommodate and we had to design learning experiences to assist in the conceptual development of our students. RMF provided a series of structured tasks for each zone level which we decided to put into use. We appreciated that teacher involvement would be huge and perhaps chaotic, especially when running several tasks in a room. We understood that the entire process would need to be supported and managed well. The decision was made to have two four-week blocks dedicated to RMF.

We began by splitting students into zone groups. Teachers made themselves aware of the purpose of the progression for each set of activities and put them into operation. All went well for the first week! It soon became obvious that we could never maintain this level of operation in our classrooms. We used funding from the RMF Project to have two teachers in each room, but although this was a better solution it was still difficult to control and support and it would be impossible to sustain this approach once the Project funding was no longer available. Running multiple groups was too demanding for the teachers so we needed to adapt. After discussing what it was we were aiming to achieve, we decided to use a targeted approach to fundamental concepts suitable for early, middle and late zones, as we called them. Teaching specific strategies to a group put the teachers back into an area in which they felt comfortable.

Additive thinkers were encouraged to group and partition, two essential skills that were missing for many students. Those acquiring multiplicative skills were encouraged to develop multiplication strategies to better understand the process of multiplication. At no stage were any algorithms taught or used, rather students were encouraged to use arrays and partial product strategies with partitions determined by the students. Those who were multiplicative thinkers were stretched into using and describing ratio and proportion problems involving rates or movements.

Professional Development and Learning Teams

One of the difficulties for secondary teachers of starting where the students are developmentally, is that they are not trained in primary mathematics. Many secondary teachers do not have the understanding of how early mathematics is conceptually developed and many do not have the language understanding to follow a primary developmental progression and understand it. Sessions were spent with the mathematics staff demonstrating the conceptual development necessary for numeracy. We drew on the teaching advice in the Learning Assessment Framework and other material available through the RMF Project and came to an agreement about how to approach specific topics, for example when teaching place value how decimal points remain static and it is the numbers that move when multiplying or dividing by ten. We have agreed on a set of multiplication strategies which has helped to reinforce strategies rather than memory skills as important in mathematics. There is still an emphasis on fluency, but with numbers other than one to ten. An important question we adopted was "Can you draw this for me to help me understand your thinking?"

As well as sharing strategies we agreed on a common approach in the classroom. Each lesson begins with five to ten questions using a particular RMF strategy, for example using a partial product when multiplying. The questions are left on the board so students may return to them as necessary. Teachers then take a particular group and make the targeted teaching point to be achieved in that session. This is repeated around other groups until every student is working on a specific task or series of tasks. Teachers circulate around the room checking on progress and questioning students to determine understanding.

Some of the teachers felt more confident than others when it came to the organisational aspects of teaching students in groups. This was a pedagogical leap for some of the teachers who had been used to teaching the entire class as a group. We have a few teachers with primary teaching experience and their experience has been beneficial in assisting to organise and structure the activity based learning in the classes. We determined that some lessons would be task based and others would be based on text book work. The use of task based mathematics offers teachers a way to engage students in reasoning activities and problem solving. Better tasks will have a low entry point allowing most students to begin the problem and have multiple exit points (Boaler, 2016; Day, 2012; Sullivan 2011) allowing teachers and students to access the useful mathematics embedded in the task. One of the problems some of the teachers faced, at least initially, was the transition from the task to the specific mathematical knowledge to be gleaned from the task. Making the mathematical point requires skill from the teacher in connecting the mathematical ideas and making the teaching point explicit. It was important for us to spend time on seeing what mathematics could be 'mined' from each task to ensure that there were opportunities for deeper learning (Boaler, 2016).

Results after Intervention

After the two four-week blocks of intervention the question was whether our efforts had been worthwhile. Hattie (2012) suggested an effect size of 0.1 equated to a natural improvement of approximately one term of learning. Our figures indicated an average effect size of 0.55 after our two four-week blocks (see Figures 3 and 4). This was encouraging, and in our view worth pursuing further.

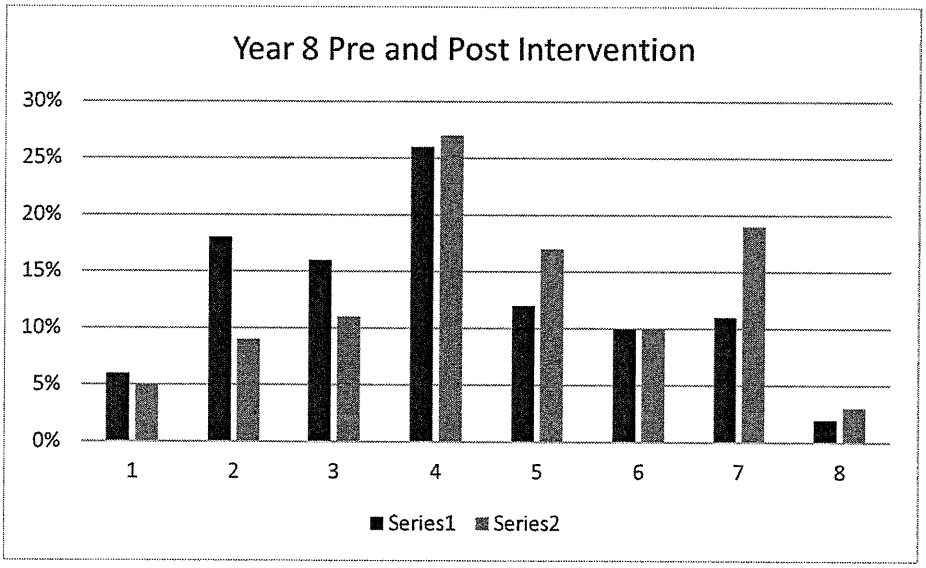

Figure 3. LAF zone distribution before and after intervention for Year 8 students.

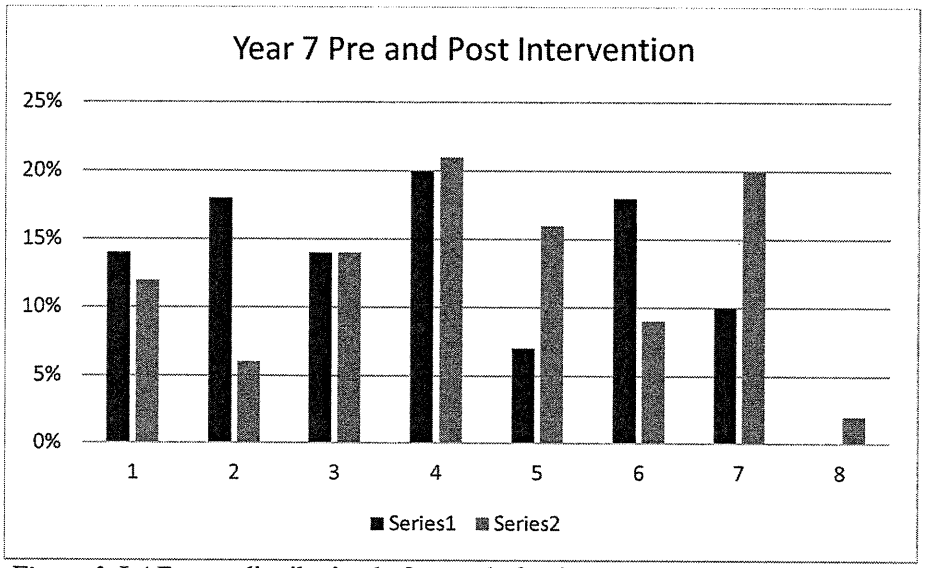

Figure 3. LAF zone distribution before and after intervention for Year 7 students

The graphs indicate that there has been a general shift to the right which is the desired outcome. Comparative percentages in the three bands of zones are provided in Table 1. Interestingly our data reinforce the national data that shows that zone 4 is the most difficult zone for students to move out of. Recent NAPLAN results for Year 9 were also encouraging, with us reaching our target of surpassing the national average for the first time in four years. The target has now been revised to surpass the state average, which is higher, in 2017.

Table 1.
Comparative Percentages Pre and Post Intervention

	Pre Intervention	Post Intervention
Zones 1 - 3	44%	28%
Zones 4 & 5	32%	40%
Zones 6 - 8	23%	31%

Student Feedback

Student surveys indicate that the students enjoy the practical, hands-on aspects of using tasks. They seem to enjoy the collaboration between groups and the challenge to solve particular problems. The students do not always appreciate why it is important to proceed past the original problem to generalise a solution, but that will hopefully come in time. Since adopting this methodology in the school we have seen better engagement with all levels of students. Students who are viewed as more able have often been surprised by the practical insights offered by those considered as less able, which has contributed to a better equality in the learning process. The students enjoy the freedom that tasks bring to their mathematics and they do not always want to practise the new skills they have learned by returning to text based exercises. From our perspective improving the students' understanding of the whole learning and doing process still requires work. Although most students prefer the task based approach, some still prefer a text based approach to learning mathematics through being taught specific algorithmic approaches. Interestingly, these students tend to sit in the lower to middle multiplicative thinking zones.

Sustainability

Over the last two years, we have scheduled our task activities into two four-week blocks per year. Both teachers and students have suggested that this should be extended into three or four blocks next year, so we are currently planning for four blocks in 2017. Although it is tempting to change to more and more task based learning, it is important that we balance this temptation with patience. We have adopted the notion that it is better to do a small amount well than take on too much. The increased engagement with students needs to be balanced against the stress on the mathematics staff, who have found it tough going but rewarding.

Initially, it was a challenge to organise and create a suite of resources suitable for such a diverse range of abilities. Having access to the RMF materials was critical in this aspect. Not all tasks that we have tried have worked as well as we had hoped, some have failed to engage students and others have had an entry point that it too high for all students to access the task. Using this knowledge and the RMFII Project resources that move into algebraic, geometrical and statistical reasoning has led to an engaging collection of activities for students and for most staff members a renewal of spirit. Several staff have indicated that this is why they wanted to be teachers of mathematics in the first place.

To be most effective the teachers have recognised that often they need to use a concrete model to assist students to understand a concept and that these times are not limited to our task based learning blocks. It is become a priority to build a bank of resources in each mathematics classroom, which can become quite expensive. We have found that along with concrete experiences students need to develop a deep understanding of the mathematics and to this end, we have found that interrogating incorrect solutions to problems has been a useful tool to help to improve student understanding.

Teachers now having an awareness of the additive and multiplicative thinking required in the content strands of the Australian Curriculum: Mathematics [ACM] (ACARA, 2016), has benefited the teaching of mathematics in our school. Teachers are working better to differentiate lessons depending on where students are on the continuum of additive to multiplicative thinking. The real litmus test though has

been the level of engagement from the students. They have shown a willingness to solve practical problems, they are more confident in using numbers including fractions, and we are beginning to see these changes reflected in results. We are hopeful that our hard working staff see the benefits of their efforts over a long period of time.

Conclusion

Our progress at Kalamunda Senior High School has been slow but marked. Our students tell us that they are better engaged and seeing mathematics as a problem solving and interesting subject. They have enjoyed the change of emphasis away from the memorization of facts into actually doing mathematics. Teacher skill in understanding the conceptual acquisition of number has improved our own professional development and often takes place over the staff room table. We had some teething problems initially, mostly in the setting up and organization of the program, but these have been overcome. The benefits in student learning, in doing some mathematics and enjoying it are much more important. Our progress is pleasing, but needs to be maintained and our results need to show a level of consistency. It is early days to predict long term success, but the signs are very encouraging.

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